Indian Institute of Technology Patna
Department of Electrical Engineering
EE381 - Power Systems
Autumn - 2023
Quiz - I — Solution.
September 20, 2023

There are 5 problems. They carry equal marks.

$(5 \times 2 = 10)$

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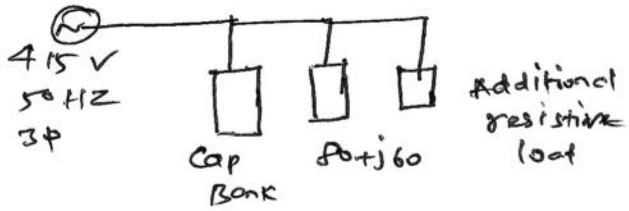
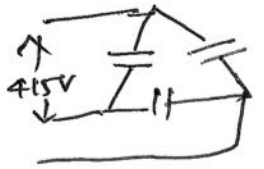
1. A three-phase cable is supplying 80 kW and 60 kVAr to an inductive load. It is intended to supply an additional resistive load through the same cable without increasing the heat dissipation in the cable, by providing a three-phase bank of capacitors connected in delta across the load. Given the line voltage is 415 V, 50 Hz, and the capacitance per phase of the bank is 200μF, find the additional resistive load in kW which can be supplied.



To have the same heat dissipation, the apparent power (kVA) of the total loads must be the same.



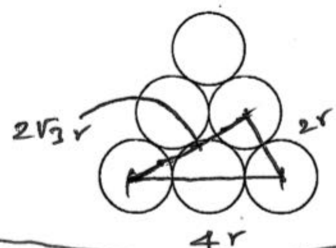
Q_c (3P) $= 3 \times 415^2 \times \omega \times 200 \times 10^{-6}$
$= 32.5$ kVAr

$S = 100$ kVA

$S^2 = (80 + P_{add})^2 + (60 - 32.5)^2$

$\boxed{\therefore P_{additional} = 16.13 \text{ kW}}$

2. Determine the geometric mean radius (GMR) of the following configurations for inductance in terms of the radius r of the individual strand.



$$GMR_L = \sqrt[36]{(0.7788\,r \times 2r \times 4r \times 2\sqrt{3}r \times 2r \times 4r)^3 \times (0.7788\,r \times 2r \times 2r \times 2r \times 2r \times 2\sqrt{3}r)^3}$$

$$= (0.7788)^{1/6} \times r \times 2 \times (3)^{1/12}$$

$$\boxed{GMR_L = 2.1024\, r}$$

3. For a 400 km long transmission line, the series impedance is $(0 + j0.5)\ \Omega/\text{km}$ and the shunt admittance is $(0 + j5)\ \mu℧/\text{km}$. The magnitude of the series impedance (in Ω) of the equivalent-π circuit of the transmission line is

$Z' = Z_c \sinh \gamma l$ (Equivalent π-model)

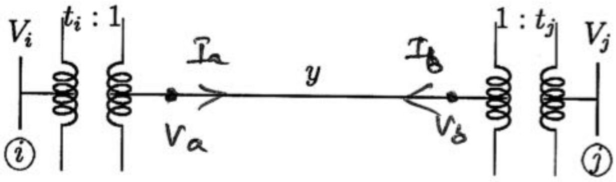
$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{j0.5}{j5\times10^{-6}}} = 316.22\,\Omega$

$\gamma = \sqrt{zy} = \sqrt{j0.5\times5\times10^{-6}} = j0.0016$ rad/km

$Z' = 316.23 \times \sinh(j0.0016\times400)$

$Z' = j186.93\,\Omega$ $\boxed{|Z| \approx 187\,\Omega}$

4. Two buses, i and j are connected with a transmission line of admittance Y, at the two ends of which there are ideal transformers with turns ratios as shown below.



Find the bus admittance matrix.

$V_a = \frac{V_i}{t_i}$; $V_b = \frac{V_j}{t_j}$; $I_a = I_i t_i$; $I_b = I_j t_j$

$I_a = (V_a - V_b)y = \frac{V_i}{t_i}y - \frac{V_j}{t_j}y$

$I_b = (V_b - V_a)y = \frac{V_j}{t_j}y - \frac{V_i}{t_i}y$

$I_i = \frac{I_a}{t_i} = \frac{V_i}{t_i^2}y - \frac{V_j}{t_i t_j}y$ —(1)

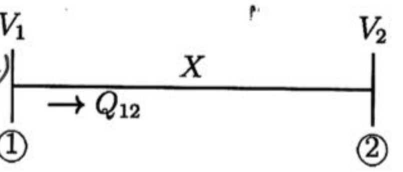
$I_j = \frac{I_b}{t_j} = \frac{-V_i y}{t_i t_j} + \frac{V_j}{t_j^2}y$ —(2)

From (1) and (2),

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{y}{t_i^2} & \frac{-y}{t_i t_j} \\ \frac{-y}{t_i t_j} & \frac{y}{t_j^2} \end{bmatrix}}_{Y_{Bus}} \begin{bmatrix} V_i \\ V_j \end{bmatrix}$$

5. Bus 1 with voltage magnitude $V_1 = 1$ p.u. is sending reactive power Q_{12} to bus 2 with voltage magnitude $V_2 = 0.9$ p.u. through a lossless transmission line of reactance X. Keeping V_2 at 0.9 p.u., V_1 is changed so that Q_{12} is increased by 10 %. Real power flow under both the conditions is zero. What is the new V_1 in p.u.?

$Q_{12} = \frac{V_1^2}{X} - \frac{V_1V_2\cos(\delta_1-\delta_2)}{X}$



Since $P_{12} = 0$, $\delta_1 - \delta_2 = 0$

$\therefore Q_{12} = \frac{1\times0.1}{X} = \frac{0.1}{X}$

$Q_{12\,new} = 1.1\times Q_{12}(old)$

$Q_{12\,new} = \frac{1.1\times0.1}{X}$

$\frac{1.1\times0.1}{X} = \frac{V_1^2}{X} - \frac{V_1\times0.9}{X}$

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Solving this $V_1 = 1.01, -0.11$, Picking the feasible solution

$\boxed{V_{1\,new} = 1.01 \text{ p.u.}}$