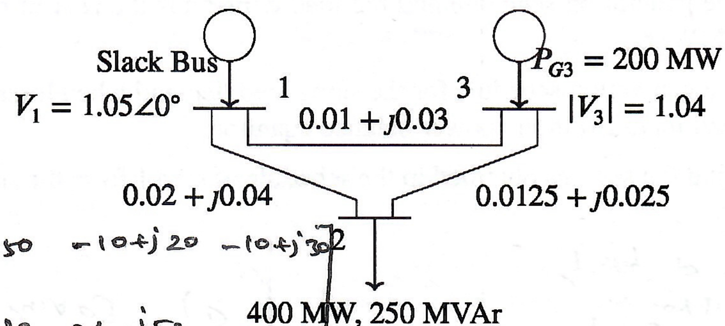
Indian Institute of Technology Patna
Department of Electrical Engineering
EE381 - Power Systems
Autumn - 2023
End Semester Exam — Solution
December 3, 2023

There are 5 questions. They carry equal marks.

$(5 \times 10 = 50)$

---

1. The figure below shows the single line diagram of a simple three bus power system with generators at buses 1 and 3. Line impedances are in p.u on a 100 MVA base. Assuming a flat voltage start, determine the voltage at the end of the first iteration using Gauss-Seidal method. Choose Base MVA as 100. Neglect Q limits for Bus 3. Take $\alpha = 1.6$.



(i) $Y_{Bus} = \begin{bmatrix} 20-j50 & -10+j20 & -10+j30 \\ -10+j20 & 26-j52 & -16+j32 \\ -10+j30 & -16+j32 & 26-j62 \end{bmatrix}$

(ii) Assume $V_2^0 = 1+j0$; $\delta_3^0 = 0$ $\quad V_3^0 = 1.04\angle 0°$

(iii) Bus ② is a PQ bus. $P_2 = -4$, $Q_2 = -2.5$

$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1 - Y_{23}V_3^0\right]$

$V_2^1 = 0.9746 - j0.0423$ $\quad V_{2acc}^1 = \underline{0.9594 - j0.0677} = \underline{0.9618\angle -4.036°}$

(iv) Bus ③ is a PV bus.

$Q_3^1 = -\text{Imag}\left\{(V_3^0)^* \times (Y_{31}V_1 + Y_{32}V_{2acc}^1 + Y_{33}V_3^0)\right\}$

$\underline{Q_3^1} = \underline{1.2438}$; $P_3 = 2$

$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1 - Y_{32}V_{2acc}^1\right]$

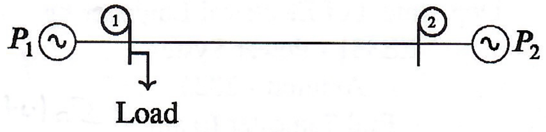
$V_3^1 = 1.0318 - j0.0197$ $\quad$ Since $|V_3|$ is fixed,

$V_{3\,corr}^1 = |V_3| \times \frac{V_3^1}{|V_3^1|}$

$V_{3\,corr}^1 = \underline{1.0398 - j0.0198}$

$V_{3\,corr}^1 = \underline{1.04\angle -1.09°}$

2. Consider a two bus system.



The incremental fuel cost characteristics of plant 1 and plant 2 are given by

$$\frac{dF_1}{dP_1} = 0.05P_1 + 20 \text{ Rs/MWHr}$$
$$\frac{dF_2}{dP_2} = 0.025P_2 + 15 \text{ Rs/MWHr}$$

If 100 MW of power is transmitted from plant 2 to the load, a transmission loss of 10 MW will be incurred.

(a) Find the generation schedule and the load demand if the cost of received power is 24 Rs/MWhr.

(b) Find the generation schedule for the same load demand when losses are not coordinated but included in the power balance equation.

(c) Also, find the savings obtained in the schedule of step 1 from the schedule of step 2.

a) $P_L = 0.001 P_2^2$

(Since the load is at bus 1, the losses will not be affected by P_1)

$L_1 \frac{dF_1}{dP_1} = L_2 \frac{dF_2}{dP_2} = \lambda$

$L_1 = 1$; $L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \frac{1}{1 - 0.002P_2}$

For optimum dispatch,

$\frac{dF_1}{dP_1} = L_2 \frac{dF_2}{dP_2} = \lambda$

$0.05P_1 + 20 = 24 \rightarrow$ $P_1 = 80 \text{MW}$

$\frac{1}{1-0.002P_2}$ $0.025P_2 + 15 = 24$ $P_2 = 123.28 \text{MW}$

$P_L = 15.19 \text{ MW}$

$P_D = P_1 + P_2 - P_L = 188.08 \text{ MW}$

b) when losses are not coordinated but included.

$0.05P_1 + 20 = 0.025P_2 + 15$ — (1)

$P_1 + P_2 = 0.001P_2^2 + 188.08$ — (2)

on solving (1) and (2),

$P_1 = 13.07 \text{ MW}$

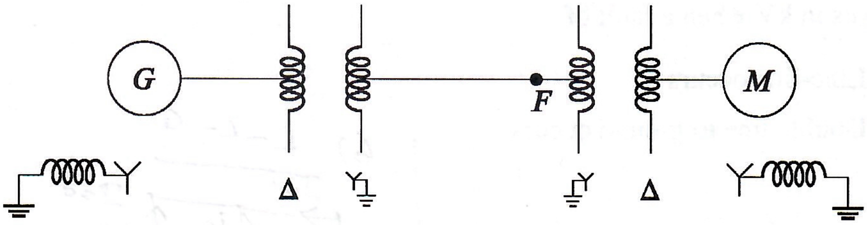
$P_2 = 226.15 \text{ MW}$

c) Savings

$$= \int_{80}^{13.07} \frac{dF_1}{dP_1} dP_1 + \int_{123.28}^{226.15} \frac{dF_2}{dP_2} dP_2$$

$= 498.05 \text{ Rs/hr}$

3. A single line to ground fault occurs on phase a at point F in the system shown below.



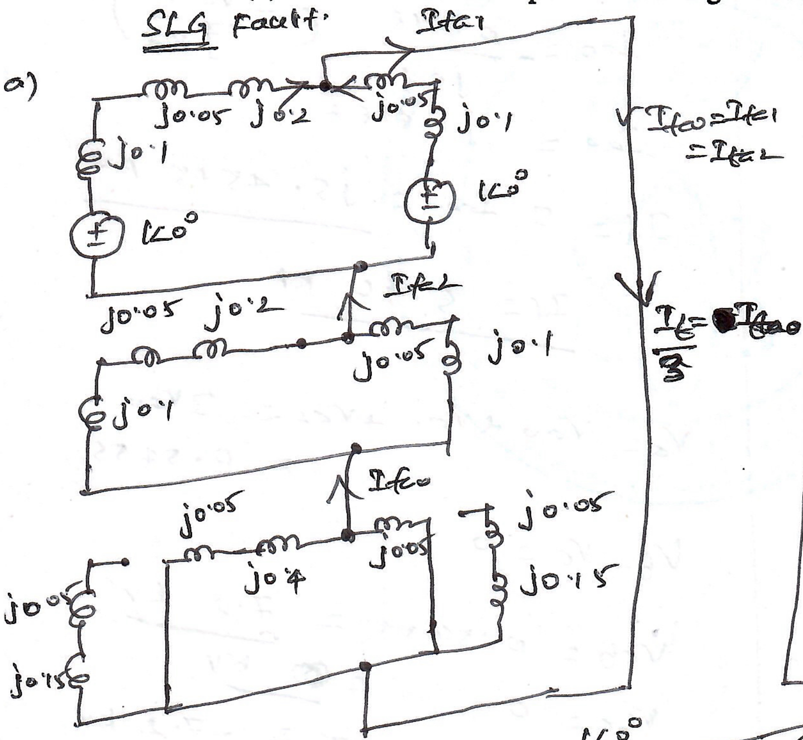
Both machines are rated 1200 kVA, 600 V with reactances of $X''_d = X_1 = X_2 = 10\%$ and $X_0 = 5\%$. Each three phase transformer is rated 1200 kVA, 600 V - Δ / 3300 V - Y with leakage reactance of 5 %. The reactance of the transmission lines are $X_1 = X_2 = 20\%$ and $X_0 = 40\%$ on a base of 1200 kVA and 3300 V. The reactances of the neutral grounding reactors are 5 % on the base of machines. Assume the prefault current to be zero. Find

(a) the fault current at point F in Amperes.

(b) the fault current in phase a of the generator in Amperes.

SLG Fault:

a)



$$I_{fa0} = I_{fa1} = I_{fa2} = \frac{I_f}{3} = \frac{1\angle 0^\circ}{j0.105 + j0.105 + j0.045}$$

$$I_{fa0} = 3.92\angle -90^\circ \text{ p.u}$$

$$I_f = 3 I_{fa0} = 11.76 \angle -90^\circ \text{ p.u}$$

$$I_f = 11.76 \times \frac{1200 \times 10^3}{\sqrt{3} \times 3300}$$

$$\underline{I_f = 2.5 \text{ KA}}$$

b) $I_{faG} = I_{fa1G} + I_{fa2G} + I_{fa0G}$

$I_{fa0G} = 0$ (Since T_1's primary is Δ connected)

$$I_{fa1G} = I_{fa1} \times \frac{j0.15}{j0.5}$$

(Current division)

$$I_{fa1G} = -j1.176$$

$$I_{fa2G} = I_{fa2} \times \frac{j0.15}{j0.5}$$

$$I_{fa2G} = -j1.176$$

$$I_{faG} = -j1.176 - j1.176$$

$$\underline{I_{faG} = -j2.352 \text{ p.u}}$$

$$I_{faG} = 2.352 \times \frac{1200 \times 10^3}{\sqrt{3} \times 600}$$

$$\underline{I_{faG} = 2.72 \text{ KA}}$$

4. The positive, negative and zero sequence reactances of a 25 MVA, 13.2 kV alternator are 0.3 p.u, 0.2 p.u and 0.1 p.u, respectively. The generator is star connected and neutral is solidly grounded. When it is unloaded, find the fault current in Ampere and line-line voltages in kV when a fault of

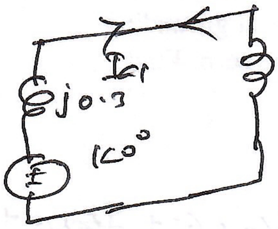
(a) Line-line occurs

(b) Double line to ground occurs

a) L-L Fault:

$I_{a1} + I_{a2} = 0$

$V_{a1} = V_{a2}$



$I_{a1} = \frac{1}{j0.5} = -2j$

$I_b = -I_c = (\alpha^2 - \alpha) I_{a1}$

$= -3.4641$ p.u

$I_b = 3.4641 \times \frac{25 \times 10^3}{\sqrt{3} \times 13.2}$

$I_b = 3.8$ KA

$V_a = V_{a0} + V_{a1} + V_{a2}$

($V_{a0} = 0$ L-L fault)

$V_a = V_{a1} + V_{a2}$

$V_{a1} = 1\angle 0^\circ - I_{a1} \times j0.3$

$V_{a1} = 0.4 = V_{a2}$

$\therefore V_a = 0.8$ p.u

$V_b = \alpha^2 V_{a1} + \alpha V_{a2}$ ($V_{b0} = 0$)

$V_c = \alpha V_{a1} + \alpha^2 V_{a2}$ ($V_{c0} = 0$)

$V_b = -0.4$ p.u

$V_c = -0.4$ p.u

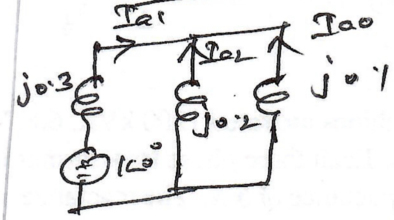
$V_{ab} = 1.2$ p.u $= 1.2 \times 13.2 = 15.84$ kV

$V_{bc} = 0$ $= 0$

$V_{ca} = -1.2$ p.u $= -1.2 \times 13.2 = -15.84$ kV

④

b) L-L-G



$I_{a1} = \frac{1}{j0.3 + \frac{j0.2 \times j0.1}{j0.3}}$

$I_{a1} = -2.7273j$

$V_{a1} = V_{a2} = V_{a0} = 1 - I_{a1} \times j0.3$

$V_{a1} = 0.1818$

$I_{a0} = -\frac{0.1818}{j0.1}$ $\left(-\frac{V_{a0}}{jX_0}\right)$

$I_{a0} = j1.8182$

$I_f = 3 I_{a0} = j5.4545$ p.u

$I_f = 5.96$ KA

$V_a = V_{a0} + V_{a1} + V_{a2} = 3V_{a0}$

$= 0.5455$

$V_b = V_c = 0$

$V_{cb} = 0.5455 = 7.2$ kV

$= 0$ kV

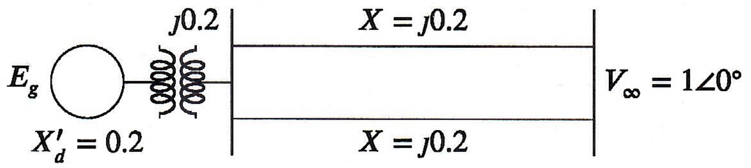
$V_{bc} = 0$

$V_{ca} = -0.5455 = -7.2$ kV

$V_{cb} = 0.5455 \times 13.2$ kV

$V_{cb} = 7.2$ kV

5. A 50 Hz synchronous generator having inertia constant $H = 5$ sec and a direct axis reactance $X'_d = 0.2$ p.u. is connected to an infinite bus through a transformer and a double circuit line. The network is purely reactive. The synchronous generator is delivering real power $P = 1$ p.u. at 0.8 pf lagging to the infinite bus.



A three phase fault occurs at one of the lines near the sending end of the line. The fault is cleared by opening the faulted line. Determine the critical clearing angle and the critical fault clearing time.

$|I| = \frac{P}{V \times \cos\phi} = 1.25$

(i) Prefault:

$X_I = j0.5$; $\bar{I} = 1.25 \angle -\cos^{-1}(0.8)$

$E\angle\delta = 1\angle 0° + 1.25 \angle -\cos^{-1}(0.8) \times j0.5 = 1.46 \angle 19.98° \text{ p.u}$

$= 1.46 \angle 0.35 \text{ rad}$

$|\bar{E}| = 1.46$; $\delta_0 = 19.98° = 0.35 \text{ rad}$

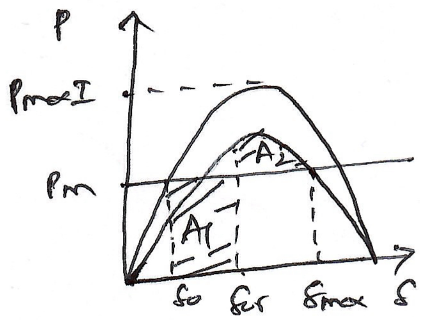
$P_{max\,I} = \frac{EV}{X_I} = \frac{1.46 \times 1}{0.5} = 2.92 \text{ p.u}$

(ii) $P_{max\,II} = 0$ (Since the fault is on/near a bus)

(iii) $P_{max\,III} = \frac{EV}{X_{III}} = \frac{1.46 \times 1}{0.6} = 2.433 \text{ p.u}$

$\delta_{max} = \pi - \sin^{-1}\left(\frac{P_m}{P_{max\,III}}\right) = 2.718 \text{ rad}$

$= 155.73°$



$A_1 = A_2$

$\delta_{cr} = \cos^{-1}\left(\frac{P_m(\delta_{max} - \delta_0) + P_{max\,III}\cos\delta_{max}}{P_{max\,III}}\right)$

$\delta_{cr} = 1.509 \text{ rad}$

$\boxed{\delta_{cr} = 86.46°}$

$t_{cr} = \sqrt{\frac{4H(\delta_{cr} - \delta_0)}{\omega_s \cdot P_m}} = \sqrt{\frac{4 \times 5\,(1.5 - 0.35)}{2\pi \times 50 \times 1}}$

$\boxed{t_{cr} = 0.2717 \text{ sec}}$