Economic Load Dispatch

- The idea is to minimize the cost of electricity generation without sacrificing quality and reliability.
- Therefore, the production cost is minimized by operating plants economically.
- Since the load demand varies, the power generation must vary accordingly to maintain the power balance.
- The turbine-governor must be controlled such that the demand is met economically.
- *This arises when there are multiple choices*.

Economic Distribution of Loads between the units in a Plant:

- To determine the economic distribution of load between various generating units, the variable operating costs of the units must be expressed in terms of the power output.
- Fuel cost is the principle factor in thermal and nuclear power plants. It must be expressed in terms of the power output.
- Operation and Maintenance costs can also be expressed in terms of the power output.
- Fixed costs, such as the capital cost, depreciation etc., are not included in the fuel cost.

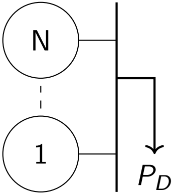
Let us define the input cost of an unit i ,F_i in Rs./h and the power output of the unit as P_i . Then the input cost can be expressed in terms of the power output as

$$F_i = a_i P_i^2 + b_i P_i + c_i \text{ Rs/h}$$

Where a_i, b_i and c_i are fuel cost coefficients.
The incremental operating cost of each unit is

$$\lambda_i = \frac{dF_i}{dP_i} = 2a_i P_i + b_i \text{ Rs./MWh}$$

Let us assume that there ar N units in a plant.



The total fuel cost is

$$F_T = F_1 + F_2 + \cdots + F_N = \sum_{i=1}^{N} F_i \text{ Rs./h}$$

All the units have to supply a load demand of P_D MW.

$$P_1 + P_2 + \cdots + P_N = P_D$$

$$\sum_{i=1}^{N} P_i = P_D$$

$$\min F_T = \sum_{i=1}^{N} F_i$$

Subject to

$$\sum_{i=1}^{N} P_i = P_D$$

It is a constrained optimization problem. Let us form the Lagrangian function.

$$L = F_T + \lambda(P_D - \sum_{i=1}^{N} P_i)$$

To find the optimum,

$$\frac{\partial L}{\partial P_i} = 0 \quad i = 1, 2, \cdots, N$$

$$\frac{\partial L}{\partial \lambda} = 0$$

$$\frac{dF_i}{dP_i} = \lambda \quad i = 1, 2, \cdots, N$$

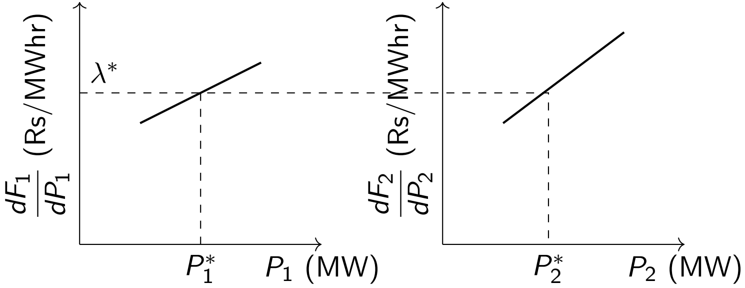
$$\sum_{i=1}^{N} P_i = P_D$$

$N+1$ linear equations need to be solved for $N+1$ variables.

For economical division of load between units within a plant, the criterion is that *all units must operate at the same incremental fuel cost*.

$$\boxed{\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \cdots = \frac{dF_n}{dP_n} = \lambda}$$

This is called the coordination equation.



Example : Consider two units of a plant that have fuel costs of

$$F_1 = 0.2P_1^2 + 40P_1 + 120 \text{ Rs./h}$$
$$F_2 = 0.25P_2^2 + 30P_2 + 150 \text{ Rs./h}$$

1. Determine the economic operating schedule and the corresponding cost of generation for the demand of 180 MW.
2. If the load is equally shared by both the units, determine the savings obtained by loading the units optimally.

1. For economical dispatch,

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

$$0.4P_1 + 40 = 0.5P_2 + 30$$

and

$$P_1 + P_2 = 180$$

On solving the above two equations,

$$P_1 = 88.89 \text{ MW}; \quad P_2 = 91.11 \text{ MW}$$

The cost of generation is

$$F_T = F_1 + F_2 = 10,214.43 \text{ Rs./h}$$

2. If the load is shared equally,

$$P_1 = 90 \text{ MW}; \quad P_2 = 90 \text{ MW}$$

The cost of generation is

$$F_T = 10,215 \text{ Rs./h}$$

Therefore, the saving will be 0.57 Rs./h

Generator Limits:

The power generation limit of each unit is given by the inequality constraints

$$P_{i,min} \leq P_i \leq P_{i,max} \quad i = 1, \cdots, N$$

- The maximum limit P_{max} is the upper limit of power generation capacity of each unit.
- Whereas, the lower limit P_{min} pertains to the thermal consideration of operating a boiler in a thermal or nuclear generating station.

How to consider the limits

- If any one of the optimal values violates its limits, fix the generation of that unit to the violated value.
- Optimally dispatch the *reduced load* among the remaining generators.

Example: The fuel cost functions for three thermal plants are

$$F_1 = 0.4P_1^2 + 10P_1 + 25 \text{ Rs./h}$$
$$F_2 = 0.35P_2^2 + 5P_2 + 20 \text{ Rs./h}$$
$$F_3 = 0.475P_3^2 + 15P_3 + 35 \text{ Rs./h}$$

The generation limits of the units are

$$30 \text{ MW} \le P_1 \le 500 \text{ MW}$$
$$30 \text{ MW} \le P_2 \le 500 \text{ MW}$$
$$30 \text{ MW} \le P_3 \le 250 \text{ MW}$$

Find the optimum schedule for the load of 1000 MW.

For optimum dispatch,

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \frac{dF_3}{dP_3}$$

$$0.8P_1 + 10 = 0.7P_2 + 5$$
$$0.7P_2 + 5 = 0.9P_3 + 15$$

and

$$P_1 + P_2 + P_3 = 1000$$

On solving the above three equations,

$$P_1 = 334.3829 \text{ MW}; \quad P_2 = 389.2947 \text{ MW}; \quad P_3 = 276.3224 \text{ MW}$$

Since the unit 3 violates its maximum limit,

$$P_3 = 250 \text{ MW}$$

The remaining load (750 MW) is scheduled optimally among 1 and 2 units.

$$0.8P_1 + 10 = 0.7P_2 + 5$$
$$P_1 + P_2 = 750$$

On solving the above equations,

$$P_1 = 346.6667 \text{ MW}; \quad P_2 = 403.3333 \text{ MW}$$

Therefore, the final load distribution is

$$P_1 = 346.6667 \text{ MW}; \quad P_2 = 403.3333 \text{ MW}; \quad P_3 = 250 \text{ MW}$$

Economic Distribution of Loads between different Plants:

- If the plants are spread out geographically, line losses must be considered.
- The line losses are expressed as a function of generator outputs.

$$\min F_T = \sum_{i=1}^{N} F_i$$

Subject to

$$\sum_{i=1}^{N} P_i = P_L + P_D$$

where $P_L = f(P_i)$. It is a nonlinear function of P_i. Let us form the Lagrangian function.

$$L = F_T + \lambda(P_D + P_L - \sum_{i=1}^{N} P_i)$$

To find the optimum,

$$\frac{\partial L}{\partial P_i} = 0 \quad i = 1, 2, \cdots, N$$

$$\frac{\partial L}{\partial \lambda} = 0$$

$$\frac{dF_i}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} = \lambda \quad i = 1, 2, \cdots, N$$

$$\frac{dF_i}{dP_i} = \lambda \left(1 - \frac{\partial P_L}{\partial P_i}\right)$$

$$\frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \frac{dF_i}{dP_i} = \lambda$$

Let us define the penalty factor L_i for i^{th} generator.

$$L_i \frac{dF_i}{dP_i} = \lambda$$

where $L_i = \dfrac{1}{1 - \dfrac{\partial P_L}{\partial P_i}}$.

For economical division of load between plants, the criterion is

$$\boxed{L_1 \frac{dF_1}{dP_1} = L_2 \frac{dF_2}{dP_2} = \cdots = L_n \frac{dF_n}{dP_n} = \lambda}$$

This is called the exact coordination equation.

Since P_L is a nonlinear function of P_i, the following $N+1$ equations need to be solved numerically for $N+1$ variables.

$$L_i \frac{dF_i}{dP_i} = \lambda \quad i = 1, 2, \cdots, N$$

$$\sum_{i=1}^{N} P_i = P_L + P_D$$

The transmission losses are usually expressed as

$$P_L = \mathrm{P}^\mathrm{T}\mathrm{BP}$$

where $P = [P_1, P_2, \cdots P_n]$ and B is a symmetric matrix given by

$$B = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1n} \\ B_{21} & B_{22} & \cdots & B_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n1} & B_{n2} & \cdots & B_{nn} \end{bmatrix}$$

The elements of the matrix B are called the **loss coefficients**.

Example: Consider a two bus system.

The incremental fuel cost characteristics of plant 1 and plant 2 are given by

$$\frac{dF_1}{dP_1} = 0.025P_1 + 14 \text{ Rs/MWHr}$$
$$\frac{dF_2}{dP_2} = 0.05P_2 + 16 \text{ Rs/MWHr}$$

If 200 MW of power is transmitted from plant 1 to the load, a transmission loss of 20 MW will be incurred. Find the optimum generation schedule and the cost of received power for a load demand of 204.41 MW.

$$P_L = \begin{bmatrix} P_1 & P_2 \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

Since the load is at bus 2, P_2 will not have any effect on P_L.

$$B_{12} = B_{21} = 0;\ B_{22} = 0$$

Therefore,

$$P_L = B_{11} P_1^2$$

For 200 MW of P_1, $P_L = 20$ MW.

$$20 = B_{11} 200^2$$

$$B_{11} = 0.0005 \text{ MW}^{-1}$$

$$P_L = 0.0005 P_1^2$$

For optimum dispatch,

$$L_1 \frac{dF_1}{dP_1} = L_2 \frac{dF_2}{dP_2} = \lambda$$

Since P_L is a function of P_1 alone,

$$L_1 = \frac{1}{1 - \dfrac{\partial P_L}{\partial P_1}} = \frac{1}{1 - 0.001P_1}$$

$$L_2 = 1$$

$$\left(\frac{1}{1 - 0.001P_1}\right) 0.025P_1 + 14 = 0.05P_2 + 16$$

On simplification,

$$0.041P_1 - 0.05P_2 + 0.00005P_1P_2 = 2$$

and

$$P_1 + P_2 - 0.0005P_1^2 = 204.41$$

$$f_1(P_1, P_2) = 2$$
$$f_2(P_1, P_2) = 204.41$$

Let us solve them by N-R method.

$$\Delta \mathrm{f} = \mathrm{J} \Delta \mathrm{P}$$

where

$$\Delta \mathrm{f} = \begin{bmatrix} 2 - f_1(P_1, P_2) \\ 204.41 - f_2(P_1, P_2) \end{bmatrix}$$

$$\mathrm{J} = \begin{bmatrix} \frac{\partial f_1}{\partial P_1} & \frac{\partial f_1}{\partial P_2} \\ \frac{\partial f_2}{\partial P_1} & \frac{\partial f_2}{\partial P_2} \end{bmatrix}$$

To find the initial estimate : Let us solve the problem with out loss.

$$0.025P_1 + 14 = 0.05P_2 + 16$$

$$P_1 + P_2 = 204.41$$

$$P_1^0 = 162.94;\ P_2^0 = 41.47$$

First Iteration :

$$\Delta \mathrm{f}^0 = \begin{bmatrix} 2 - f_1(P_1^0, P_2^0) \\ 204.41 - f_2(P_1^0, P_2^0) \end{bmatrix} = \begin{bmatrix} -2.9449 \\ 13.2747 \end{bmatrix}$$

$$\mathrm{J}^0 = \begin{bmatrix} 0.0431 & -0.0419 \\ 0.8371 & 0.9585 \end{bmatrix}$$

$$\begin{bmatrix} \Delta P_1^0 \\ \Delta P_2^0 \end{bmatrix} = \begin{bmatrix} -29.7060 \\ 39.7906 \end{bmatrix}$$

$$\begin{bmatrix} P_1^1 \\ P_2^1 \end{bmatrix} = \begin{bmatrix} P_1^0 \\ P_2^0 \end{bmatrix} + \begin{bmatrix} \Delta P_1^0 \\ \Delta P_2^0 \end{bmatrix} = \begin{bmatrix} 133.2340 \\ 81.2606 \end{bmatrix}$$

It took 6 iterations to converge.

$$P_1 = 133.3153 \text{ MW} \quad P_2 = 79.9812 \text{ MW}$$

The cost of received power is

$$\lambda = L_2 \frac{dF_2}{dP_2} = 1 \times (0.05 \times 79.9812 + 16) = 19.9991 \text{ Rs./MWh}$$

λ-iteration Method

$$P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i B_{ij} P_j$$

The exact coordination equation is

$$\frac{dF_i}{dP_i} + \lambda \frac{\partial P_L}{\partial P_i} = \lambda$$

It can be written as

$$2a_i P_i + b_i + 2\lambda \sum_{j=1}^{N} B_{ij} P_j = \lambda$$

$$2a_i P_i + b_i + 2\lambda B_{ii} P_i + 2\lambda \sum_{\substack{j=1 \\ j \neq i}}^{N} B_{ij} P_j = \lambda$$

$$P_i = \frac{\lambda - b_i - 2\lambda \sum_{\substack{j=1\\ j\neq i}}^{N} B_{ij} P_j}{2(a_i + \lambda B_{ii})}$$

On substituting this in the power balance equation,

$$\sum_{i=1}^{N} P_i = P_D + P_L$$

$$\sum_{i=1}^{N} \frac{\lambda - b_i - 2\lambda \sum_{\substack{j=1\\ j\neq i}}^{N} B_{ij} P_j}{2(a_i + \lambda B_{ii})} = P_D + P_L$$

$$f(\lambda) = P_D + P_L$$

This needs to be solved repeatedly for different values of λ.

Expanding it using Taylor's series about an initial point (λ^0) and neglecting the higher order terms.

$$f(\lambda^0) + (\frac{df(\lambda)}{d\lambda})^0 \Delta\lambda^0 \approx P_D + P_L^0$$

$$\Delta\lambda^0 = \frac{P_D + P_L^0 - f(\lambda^0)}{(\frac{df(\lambda)}{d\lambda})^0}$$

where

$$f(\lambda^0) = \sum_{i=1}^{N} P_i^0$$

$$(\frac{df(\lambda)}{d\lambda})^0 = \sum_{i=1}^{N} (\frac{dP_i}{d\lambda})^0 = \sum_{i=1}^{N} \left(\frac{a_i + b_i B_{ii} - 2a_i \sum_{\substack{j=1 \\ j \neq i}}^{N} B_{ij} P_j^0}{2(a_i + \lambda^0 B_{ii})^2} \right)$$

Therefore,

$$\lambda^1 = \lambda^0 + \Delta\lambda^0$$

In general,

$$\lambda^{k+1} = \lambda^k + \Delta\lambda^k$$

where

$$\Delta\lambda^k = \frac{P_D + P_L^k - \sum_{i=1}^{N} P_i^k}{\sum_{i=1}^{N}\left(\frac{a_i + b_i B_{ii} - 2a_i \sum_{\substack{j=1\\ j\neq i}}^{N} B_{ij} P_j^k}{2(a_i + \lambda^k B_{ii})^2}\right)}$$

- Start with λ^k.
- Find P_i^k as follows:

$$P_i^k = \frac{\lambda^k - b_i - 2\lambda^k \sum_{\substack{j=1\\ j\neq i}}^{N} B_{ij} P_j^k}{2(a_i + \lambda^k B_{ii})}$$

- Find P_L^k using the following equation.

$$P_L^k = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i^k B_{ij} P_j^k$$

- Repeat the above steps till $|P_D + P_L^k - \sum_{i=1}^{N} P_i^k| \leq \epsilon$.
- To start with, assume λ^0 such that it is greater than the largest value of the coefficients b.

Example : Let us take the same example. The incremental fuel cost characteristics of plant 1 and plant 2 are given by

$$\frac{dF_1}{dP_1} = 0.025P_1 + 14 \text{ Rs/MWHr}$$
$$\frac{dF_2}{dP_2} = 0.05P_2 + 16 \text{ Rs/MWHr}$$

$$P_L = 0.0005P_1^2$$

1. Assume $\lambda^0 = 17$.
2. P_1^0 and P_2^0 are

$$P_1^0 = \frac{\lambda^0 - b_1}{2(a_1 + \lambda^0 B_{11})} = 71.4286 \text{ MW}$$

$$P_2^0 = \frac{\lambda^0 - b_2}{2(a_2)} = 20 \text{ MW}$$

3. It took 8 iterations to converge.

$$\lambda = 19.9991 \text{ Rs/MWhr } P_1 = 133.3152 \text{ MW } P_2 = 79.9812 \text{ MW}$$