Power System Stability

Power system stability is defined as the property of a power system that enables it to remain in a state of operating equilibrium under normal operating conditions and to regain an acceptable state of equilibrium after being subjected to a disturbance.

Disturbances can be small or large.

1. Small Disturbances
   - Incremental changes in load
   - Incremental changes in generation
2. Large Disturbances
   - Loss of a large generator or load
   - Faults on transmission lines

Classification of Power System Stability

1. Rotor Angle Stability
   - Ability to maintain synchronism after being subjected to a disturbance.
   - Torque balance of synchronous machines.
2. Voltage Stability
   - Ability to maintain steady acceptable voltage at all buses after being subjected to a disturbance.
   - Reactive power balance.

We study Rotor angle stability in this course.

Rotor Angle Stability

Rotor angle stability is the ability of interconnected synchronous machines of a power system to remain in synchronism after being subjected to a disturbance.

1. Small disturbance (small signal) stability
   - Ability to maintain synchronism under small disturbances.
   - Since disturbances are small, nonlinear differential equations can be linearized.
   - It is easy to solve.
2. Large disturbance (Transient) stability
   - Ability to maintain synchronism under large disturbances.
   - Since disturbances are large, nonlinear differential equations can not be linearized.
   - It has to be solved numerically. It is difficult..
   - However, we use a direct approach called *Equal Area Criterion* for analyzing the stability of a single machine connected to an infinite bus.

Power-Angle Relationship:

Consider a single machine infinite bus (SMIB) system:

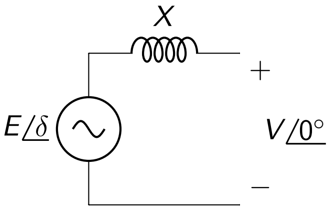


Figure: Per phase equivalent circuit

Where $X = X_g + X_{Tr} + X_{TL}$ in p.u.

To find the real power output of the machine:

$$I = \frac{E\angle\delta - V\angle 0^\circ}{\jmath X}$$

$$S_S = EI^*$$

$$S_S = E\angle\delta \left(\frac{E\angle -\delta - V\angle 0^\circ}{-\jmath X} \right)$$

$$S_S = \frac{E^2\angle 90^\circ}{X} - \frac{EV\angle 90^\circ + \delta}{X}$$

$$P_S = \frac{EV \sin\delta}{X}$$

$$Q_S = \frac{E^2}{X} - \frac{EV \cos\delta}{X}$$

Since the system is lossless, the real power delivered at the infinite bus is also the same.

$$P_R = P_S = \frac{EV \sin\delta}{X} = P_e$$

$$P_e = P_{max} \sin \delta$$

where $P_{max} = \dfrac{EV}{X}$.

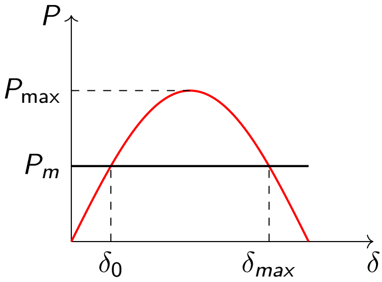


Figure: Power angle curve

For a given mechanical power (P_m), there are two operating angles.

$$\delta_0 = \sin^{-1}(\frac{P_m}{P_{max}})$$

$$\delta_{max} = \pi - \delta_0$$

- δ_0 is a stable equilibrium point.
- δ_{max} is an unstable equilibrium point.

Rotor Dynamics - Swing Equation

The equation governing rotor motion of a synchronous machine is given as

$$J\frac{d^2\theta_m}{dt^2} = T_a = T_m - T_e \text{ N-m}$$

where

J = the total moment of inertia of the rotor masses in kg-m^2

θ_m = the angular displacement of the rotor with respect to a stationary axis in mechanical radians (rad)

t = time in seconds (s)

T_m = the mechanical or shaft torque supplied by the prime mover in N-m

T_e = the net electrical or electromagnetic torque in N-m

T_a = the net accelerating torque in N-m

- T_m and T_e are considered positive for the synchronous generator.
- T_m accelerates the rotor in the positive θ_m in the direction of rotation.
- For a motor, T_m and T_e are reversed in sign.
- In the steady state, $T_m = T_e$. Hence, $T_a = 0$.

θ_m is measured with respect to a stationary reference axis on the stator. To represent it with respect to the synchronously rotating frame, let us define

$$\theta_m = \omega_{sm} t + \delta_m$$

where
ω_{sm} is the synchronous speed of the machine in mechanical radians per second
δ_m is the angular displacement of the rotor in mechanical radians from the synchronously rotating reference axis.

$$\frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt}$$

$$\omega_m - \omega_{sm} = \frac{d\delta_m}{dt}$$

where $\omega_m = \dfrac{d\theta_m}{dt}$ is the angular velocity of the rotor in mechanical radians per second.
Differentiating it again,

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

Substituting it,

$$J\frac{d^2\delta_m}{dt^2} = T_a = T_m - T_e \text{ N-m}$$

On multiplying by ω_m,

$$J\omega_m\frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e$$

where

P_m = shaft power input in MW
P_e = electrical power output in MW
P_a = accelerating power in MW

Let us define *inertia constant H*.

$$H = \frac{\text{stored kinetic energy in megajoules at synchronous speed}}{\text{Machine rating in MVA}}$$

$$H = \frac{\frac{1}{2} J \omega_{sm}^2}{S_{mach}} \text{ MJ/MVA}$$

Substituting it,

$$\frac{2H}{\omega_{sm}^2} \omega_m \frac{d^2 \delta_m}{dt^2} = \frac{P_a}{S_{mach}} = \frac{P_m - P_e}{S_{mach}}$$

In practice, ω_m does not differ significantly from the synchronous speed. $\omega_m \approx \omega_{sm}$

$$\frac{2H}{\omega_{sm}} \frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e \text{ per unit}$$

It can be written as

$$\boxed{\frac{2H}{\omega_s} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \text{ per unit}}$$

δ and ω_s have consistent units which can be mechanical or electrical degrees or radians.

- This equation is called the *swing equation* of the machine.
- It is a second-order nonlinear differential equation.
- When it is solved, we obtain δ as a function of t. This is called the *swing curve*.

It can be written as two first-order differential equations.

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = P_a = P_m - P_e \text{ per unit}$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

- ω, ω_s and δ involve electrical radians or electrical degrees.
- δ is the **load angle**.

Example 1 : A 50 Hz, 4-pole, turbo-alternator rated 500 MVA, 22 kV has an inertia constant of 7.5 sec. Find

1. the rotor acceleration if the input to the generator is suddenly raised to 400 MW for an electrical load of 350 MW.
2. the speed of rotor in rpm if the rotor acceleration is constant for 10 cycles and the change in torque angle δ in elect degrees.

1.

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_a = P_m - P_e \text{ per unit}$$

$$\frac{2 \times 7.5}{2 \times \pi \times 50}\frac{d^2\delta}{dt^2} = \frac{400 - 350}{500}$$

$$\frac{d^2\delta}{dt^2} = 2.0944 \text{ elect. rad/s}^2$$

$$\frac{d^2\delta}{dt^2} = 2.0944 \times \frac{180}{\pi} = 120 \text{ elect. degree/s}^2$$

For a 4-pole machine,

$$\frac{d^2\delta}{dt^2} = 60 \text{ mech. degree/s}^2$$

Since 1 revolution = 360 mech. degree,

$$\frac{d^2\delta}{dt^2} = \frac{60}{360} \;\text{ revolution/s}^2$$

$$\frac{d^2\delta}{dt^2} = \frac{60 \times 60}{360} = 10 \;\text{ rpm/s}$$

2. If the acceleration is constant for 10 cycles, the duration of acceleration will be

$$t = 10 \times \frac{1}{50} = 0.2 \text{ s}$$

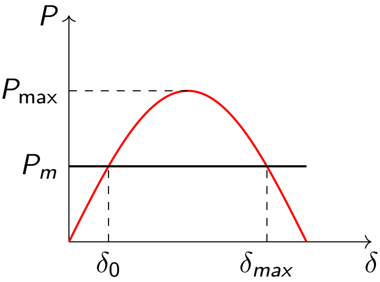
$$N = 1500 + 10 \times 0.2 = 1502 \text{ rpm}$$

To find the change in δ, twice integrate $\frac{d^2\delta}{dt^2}$.

$$\delta = \delta_0 + \frac{1}{2}(\frac{d^2\delta}{dt^2})t^2$$

$$\Delta\delta = \frac{1}{2} \times 120 \times (0.2)^2 = 2.4 \text{ elect. degree}$$

Synchronizing Power Coefficients



Let us assume that P_m is constant. Consider small incremental changes in δ,

$$\delta = \delta_0 + \delta_\Delta \quad P_e = P_{e0} + P_{e\Delta}$$

$$\begin{aligned} P_{e0} + P_{e\Delta} &= P_{max} \sin(\delta_0 + \delta_\Delta) \\ &= P_{max}(\sin\delta_0 \cos\delta_\Delta + \cos\delta_0 \sin\delta_\Delta) \end{aligned}$$

Since δ_Δ is small,

$$\sin \delta_\Delta \approx \delta_\Delta \quad \cos \delta_\Delta \approx 1$$

$$P_{e0} + P_{e\Delta} = P_{max} \sin \delta_0 + (P_{max} \cos \delta_0)\delta_\Delta$$

At δ_0,

$$P_m = P_{e0} = P_{max} \sin \delta_0$$

Therefore,

$$P_m - (P_{e0} + P_{e\Delta}) = -(P_{max} \cos \delta_0)\delta_\Delta$$

Substituting the incremental variables in the swing equation,

$$\frac{2H}{\omega_s} \frac{d^2(\delta_0 + \delta_\Delta)}{dt^2} = P_m - (P_{e0} + P_{e\Delta})$$

On simplification, we get

$$\frac{2H}{\omega_s} \frac{d^2\delta_\Delta}{dt^2} + (P_{max} \cos \delta_0)\delta_\Delta = 0$$

Since δ_0 is constant, $P_{max} \cos \delta_0$ is the slope of the curve at δ_0. Let

$$S_p = \left. \frac{dP}{d\delta} \right|_{\delta=\delta_0} = P_{max} \cos \delta_0$$

where S_p is called the *synchronizing power coefficient*.

$$\frac{d^2\delta_\Delta}{dt^2} + \frac{\omega_s S_p}{2H}\delta_\Delta = 0$$

- It is a linear second-order differential equation.
- When S_p is positive, the solution $\delta_\Delta(t)$ is an undamped sinusoid.
- When S_p is negative, the solution $\delta_\Delta(t)$ increases exponentially without limit.
- Therefore δ_0 is a stable equilibrium point and δ_{max} is an unstable equilibrium point.

Equal Area Criterion

- Since the swing equation is nonlinear, it has to be numerically integrated to obtain solutions.
- If the disturbances are large, the equation can not be linearised.
- It is very difficult to obtain solutions.
- However, for a single machine connected to an infinite bus system, a direct approach without solving it is possible.
- The approach is called the Equal Area Criterion method.

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H}(P_m - P_e)$$

Let us multiply both sides of the above equation by $2d\delta/dt$,

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = \frac{\omega_s(P_m - P_e)}{H}\frac{d\delta}{dt}$$

$$\frac{d}{dt}\left[\frac{d\delta}{dt}\right]^2 = \frac{\omega_s(P_m - P_e)}{H}\frac{d\delta}{dt}$$

On integration,

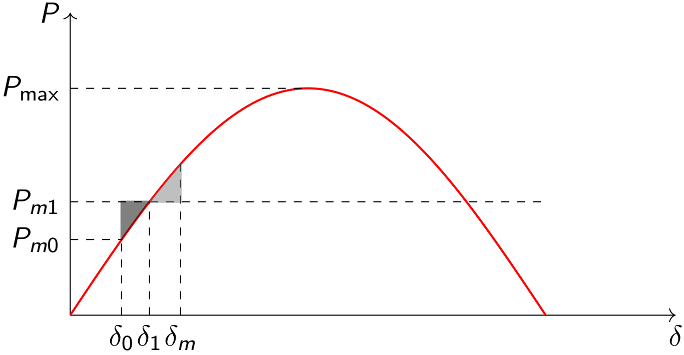
$$\left[\frac{d\delta}{dt}\right]^2 = \int \frac{\omega_s(P_m - P_e)}{H}d\delta$$

For a system to be stable, $\frac{d\delta}{dt} = 0$ after a disturbance.

$$\int \frac{\omega_s(P_m - P_e)}{H} d\delta = 0$$

$$\int (P_m - P_e) d\delta = 0$$

Sudden change in P_m



$$\int_{\delta_0}^{\delta_m} (P_{m1} - P_e) d\delta = 0$$

δ_0 is the initial rotor angle. δ_m is the maximum rotor angle during oscillation.

$$\int_{\delta_0}^{\delta_1} (P_{m1} - P_e) d\delta + \int_{\delta_1}^{\delta_m} (P_{m1} - P_e) d\delta = 0$$

$$\int_{\delta_0}^{\delta_1} (P_{m1} - P_e) d\delta = \int_{\delta_1}^{\delta_m} (P_e - P_{m1}) d\delta$$

Therefore for the system to be stable

$$\text{Area}(A_1) = \text{Area}(A_2)$$

$$\text{Energy Gained } = \text{Energy Lost}$$

If $A_1 > A_2$, the system will be unstable.

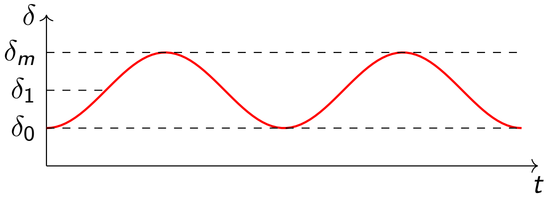
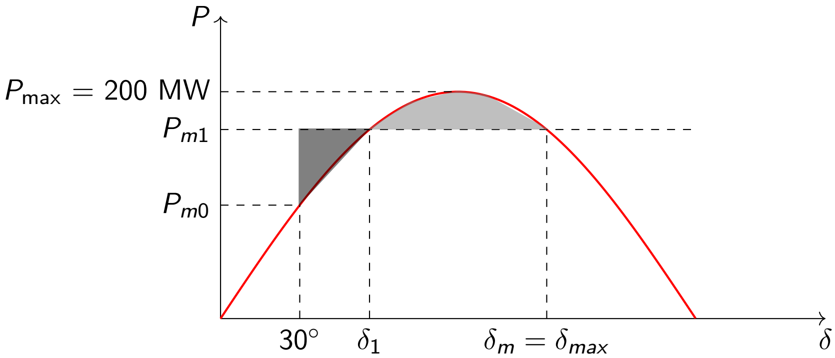


Figure: Swing Curve

- There is no damping in our model.
- In practice, damper or amortisseur windings produce damping.

Example 2 : A synchronous generator capable of developing 200 MW is operating at an angle of 30°. By how much can the input shaft power be increased suddenly without loss of stability?



P_m can be increased suddenly without losing stability such that δ_m is equal to δ_{max}.

$$\int_{\delta_0}^{\delta_1} (P_{m1} - P_e) d\delta = \int_{\delta_1}^{\delta_{max}} (P_e - P_{m1}) d\delta$$

where $P_e = P_{max} \sin \delta$, $P_{m1} = P_{max} \sin \delta_1$ and $\delta_{max} = \pi - \delta_1$. Integrating and simplifying, we get

$$200 \cos \delta_1 - \pi \times 200 \sin \delta_1 + \delta_1 \times 200 \sin \delta_1 + (\pi/6) \times 200 \sin \delta_1 + 100\sqrt{3} = 0$$

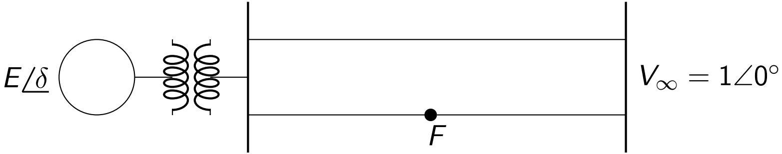
Solving this numerically,

$$\delta_1 = 1.0545 \text{ rad} = 60.4174 \text{ degree}$$

$$P_{m1} = P_{max} \sin \delta_1 = 173.93 \text{ MW}$$

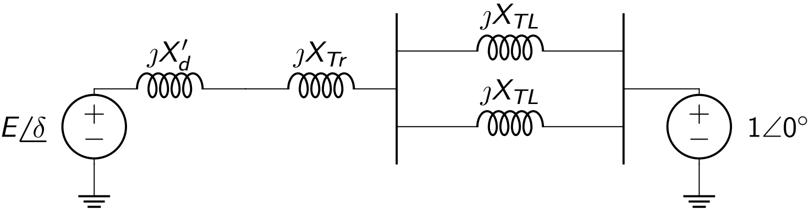
The shaft power can be increased from 100 MW to 173.93 MW suddenly without losing stability.

Short Circuit Faults



At point F, a three phase fault occurs. To analyze this, we need to understand the physical conditions before, during and after the fault.

1. Before Fault :



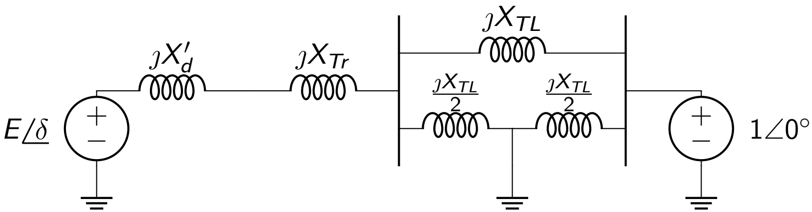
$$X_1 = X'_d + X_{Tr} + \frac{X_{TL}}{2}$$

$$P_{e1} = P_{max1} \sin \delta$$

where

$$P_{max1} = \frac{EV}{X_1}$$

2. During Fault :



- The total reactance X between two nodes can be found using $Y - \Delta$ conversion.
- X_2 during fault will be higher than before fault X_1.

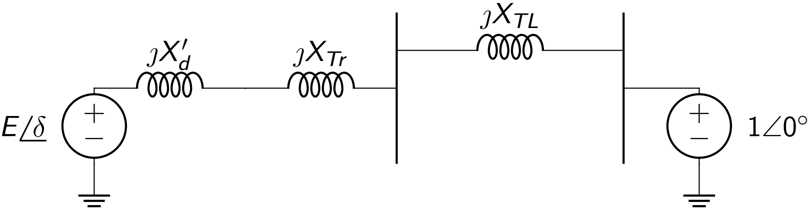
Hence,

$$P_{e2} = P_{max2} \sin \delta$$

where

$$P_{max2} = \frac{EV}{X_2}$$

3. After Fault :

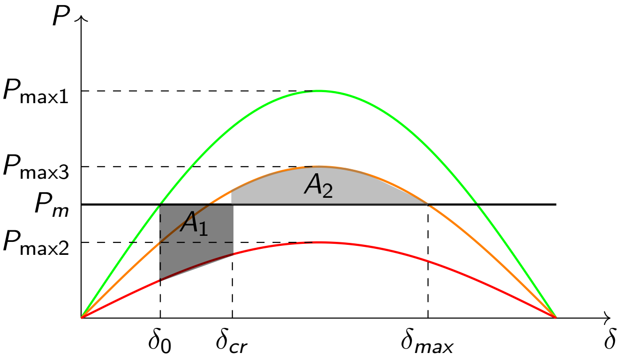


$$X_3 = X'_d + X_{Tr} + X_{TL}$$

$$P_{e3} = P_{max3} \sin\delta$$

where

$$P_{max3} = \frac{EV}{X_3}$$



$\delta_{max} = \pi - \sin^{-1}(\frac{P_m}{P_{max3}})$.
For the system to be stable, $A_1 = A_2$. There is a critical clearing angle δ_{cr} before which the fault has to be cleared.

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_{max2} \sin \delta) d\delta = \int_{\delta_{cr}}^{\delta_{max}} (P_{max3} \sin \delta - P_m) d\delta$$

Integrating and simplifying the above equation, we get

$$\boxed{\cos\delta_{cr} = \frac{P_m(\delta_{max} - \delta_0) + P_{max3}\cos\delta_{max} - P_{max2}\cos\delta_0}{(P_{max3} - P_{max2})}}$$

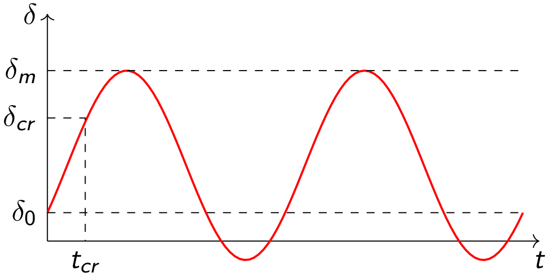


Figure: Swing Curve

t_{cr} is the critical clearing time in seconds.
It will settle at $\delta_{new} = \sin^{-1}(\frac{P_m}{P_{max3}})$ if damping is present.

Example 3: A three phase generator delivers 1.0 p.u. power to an infinite bus through a transmission network when a fault occurs. The maximum power which can be transferred during pre-fault, fault and post-fault conditions are 1.75 p.u, 0.4 p.u. and 1.25 p.u.respectively. Find the critical clearing angle.

$$P_m = 1 \quad P_{max1} = 1.75 \quad P_{max2} = 0.4 \quad P_{max3} = 1.25$$

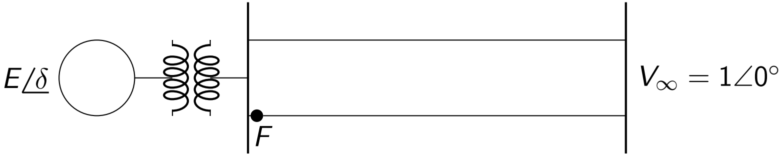
$$\delta_0 = \sin^{-1}(\frac{P_m}{P_{max1}}) = \sin^{-1}(\frac{1}{1.75}) = 0.61 \text{ rad}$$

$$\delta_{max} = \pi - \sin^{-1}(\frac{P_m}{P_{max3}}) = \pi - \sin^{-1}(\frac{1}{1.25}) = 2.2143 \text{ rad}$$

Substituting,

$$\delta_{cr} = 0.903 \text{ rad} = 51.73 \text{ degree}$$

Short Circuit Faults at the end of Transmission Lines (Near the bus)



1. Before Fault :

$$X_1 = X'_d + X_{Tr} + \frac{X_{TL}}{2}$$

$$P_{e1} = P_{max1} \sin\delta$$

where

$$P_{max1} = \frac{EV}{X_1}$$

2. During Fault :
   - Since the fault is near the bus, the bus voltage is zero.
   - The power transfer during fault is zero.

   Hence,

$$P_{e2} = 0$$

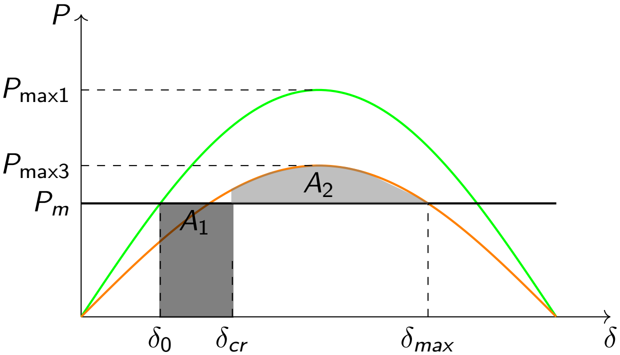
3. After Fault :

$$X_3 = X'_d + X_{Tr} + X_{TL}$$

$$P_{e3} = P_{max3} \sin \delta$$

   where

$$P_{max3} = \frac{EV}{X_3}$$



$\delta_{max} = \pi - \sin^{-1}(\frac{P_m}{P_{max3}})$.
For the system to be stable, $A_1 = A_2$.

$$\int_{\delta_0}^{\delta_{cr}} P_m d\delta = \int_{\delta_{cr}}^{\delta_{max}} (P_{max3} \sin \delta - P_m) d\delta$$

Integrating and simplifying the above equation, we get

$$\boxed{\cos\delta_{cr} = \frac{P_m(\delta_{max} - \delta_0) + P_{max3}\cos\delta_{max}}{P_{max3}}}$$

We can find the critical clearing time for this case as follows:

$$\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H}(P_m - P_e)$$

Since $P_e = 0$ during fault,

$$\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H}P_m$$

Integrating this,

$$\frac{d\delta}{dt} = \int_0^t \frac{\omega_s}{2H}P_m = \frac{\omega_s}{2H}P_m t$$

On further integration,

$$\delta = \frac{\omega_s}{4H} P_m t^2 + \delta_0$$

If $\delta = \delta_{cr}$,

$$\delta_{cr} = \frac{\omega_s}{4H} P_m t_{cr}^2 + \delta_0$$

$$t_{cr} = \sqrt{\frac{4H(\delta_{cr} - \delta_0)}{\omega_s P_m}}$$

Factors Influencing Transient Stability

1. How heavily the generator is loaded.
2. The generator output during the fault. This depends on the fault location and type.
3. The fault-clearing time.
4. The post fault transmission system reactance.
5. The generator inertia. The higher the inertia, the slower the rate of change in angle. This reduces A_1.
6. The generator internal voltage magnitude E. This depends on the field excitation.