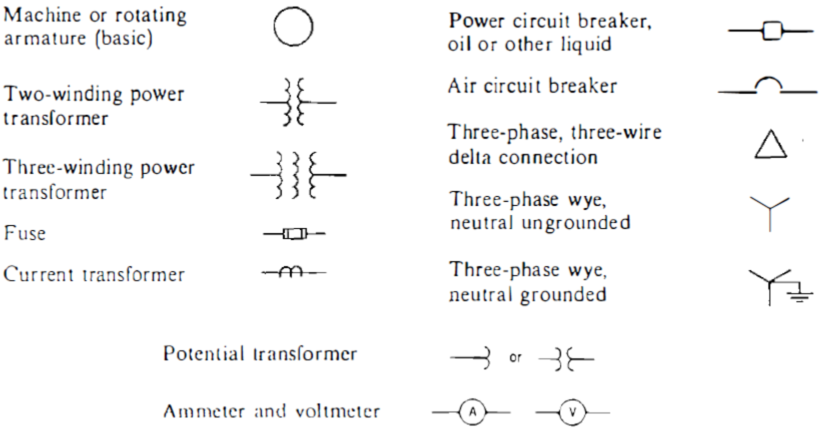
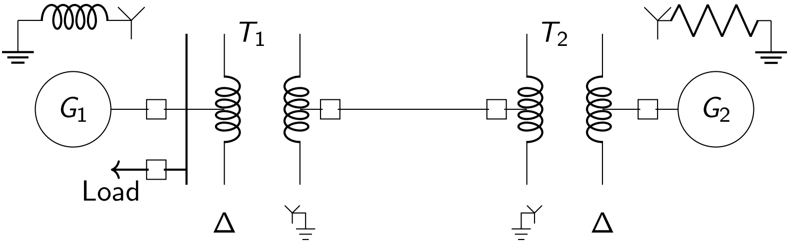


Figure: Symbols

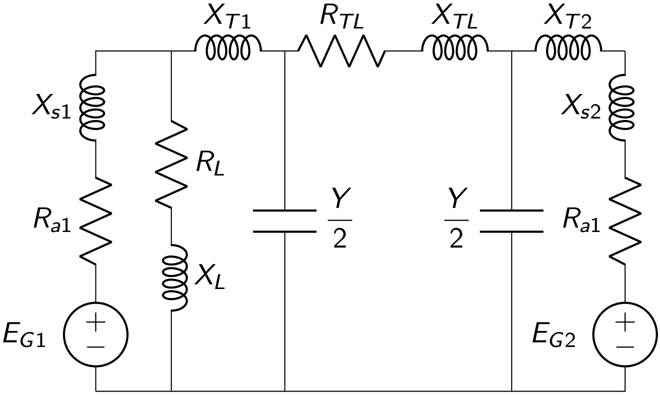
Source: John Grainger, "Power System Analysis"

Single Line Diagram / One Line Diagram



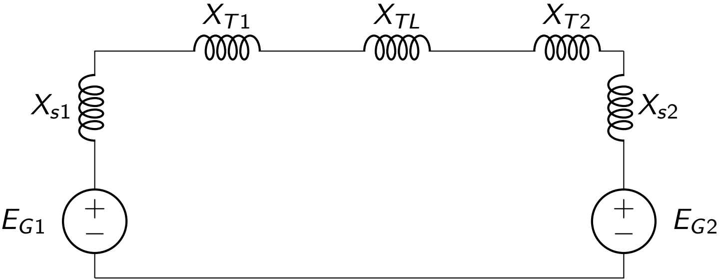
Impedance Diagram:

The per phase impedance diagram of the circuit under balanced condition is



Reactance Diagram:

- Resistance is often neglected in fault calculations.
- Capacitance is also neglected in the fault analysis.
- Static loads are omitted as they do not contribute during a fault.
- Synchronous motor loads are included in the analysis.



per unit

The per unit is defined as

$$\text{per unit} = \frac{\text{actual value in any unit}}{\text{base value in the same unit}}$$

There are normally four quantities associated with a power system.

$$S, V, I, Z$$

How to find base quantities?

- Choose any two. Normally S_{base} and V_{base} are chosen.
- Find the remaining two using their relations.

Let us start with single phase.

$$S_b = S_{1\phi} \text{ MVA}; \quad V_b = V_{1\phi} \text{ kV}$$

$$I_b = \frac{S_b(\text{MVA})}{V_b(\text{kV})} \text{ kA}$$

$$Z_b = \frac{V_b(\text{kV})}{I_b(\text{kA})} \ \Omega$$

Substituting I_b in Z_b,

$$Z_b = \frac{V_b^2(\text{ in kV})}{S_b(\text{ in MVA})} \ \Omega$$

$$Z_{\text{p.u.}} = \frac{Z_{\text{actual}}(\Omega)}{Z_b(\Omega)}$$

$$\boxed{\therefore Z_{\text{p.u.}} = \frac{Z_{\text{actual}}(\Omega) \times S_b(\text{ } 1\phi \text{ MVA})}{V_b^2(\text{ L- N in kV})}}$$

For three phase.

$$S_b = S_{3\phi} \text{ MVA}; \quad V_b = V_{L-L} \text{ kV}$$

$$I_b = \frac{S_b(\text{MVA})}{\sqrt{3} V_b(\text{kV})} \text{ kA}$$

$$Z_b = \frac{V_b(\text{kV})}{\sqrt{3} \times I_b(\text{kA})} \, \Omega$$

Substituting I_b in Z_b,

$$Z_b = \frac{V_b^2(\text{ in kV})}{S_b(\text{ in MVA})} \, \Omega$$

$$Z_{\text{p.u.}} = \frac{Z_{\text{actual}}(\Omega)}{Z_b(\Omega)}$$

$$\boxed{\therefore Z_{\text{p.u.}} = \frac{Z_{\text{actual}}(\Omega) \times S_b(\text{ }3\phi\text{ MVA})}{V_b^2(\text{L-L in kV})}}$$

$$S_b^{3\phi} = 3S_b^{1\phi}; \quad V_{b_{L-L}} = \sqrt{3}V_{b_{L-N}}$$

$$S_{\text{p.u.}}^{3\phi} = \frac{S^{3\phi}}{S_b^{3\phi}} = \frac{3 \times S^{1\phi}}{3 \times S_b^{1\phi}} = S_{\text{p.u.}}$$

$$V_{\text{p.u.}} = \frac{V_{L-L}}{V_{b_{L-L}}} = \frac{\sqrt{3}V_{L-N}}{\sqrt{3}V_{b_{L-N}}} = V_{\text{p.u.}}$$

- If the voltage magnitude is 1 p.u., the line-line voltage is 1 p.u. and the line-neutral voltage is also 1 p.u.
- Similarly, three phase power in p.u. and the single phase power in p.u. are the same.

$$Z_b = \frac{(V_{b_{L-L}})^2}{S_b^{3\phi}} = \frac{(\sqrt{3}V_{b_{L-N}})^2}{3 \times S_b^{1\phi}} = \frac{(V_{b_{L-N}})^2}{S_b^{1\phi}}$$

Impedance values of a component when given in per unit without specified bases are generally understood to be based on the MVA and kV ratings of the component.

To change p.u. from one base to new base:

$$Z_{\text{p.u.}} \propto \frac{S_b}{V_b^2}$$

$$Z_{\text{p.u. (new)}} = Z_{\text{p.u. (given)}} \times \frac{S_b(\text{new in MVA})}{S_b(\text{given in MVA})} \times \frac{V_b^2(\text{given in kV})}{V_b^2(\text{new in kV})}$$

Advantages:

- The per unit values of impedance, voltage and current of a transformer are the same regardless of whether they are referred to the HV side or LV side. This is possible by choosing base voltages on either side of the transformer using the voltage ratio of the transformer.
- The factors $\sqrt{3}$ and 3 get eliminated in the per unit power and voltage the equations.

Per unit Reactance Diagram:

- Choose Base MVA and Base kV.
- Find Base kV of other sections using transformation ratios.
- Find the per unit impedance.
  1. If the impedance is given in Ω

$$Z_{\text{p.u.}} = \frac{Z_{\text{actual}}(\Omega) \times S_b(\text{ } 3\phi \text{ MVA})}{V_b^2(\text{L-L kV})}$$

  2. If the impedance is given in p.u. on a different base, convert it to the new base.

$$Z_{\text{p.u. (new)}} = Z_{\text{p.u. (given)}} \times \frac{S_b(\text{new MVA})}{S_b(\text{given MVA})} \times \frac{V_b^2(\text{given kV})}{V_b^2(\text{new kV})}$$

- Draw the per unit impedance/reactance diagram.

Example 1 (Kothari P4.2) :

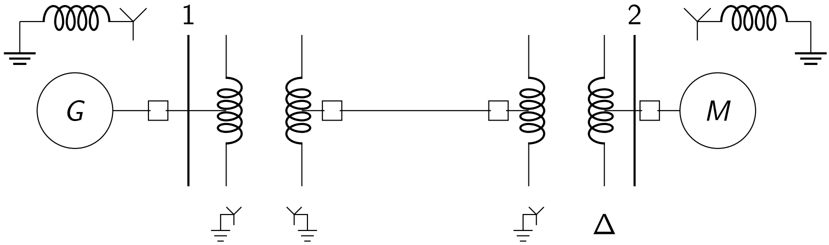
Draw the per unit impedance diagram for the power system shown here. Neglect resistance and use a base of 100 MVA and 220 kV in 50 Ω line. The ratings of the generator, motor and transformers are

Generator : 40 MVA, 25 kV, $X = 20\%$

Motor : 50 MVA, 11 kV, $X = 30\%$

Y- Y Transformer: 40 MVA, 33/220 kV, $X = 15\%$

Y- Δ Transformer: 30 MVA, 220/11 kV, $X = 15\%$.



- Base MVA is the same across the system.
- Base kV of the generator and motor sections have to be found.

$$\text{Base kV of Generator} = 220 \times \frac{33}{220} = 33 \text{ kV}$$

$$\text{Base kV of Motor} = 220 \times \frac{11}{220} = 11 \text{ kV}$$

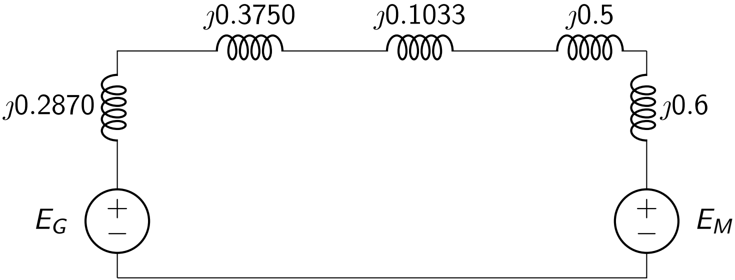
$$X_G = \jmath 0.2 \times \frac{100}{40} \times \frac{25^2}{33^2} = \jmath 0.2870 \text{ p.u.}$$

$$X_{T1} = \jmath 0.15 \times \frac{100}{40} \times \frac{220^2}{220^2} = \jmath 0.3750 \text{ p.u.}$$

$$X_{\text{Line}} = \jmath 50 \times \frac{100}{220^2} = \jmath 0.1033 \text{ p.u.}$$

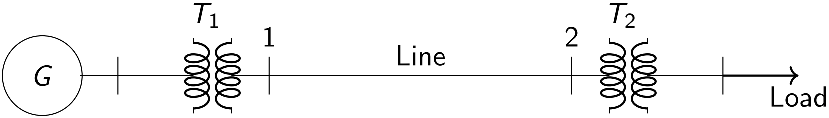
$$X_{T2} = \jmath 0.15 \times \frac{100}{30} \times \frac{220^2}{220^2} = \jmath 0.5 \text{ p.u.}$$

$$X_M = \jmath 0.3 \times \frac{100}{50} \times \frac{11^2}{11^2} = \jmath 0.6 \text{ p.u.}$$



j0.3750
j0.1033
j0.5
j0.2870
j0.6
E_G
E_M

Example 2 :



Generator: G : 10 MVA, 11 kV, X=0.15 p.u.
T_1: 50 MVA, 11/220 kV. X = 0.1 p.u.
T_2: 50 MVA, 33/220 kV. X = 0.1 p.u.
Line: $30+\jmath 100\ \Omega$.
Load: 2 MW at 0.9 pf lagging .
If the load voltage is to be maintained at 33 kV, what will be the terminal voltage in kV of the generator? Select a base of 100 MVA and 220 kV in the transmission line.

- Base MVA is the same across the system.
- Base kV of the generator and load sections have to be found.

$$\text{Base kV of Generator} = 220 \times \frac{11}{220} = 11 \text{ kV}$$

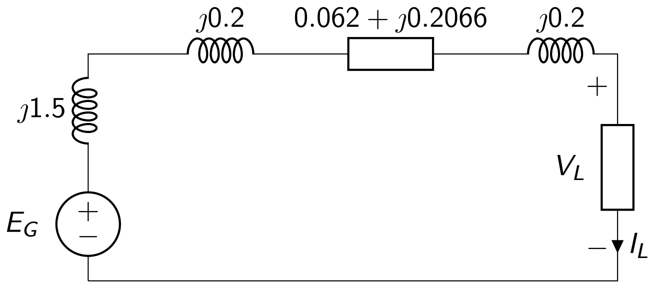
$$\text{Base kV of Load} = 220 \times \frac{33}{220} = 33 \text{ kV}$$

$$X_G = \jmath 0.15 \times \frac{100}{10} \times \frac{11^2}{11^2} = \jmath 1.5 \text{ p.u.}$$

$$X_{T1} = \jmath 0.1 \times \frac{100}{50} \times \frac{220^2}{220^2} = \jmath 0.2 \text{ p.u.}$$

$$X_{\text{Line}} = 30 + \jmath 100 \times \frac{100}{220^2} = 0.062 + \jmath 0.2066 \text{ p.u.}$$

$$X_{T2} = \jmath 0.1 \times \frac{100}{50} \times \frac{220^2}{220^2} = \jmath 0.2 \text{ p.u.}$$



$$P_L = \frac{2}{100} = 0.02 \text{ p.u.} \quad V_L = \frac{33}{33} = 1 \text{ p.u.}$$

$$I_L = \frac{P_L}{V_L \times pf} = \frac{0.02}{1 \times 0.9} = 0.022 \text{ p.u.}$$

$$V_G = 1.0\underline{/0^\circ} + 0.022\underline{/-25.84^\circ}(\jmath 0.2 + 0.062 + \jmath 0.2066 + \jmath 0.2)$$

$$V_G = 1.0075\angle 0.65^\circ \text{ p.u.}$$

$$V_G = 1.0075 \times 11 = 11.08 \text{ kV}$$