Indian Institute of Technology Patna
Department of Electrical Engineering
EE381 - Power Systems
Autumn - 2023
Mid Semester Exam — Solution.
September 30, 2023

There are 5 questions. They carry equal marks.

$(5 \times 6 = 30)$

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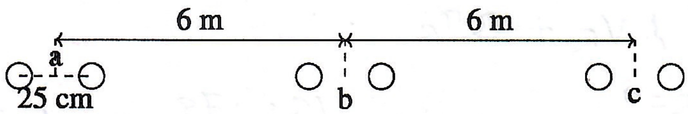
1. Determine the total inductance and capacitance of the single circuit, two-bundle and 200 km long line as shown below. The diameter of each conductor is 5 cm. ($\epsilon_0 = 8.854 \times 10^{-12} F/m$)



$GMD = \sqrt[3]{D_{ab} D_{bc} D_{ca}} = \sqrt[3]{6 \times 6 \times 12} = 7.56\ m$

$GMR_L = \sqrt[2]{r' \times d} = \sqrt[2]{0.7788 \times \frac{5}{2} \times 10^{-2} \times 25 \times 10^{-2}}$

$GMR_L = 0.07\ m$

$GMR_C = \sqrt[2]{r \times d} = \sqrt[2]{5/2 \times 10^{-2} \times 25 \times 10^{-2}} = 0.08\ m$

(2)

$L = 2 \times 10^{-7} \ln \left(\frac{GMD}{GMR_L}\right) = 9.4 \times 10^{-7}\ H/m$

$\underline{L = 0.94\ mH/km}$

(2)

$C = \frac{2\pi\epsilon_0}{\ln\left(\frac{GMD}{GMR_C}\right)} = 1.22 \times 10^{-11}\ F/m$

$\underline{C = 0.0122\ \mu F/km}$

Total Inductance

$L = L \times \text{length} = 0.94 \times 200\ mH$

$\approx 188\ mH = 0.188\ H$

$\boxed{L \approx 0.19\ H}$

(2)

$C = c \times 200 = \boxed{2.44\ \mu F}$

2. A 250 km long, three phase, 50 Hz transmission line has the following constants.

$$A = D = 0.9\angle 1^\circ$$

$$B = 120\angle 72^\circ\ \Omega$$

$$C = 0.001\angle 90^\circ\ ℧$$

The sending end voltage is 230 kV. Find

(a) the receiving end voltage and the line charging current when there is no load.

(b) the sending end voltage angle at which the maximum real power can be transferred. (Hint: Real power flow is a function of voltage angle. Take V_R as reference.)

a) $V_S/\text{phase} = \dfrac{230}{\sqrt{3}} = 132.79\ \text{KV}$

$V_S = AV_R + BI_R$

Since $I_R = 0$

$V_R = \dfrac{V_S}{A} = \dfrac{132.79}{0.9\angle 1^\circ} = 147.54\angle 1^\circ\ \text{KV/phase}$

$\boxed{V_R = 255.56\ \text{KV (Line)}}$ (3)

$I_S = CV_R = \boxed{147.55\angle 89^\circ\ \text{A}}$

b) Let $V_S = |V_S|\angle\delta$, $V_R = |V_R|\angle 0^\circ$, $A = |A|\angle a$,

$B = |B|\angle b$

$V_S = AV_R + BI_R$

From this eqn, $I_R = \dfrac{V_S - AV_R}{B}$

$S_R = \{V_R I_R^*\}$ $\quad P_R = \text{Real}\{S_R\} = \text{Real}\{V_R I_R^*\}$

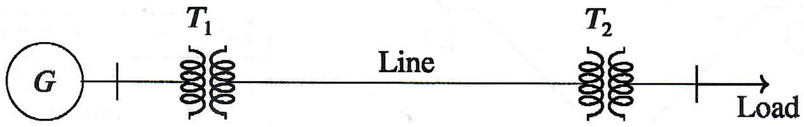
$P_R = \text{Real}\left\{V_R\angle 0 \left(\dfrac{V_S\angle\delta - A\angle a\ V_R\angle 0}{B\angle b}\right)^*\right\}$ (2)

$P_R = \dfrac{V_R V_S}{B}\cos(b-\delta) - \dfrac{V_R^2 A}{B}\cos(b-a)$

$\dfrac{dP_R}{d\delta} = 0$; $\boxed{b = \delta}$ (1) Hence $\boxed{\delta = 72^\circ}$

for maximum power transfer.

3. The schematic diagram of a radial transmission system is shown below.



T_1: 100 MVA, 11/220 kV. X = 12 %
T_2: 100 MVA, 220/66 kV. X = 10 %
Line: $j100\ \Omega$.
Load: 60 MW at 0.9 pf lagging .
If the load voltage is to be maintained at 60 kV, what will be the terminal voltage in kV of the generator?

Let us choose Base MVA = 100 MVA
Base kV = 220 kV (on Transmission line)

Base kV on Load = $220 \times \frac{66}{220} = 66$ kV
Base kV on G = $220 \times \frac{11}{220} = 11$ kV

$X_{T1} = j0.12 \times \left(\frac{100}{100}\right) \times \left(\frac{220}{220}\right)^2 = j0.12$

$X_{T2} = j0.1 \times \left(\frac{100}{100}\right) \times \left(\frac{66}{66}\right)^2 = j0.10$

OR $\left(j0.1 \times \left(\frac{100}{100}\right) \times \left(\frac{220}{220}\right)^2 = j0.10\right)$

$X_{Line} = j100 \times \frac{100}{220^2}$

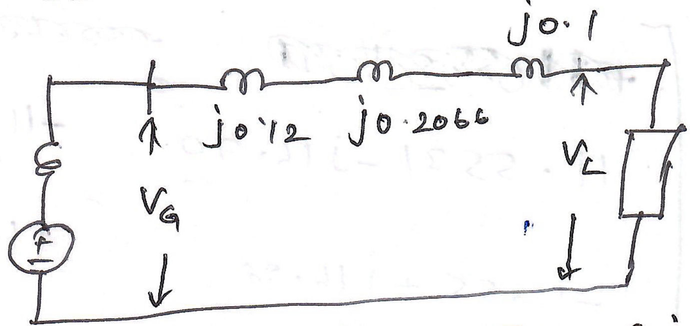
$X_{Line} = j0.2066$ p.u

3

$P_L = \frac{60}{100} = 0.6$ p.u , $V_L = \frac{60}{66} = 0.91$ p.u

$I_L = \frac{0.6}{0.91 \times 0.9} = 0.7326$ p.u

$\bar{I}_L = 0.7326 \angle -\cos^{-1}(0.9)$ p.u



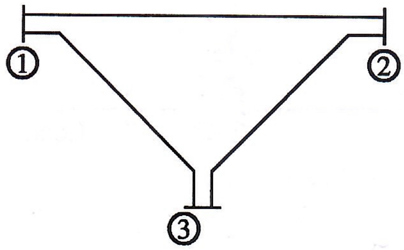
$\bar{V}_G = \bar{V}_L + \bar{I}_L \times (j0.12 + j0.2066 + j0.1)$ 1

$\bar{V}_G = 1.0834 \angle 15.04^\circ$ p.u

2

$V_G = 1.0834 \times 11 = \boxed{11.92\ \text{kV}}$

4. Consider the three node network.



Line No	Z in p.u	$\frac{Y}{2}$ in p.u
1-2	$0.02 + j0.06$	$j0.03$
1-3	$0.08 + j0.024$	$j0.025$
2-3	$0.06 + j0.018$	$j0.02$

(a) Form the $\mathbf{Y}_{bus}$ matrix.

(b) Modify the the $\mathbf{Y}_{bus}$ matrix by eliminating node 3.

a) $Y_{11} = \dfrac{1}{0.02 + j0.06} + \dfrac{1}{0.08 + j0.024} + j0.03 + j0.025 = 16.47 - j18.39$

$Y_{12} = -\dfrac{1}{0.02 + j0.06} = -5 + j15$; $Y_{13} = -11.47 + j3.44$

$Y_{21} = Y_{12}$; $Y_{22} = 20.29 - j19.54$; $Y_{23} = -15.29 + j4.6$

$Y_{31} = Y_{13}$; $Y_{32} = Y_{23}$; $Y_{33} = 26.76 - j7.98$

$$Y_{bus} = \begin{bmatrix} 16.47 - j18.39 & -5 + j15 & -11.47 + j3.49 \\ -5 + j15 & 20.29 - j19.54 & -15.29 + j4.6 \\ -11.47 + j3.44 & -15.29 + j4.6 & 26.76 - j7.98 \end{bmatrix}$$

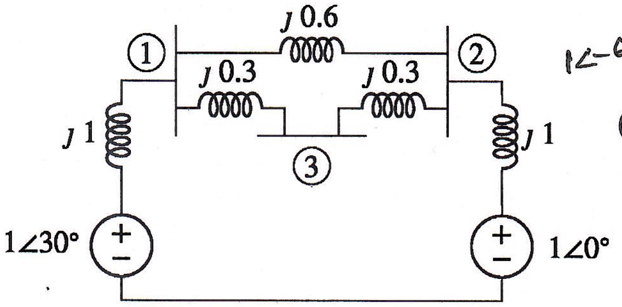
3

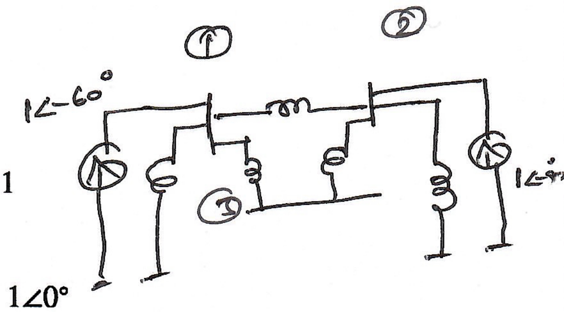
b) After elimination 3

$$Y_{bus}^{(Mod)} = \begin{bmatrix} 11.5531 - j16.90 & -11.55 + j16.98 \\ -11.55 + j16.98 & 11.5531 - j16.90 \end{bmatrix}$$

3

5. Consider the reactance network shown here.





(a) Find the voltage at node 3.

(b) Find the voltage at node 3 after connecting a capacitor having a reactance of 2.0 per unit connected between node 3 to the reference node. (Hint : Use Thevenin equivalent.)

a)

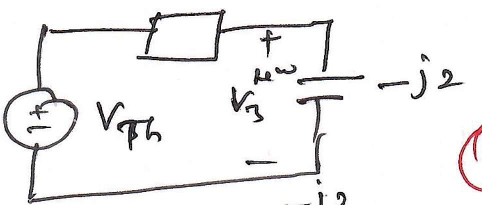
$$Y_{bus} = \begin{bmatrix} -j6 & j1.67 & j3.33 \\ j1.67 & -j6 & j3.33 \\ j3.33 & j3.33 & -j6.67 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = [Y_{bus}]^{-1} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

$$Z_{bus} = Y_{bus}^{-1} = \begin{bmatrix} j0.57 & j0.43 & j0.5 \\ j0.43 & j0.57 & j0.5 \\ j0.5 & j0.5 & j0.65 \end{bmatrix}$$

$$\boxed{\begin{aligned} V_3 &= 0.933 + j0.25 \\ V_3 &= 0.966\angle 15^\circ \end{aligned}}$$

b) By Thevenin Method :



$$V_3^{new} = V_{Th} \times \frac{-j2}{Z_{Th} - j2}$$

$$= 0.966\angle 15^\circ \times \frac{-j2}{j0.65 - j2}$$

$$V_3^{new} = 1.38 + j0.37$$

$$\boxed{V_3^{new} = 1.43\angle 15^\circ}$$

By Y_{bus} modification :

$$Y_{33} = -j6.67 + \frac{1}{-j2} = -j6.17$$

$$Z_{bus} = \begin{bmatrix} -j6 & j1.67 & j3.33 \\ j1.67 & -j6 & j3.33 \\ j3.33 & j3.33 & -j6.17 \end{bmatrix}^{-1}$$

Z_{bus} 3rd column

$$V_3^{New} = Z_{bus}\ \text{3rd row} \times \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \qquad \begin{bmatrix} j0.74 \\ j0.74 \\ j0.96 \end{bmatrix}$$

$$= 1.37 + j0.37$$

$$\boxed{V_3^{new} = 1.42\angle 15^\circ}$$