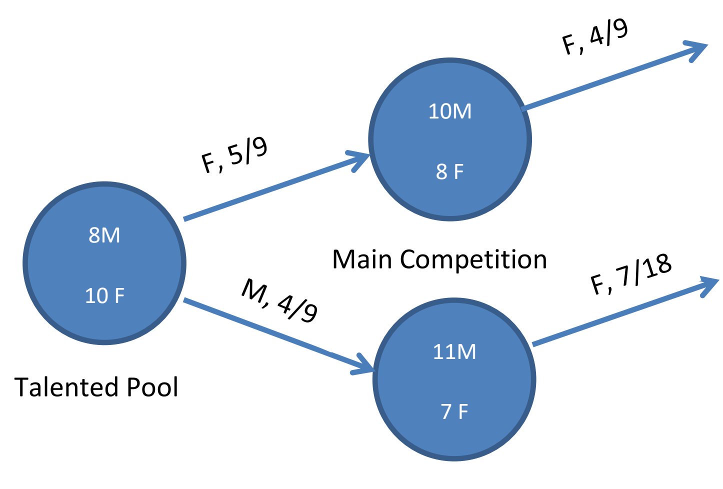
1. Following are the marks scored in statistics by students in a class (arranged according to their roll number)44, 92, 56, 64, 62, 90, 70, 72, 82, 78, 79, 80, 82, 87, 95, 96, 72, 74, 96, 98, 100, 78, 58, 72, 75, 64, 88, 86, 74, 84. Professor decided to give all such students excellent grade who got equal or more marks compared to the 90^{th} percentile. How many students of this class will get excellent grade? What is the minimum score of student who awarded excellent grade?

   Answer: Total number of students in class is 30. First we arrange the numbers in non-decreasing way.
   44 56 58 62 64 64 70 72 72 72 74 74 75 78 78 79 80 82 82 84 86 87 88 90 92 95 96 96 98 100
   0.9*(30)=27. It's a whole number. So 90^{th} percentile is average of 27^{th} and 28^{th} position. However both are 96. So 90^{th} percentile is 96. So 4 students will receive excellent grade and minimum number who awarded excellent grade is 96.

2. VP and GS are to be chosen from a student club consisting of 50 students. How many different choices of these posts are possible if Aryaa (who is very popular among students) says he might serve only if Rohit, Mukul and Divya stay out. Find out total ways by which these two positions can be filled.

   Scenario 1: Arya will serve one of the two positions when Rohit, Mukul and Divya stay out
   Aryaa will serve one of the two positions. So there are 2 choices. Another position can be filled by anyone from rest of 46 students. So there are 2*46 = 92 ways
   Scenario 2: Arya will not serve- ${}^{49}P_2$=49*48=2352
   So there are 92+2352= 2444 ways

3. A reality show on singing started with 10 male participants and 7 women participants. In that time show organizers were also conducting fresh audition for their next season. However, organizer found 8 male participants and 10 women participants from these audition who were very talented and decided to make **one** wild card entry in current competition from these talented pool of new audition. They also decided to choose this lucky one for wild card entry from talented pool in purely random way so that all participants from talented pool get similar chance for getting wild card entry. Now if we assume all who are competing for main competition have similar chance to win the first position, find out the probability that a female candidate will be winning this competition. [Note: Your final answer should be in fractional form like x/y]= 5/9*4/9+4/9*7/18=34/81



4. Calculate reliability of the system in Figure 1 if each component is operable with probability 0.92 independently of the other components. It works like a circuit, for sequential components, all components must be functional to get output through that line (for example A, B must be functional at the same time to get output through top most channel). However, for parallel component any one component needs to be functional to get output (for example out of D and E one must be functional). [5]

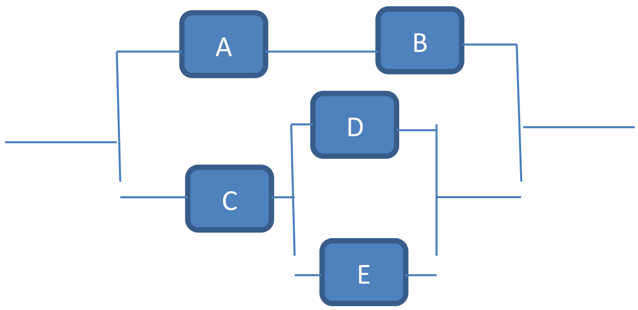


Figure 1

1. The upper link A-B works if both A and B work, which has probability

$$P\{A \cap B\} = (0.92)^2 = 0.8464.$$

We can represent this link as one component F that operates with probability 0.8464.

2. By the same token, components D and E, connected in parallel, can be replaced by component G, operable with probability

$$P\{D \cup E\} = 1 - (1 - 0.92)^2 = 0.9936,$$

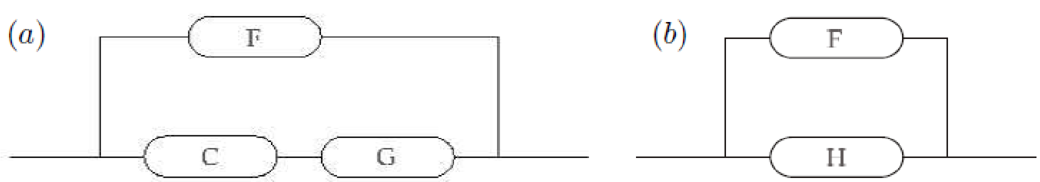


FIGURE 2.4: *Step by step solution of a system reliability problem.*

3. Components C and G, connected sequentially, can be replaced by component H, operable with probability $P\{C \cap G\} = 0.92 \cdot 0.9936 = 0.9141$, as shown in Figure 2.4b.

4. Last step. The system operates with probability

$$P\{F \cup H\} = 1 - (1 - 0.8464)(1 - 0.9141) = \underline{0.9868},$$

which is the final answer.

5. A program consists of two modules (independent modules). Random variable X1 represents the number of errors in module 1 while random variable X2 represents number of errors in module 2. Probability mass function for X1 and X2 are given in below table. Find the pmf of Y = X1 + X2, the total number of errors in both modules. [3]

x	P1(x)	P2(x)
0	0.5	0.7
1	0.3	0.2
2	0.1	0.1
3	0.1	0

Solution: We break the problem into steps. First, determine all possible values of Y, then compute the probability of each value. Clearly, the number of errors Y is an integer that can be as low as 0 + 0 = 0 and as high as 3 + 2 = 5. Since P2(3) = 0, the second module has at most 2 errors. Next,

$P_Y(0)$ = P {Y = 0} = P {X1 = X2 = 0} = P1(0)P2(0)= (0.5)(0.7) = 0.35

P_Y (1) = P {Y = 1} = P1(0)P2(1) + P1(1)P2(0)= (0.5)(0.2) + (0.3)(0.7) = 0.31

P_Y (2) = P {Y = 2} = P1(0)P2(2) + P1(1)P2(1) + P1(2)P2(0)= (0.5)(0.1) + (0.3)(0.2) + (0.1)(0.7) = 0.18

P_Y (3) = P {Y = 3} = P1(1)P2(2) + P1(2)P2(1) + P1(3)P2(0)= (0.3)(0.1) + (0.1)(0.2) + (0.1)(0.7) = 0.12

P_Y (4) = P {Y = 4} = P1(2)P2(2) + P1(3)P2(1)= (0.1)(0.1) + (0.1)(0.2) = 0.03

P_Y (5) = P {Y = 5} = P1(3)P2(2) = (0.1)(0.1) = 0.01

6. An Internet service provider launches a new plan. To promote that plan, it declares randomly selected 20% of subscriber of this plan will get 50% discount for 1st month. A group of 10 friends subscribed that plan. Find out the probability that at least 4 friends out of them will get 50% discount for the first month. [3]

Solution: We need to find the probability P {X ≥ 4}, where X is the number of people, out of 10, who receive a special promotion. This is the number of successes in 10 Bernoulli trials, therefore, X has Binomial distribution with parameters n = 10 and p = 0.2.

P {X ≥ 4} = 1 − F(3) = 1 − 0.8791 = 0.1209.

7. Assume that average age of faculty members of IIT Patna is 40 and standard deviation is 10 years. Let's say people aged 35-45 is called middle aged. Find out the percentage of middle aged faculty members assuming age distribution of faculty members follow a normal distribution.

$P\{35<X<45\}= P\{((35-\mu)/\sigma) < ((X- \mu)/\sigma) < \{((45-\mu)/\sigma)\}$

= P{((35-40)/10) < Z < {((45-40)/10)}

=P{-0.5<Z<0.5}

=0.69146 - 0.30854 = 0.38292

38.292% of faculty are middle aged.

8. Assume that rainfall of Assam across year follows a normal distribution. Also assume that standard deviation in rainfall is 0.0015 inch. A random sample of 75 days taken and it is found that average rainfall of the sample is 0.310 inch. Find 95% confidence interval for mean rainfall. [3]

$n = 75, \bar{x} = 0.310, \sigma = 0.0015$, and $z_{0.025} = 1.96$. A 95% confidence interval for the population mean is

$$0.310 - (1.96)(0.0015/\sqrt{75}) < \mu < 0.310 + (1.96)(0.0015/\sqrt{75}),$$

or $0.3097 < \mu < 0.3103$.

9. We are given that a random person will have cancer with 0.008 probabilities. We have data from cancer testing center. Test results are not very accurate. It is given that person tested positive is actually having cancer with 0.98 probabilities and a person tested negative does not have cancer with 0.97 probabilities. Given a person is tested positive what is the probability that he is actually having cancer.

P(cancer)=0.008

P(^cancer)=0.992

P(+|cancer) = 0.98

P(-|cancer)=0.02

P(+|^cancer)=0.03

P(-|^cancer)=0.97

P(Cancer|+) = P(+|cancer)*P(cancer)/(

P(+|cancer)*P(cancer)+P(+|^cancer)*p(^cancer))=0.0078/(0.0078+0.0298)=0.21

10. The Edison Electric Institute has published figures on the number of kilowatt hours used annually by various home appliances. It is claimed that a vacuum cleaner uses an average of 46 kilowatt hours per year. If a random sample of 12 homes included in a planned study indicates that vacuum cleaners use an average of 42 kilowatt hours per year with a standard deviation of 11.9 kilowatt hours, does this suggest at the 0.05 level of significance that vacuum cleaners use, on average, less than 46 kilowatt hours annually? Assume the population of kilowatt hours to be normal. [4]

1. H_0: $\mu = 46$ kilowatt hours.
2. H_1: $\mu < 46$ kilowatt hours.
3. $\alpha = 0.05$.
4. Critical region: $t < -1.796$, where $t = \frac{\bar{x}-\mu_0}{s/\sqrt{n}}$ with 11 degrees of freedom.
5. Computations: $\bar{x} = 42$ kilowatt hours, $s = 11.9$ kilowatt hours, and $n = 12$. Hence,
$$t = \frac{42 - 46}{11.9/\sqrt{12}} = -1.16, \qquad P = P(T < -1.16) \approx 0.135.$$
6. Decision: Do not reject H_0 and conclude that the average number of kilowatt hours used annually by home vacuum cleaners is not significantly less than 46.