3-Phase Transmission Line

Source:Wikipedia

Line Parameters

There are four parameters associated with lines.

1. Resistance (R)
2. Inductance (L)
3. Capacitance (C)
4. Conductance (G)

- Conductance accounts for the leakage current at the insulators of overhead lines and the insulation of cables.
- Since the leakage is small, Conductance is usually neglected.

Types of Conductors

Aluminium is used because it is cheaper in cost and lighter in weight than copper of the same resistance.

- AAC - All aluminium conductors
- AAAC - All aluminium alloy conductors
- ACSR - Aluminium conductor steel reinforced
- ACAR - Aluminium conductor alloy reinforced

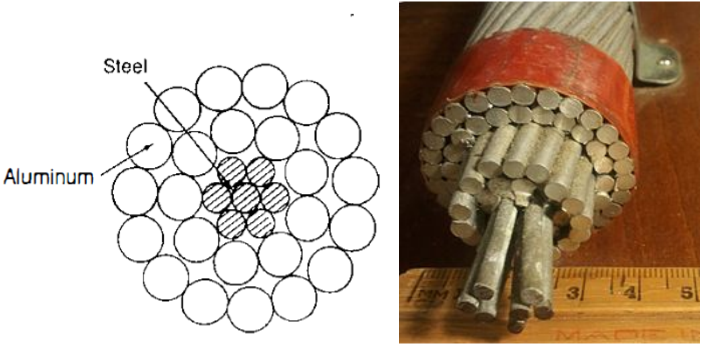


Figure: ACSR

Resistance

The dc resistance of a wire is

$$R_{dc} = \frac{\rho l}{A} \, \Omega$$

where ρ is the resistivity of the wire in Ω-m, l is the length in meter and A is the cross sectional area in m^2.

When alternating current flows through a conductor, the current density is not uniform over the entire section but higher at the surface. This is called **skin effect**.

$$R_{ac} > R_{dc}$$

As the frequency increases, the skin effect will be dominant.

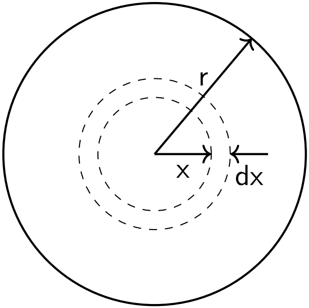
Inductance of a Single conductor:

The inductance is

$$L = \frac{\lambda}{I} \; H$$

where λ is the flux linkage in Weber-turns and I is the phasor current in Ampere.

1. Internal Inductance



Ampere's law:

$$\oint H \cdot dl = I$$

where H is the magnetic field intensity in At/m, dl is the incremental distance along the closed contour in m and I is the current in ampere.
Let H_x be the field intensity at a distance x from the center of the conductor and ds be the incremental length in the path.

$$\oint H_x \cdot ds = I_x$$

Since H_x is symmetrical,

$$H_x \times 2\pi x = I_x$$

$$H_x = \frac{I_x}{2\pi x}$$

Assuming the current density is uniform across the conductor.

$$\frac{I}{\pi r^2} = \frac{I_x}{\pi x^2}$$

$$I_x = \frac{\pi x^2}{\pi r^2} I$$

Substituting I_x in H_x,

$$H_x = \frac{I}{2\pi r^2} x$$

The flux density at x,

$$B_x = \mu_0 \mu_r H_x$$

Since μ_r is 1 for conductors,

$$B_x = \frac{\mu_0 I}{2\pi r^2} x$$

Let us consider an elementary area of $dx \times 1$ m^2.

$$d\phi_x = B_x \times dx \times 1 = \frac{\mu_0 I}{2\pi r^2} x dx$$

The entire cross section does not enclose this flux. Therefore the flux linkage is

$$d\lambda_x = \frac{\pi x^2}{\pi r^2} d\phi_x = \frac{\mu_0 I}{2\pi r^4} x^3 dx$$

The internal flux linkage is

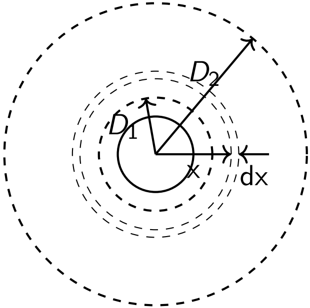
$$\lambda_{\text{int}} = \int_0^r \frac{\mu_0 I}{2\pi r^4} x^3 \, dx = \frac{\mu_0 I}{8\pi} = \frac{I}{2} \times 10^{-7} \text{ Wbt/m}$$

The internal inductance per unit length is

$$L_{\text{int}} = \frac{1}{2} \times 10^{-7} \text{ H/m}$$

It is independent of the conductor radius.

2. External Inductance



The entire current I is linked by the flux at any point outside the conductor.

$$d\lambda_x = d\phi_x = B_x \times dx \times 1 = \frac{\mu_0 I}{2\pi x} dx$$

The external flux linkages between any two points D_1 and D_2,

$$\lambda_{ext} = \int_{D_1}^{D_2} \frac{\mu_0 I}{2\pi x}\, dx = 2 \times 10^{-7} I \ln \frac{D_2}{D_1} \text{ Wbt/m}$$

The external inductance between any two points,

$$L_{\text{ext}} = 2 \times 10^{-7} \ln \frac{D_2}{D_1} \text{ H/m}$$

Similarly, the external inductance between r and D_2,

$$L_{\text{ext}} = 2 \times 10^{-7} \ln \frac{D_2}{r} \text{ H/m}$$

The total inductance of a conductor is

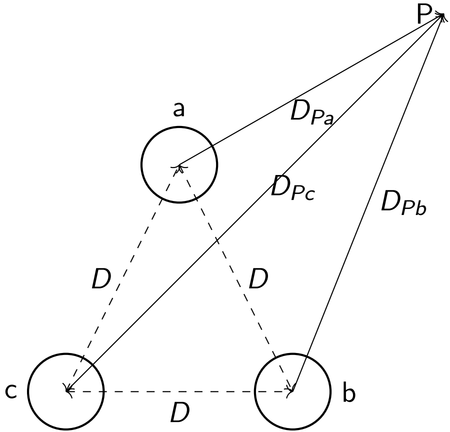
$$L = L_{\text{int}} + L_{\text{ext}} = \frac{1}{2} \times 10^{-7} + 2 \times 10^{-7} \ln \frac{D_2}{r}$$

$$L = 2 \times 10^{-7} \left(\frac{1}{4} + \ln \frac{D_2}{r} \right) \text{ H/m}$$

$$\boxed{L = 2 \times 10^{-7} \left(\ln \frac{D_2}{r'} \right) \text{ H/m}}$$

where $r' = re^{-\frac{1}{4}} = 0.7788 \times r$.

Inductance of three phase lines with symmetrical spacing:
Each conductor has a radius of r and their centers form an equilateral triangle with a distance D between them.



Assuming the currents are balanced.

$$I_a + I_b + I_c = 0$$

The flux linkage of the conductor a due to current I_a including its internal flux and the external flux till the point P is

$$\lambda_{aa} = 2 \times 10^{-7} I_a \left(\ln \frac{D_{Pa}}{r'} \right)$$

The flux linkage with the conductor a due to I_b and I_c between a and the point P, respectively are,

$$\lambda_{ab} = 2 \times 10^{-7} I_b \left(\ln \frac{D_{Pb}}{D} \right)$$

$$\lambda_{ac} = 2 \times 10^{-7} I_c \left(\ln \frac{D_{Pc}}{D} \right)$$

The total flux linkages of the conductor a is,

$$\lambda_a = \lambda_{aa} + \lambda_{ab} + \lambda_{ac}$$

$$\lambda_a = 2 \times 10^{-7} \left(I_a \ln \frac{D_{Pa}}{r'} + I_b \ln \frac{D_{Pb}}{D} + I_c \ln \frac{D_{Pc}}{D} \right)$$

$$\lambda_a = 2 \times 10^{-7} (I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D} + I_c \ln \frac{1}{D} + I_a \ln D_{Pa} + I_b \ln D_{Pb} + I_c \ln D_{Pc})$$

Since $I_b + I_c = -I_a$,

$$\lambda_a = 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} - I_a \ln \frac{1}{D} + I_b \ln \frac{D_{Pb}}{D_{Pa}} + I_c \ln \frac{D_{Pc}}{D_{Pa}} \right)$$

If we move the point P far away, $D_{Pa} \approx D_{Pb} \approx D_{Pc}$. The last two terms in the above equation will be zero.

$$\lambda_a = 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} - I_a \ln \frac{1}{D} \right) = 2 \times 10^{-7} I_a \left(\ln \frac{D}{r'} \right)$$

Hence the inductance of phase a is,

$$\boxed{L_a = 2 \times 10^{-7} \left(\ln \frac{D}{r'} \right) \text{ H/m}}$$

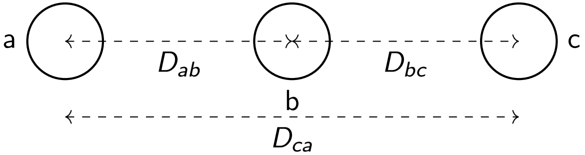
Due to symmetry, the inductances of phases b and c will be the same as a.

$$L_a = L_b = L_c$$

- There is no mutual flux linkage term in λ_a because of $I_b + I_c = -I_a$.
- The phase inductances are equal.

However, it is difficult to place transmission lines with equal spacing in practice.

Inductance of three phase lines with asymmetrical spacing:



Assume they carry balanced currents.

$$I_a + I_b + I_c = 0$$

The flux linkage of conductor *a*

$$\lambda_a = 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D_{ab}} + I_c \ln \frac{1}{D_{ca}} \right)$$

Similarly,

$$\lambda_b = 2 \times 10^{-7} \left(I_b \ln \frac{1}{r'} + I_a \ln \frac{1}{D_{ab}} + I_c \ln \frac{1}{D_{bc}} \right)$$

$$\lambda_c = 2 \times 10^{-7} \left(I_c \ln \frac{1}{r'} + I_b \ln \frac{1}{D_{bc}} + I_a \ln \frac{1}{D_{ca}} \right)$$

The flux linkages and inductance of each phase are not the same. This will result in an unbalanced circuit.

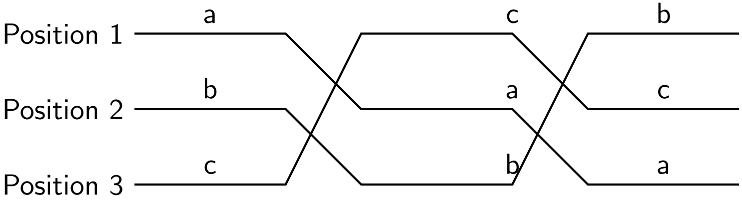


Figure: Transposition

- To avoid this, the positions of the conductors are exchanged at regular intervals. This is called *transposition*.
- This results in each conductor having the same average inductance over the whole cycle.

The average flux linkages of a over a cycle is

$$\lambda_a = \frac{\lambda_{a1} + \lambda_{a2} + \lambda_{a3}}{3}$$

where

$$\lambda_{a1} = 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D_{ab}} + I_c \ln \frac{1}{D_{ca}} \right)$$

$$\lambda_{a2} = 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D_{bc}} + I_c \ln \frac{1}{D_{ab}} \right)$$

$$\lambda_{a3} = 2 \times 10^{-7} \left(I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D_{ca}} + I_c \ln \frac{1}{D_{bc}} \right)$$

$$\lambda_a = \frac{2 \times 10^{-7}}{3} \left(3I_a \ln \frac{1}{r'} + I_b \ln \frac{1}{D_{ab}D_{bc}D_{ca}} + I_c \ln \frac{1}{D_{ab}D_{bc}D_{ca}} \right)$$

Since $I_a = -(I_b + I_c)$,

$$\lambda_a = \frac{2 \times 10^{-7}}{3} \left(3 I_a \ln \frac{1}{r'} - I_a \ln \frac{1}{D_{ab} D_{bc} D_{ca}} \right)$$

$$\lambda_a = 2 \times 10^{-7} I_a \ln \frac{\sqrt[3]{D_{ab} D_{bc} D_{ca}}}{r'} \text{ Wbt/m}$$

The average inductance per phase is

$$L_a = 2 \times 10^{-7} \ln \frac{GMD}{r'} \text{ H/m}$$

where GMD is the geometric mean distance.
$GMD = \sqrt[3]{D_{ab} D_{bc} D_{ca}}$.

Corona:

At extra-high voltage lines, that is, voltages above 220 kV, corona (It is not Covid-19) causes a large problem if the circuit has only one conductor per phase.

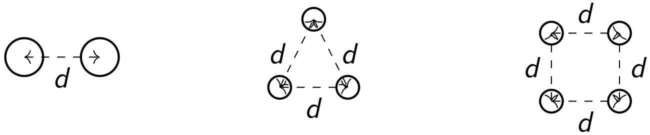
What is corona?

- It occurs when the surface potential gradient of a conductor exceeds the dielectric strength of the surrounding air.
- This causes ionization of the area near the conductor.
- It produces power loss and also causes interference with communication channels.
- It manifests itself with a hissing sound and ozone discharge.

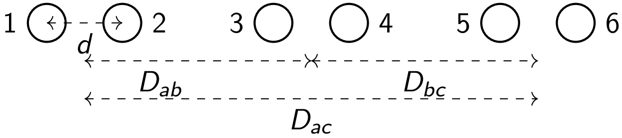
How to avoid this?

The high voltage surface gradient is reduced considerably by having two or more conductors per phase in close proximity. This is called conductor bundling .

Bundled Conductors:



Let us take two conductor bundle case:



It is assumed that the conductors in a phase carry equal current.

$$D_{13} \approx D_{14} \approx D_{ab} \text{ and } D_{15} \approx D_{16} \approx D_{ac}$$

The average flux linkages of the conductor 1 in phase *a* over a cycle is,

$$\lambda_{a1} = 2 \times 10^{-7}\Big(\frac{I_a}{2} \ln \frac{1}{r'} + \frac{I_a}{2} \ln \frac{1}{d} + \frac{I_b}{2} \ln \frac{1}{\sqrt[3]{D_{ab}D_{bc}D_{ca}}} + \frac{I_b}{2} \ln \frac{1}{\sqrt[3]{D_{ab}D_{bc}D_{ca}}} + \frac{I_c}{2} \ln \frac{1}{\sqrt[3]{D_{ab}D_{bc}D_{ca}}} + \frac{I_c}{2} \ln \frac{1}{\sqrt[3]{D_{ab}D_{bc}D_{ca}}}\Big)$$

$$\lambda_{a1} = 2 \times 10^{-7}(I_a \ln \frac{1}{\sqrt{r'd}} + I_b \ln \frac{1}{\sqrt[3]{D_{ab}D_{bc}D_{ca}}} + I_c \ln \frac{1}{\sqrt[3]{D_{ab}D_{bc}D_{ca}}}$$

Since $I_a = -(I_b + I_c)$,

$$\lambda_{a1} = 2 \times 10^{-7} I_a \ln \frac{\sqrt[3]{D_{ab}D_{bc}D_{ca}}}{\sqrt{r'd}}$$

The inductance of the conductor 1 in phase a is

$$L_{a1} = \frac{\lambda_{a1}}{Ia/2} = 4 \times 10^{-7} \ln \frac{\sqrt[3]{D_{ab}D_{bc}D_{ca}}}{\sqrt{r'd}}$$

Also $L_{a2} = L_{a1}$. Since there are two conductors per phase, the inductance per phase

$$L_a = \frac{L_{a1}}{2} = 2 \times 10^{-7} \ln \frac{\sqrt[3]{D_{ab}D_{bc}D_{ca}}}{\sqrt{r'd}} \text{ H/m}$$

Also $L_a = L_b = L_c$.
The inductance per phase is

$$\boxed{L = 2 \times 10^{-7} \ln \frac{GMD}{GMR} \text{ H/m}}$$

where GMD is the geometric mean distance in m and GMR is the geometric mean radius in m.

$$GMD = \sqrt[3]{D_{ab}D_{bc}D_{ca}}$$

For 2-conductor bundle,

$$GMR = \sqrt{r' \times d}$$

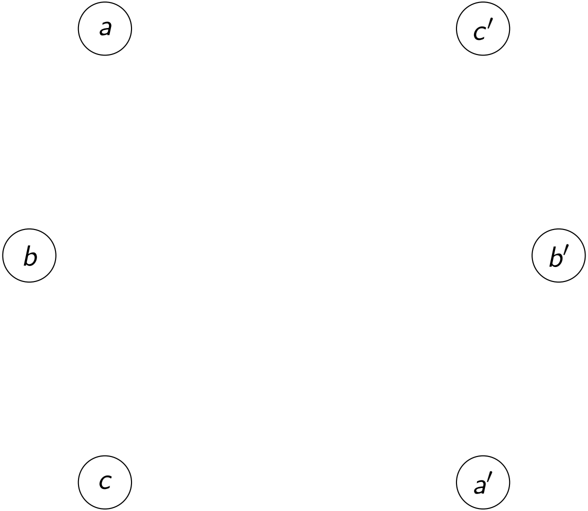
Similarly for 3-conductor bundle,

$$GMR = \sqrt[3]{r' \times d \times d}$$

For 4-conductor bundle,

$$GMR = \sqrt[4]{r' \times d \times d \times \sqrt{2}d}$$

Double Circuit Transmission Line



The inductance per phase is

$$\boxed{L = 2 \times 10^{-7} \ln \frac{GMD}{GMR} \text{ H/m}}$$

where GMD is the geometric mean distance in m and GMR is the geometric mean radius in m.

$$GMD = \sqrt[3]{D_{AB} D_{BC} D_{CA}}$$

where

$$D_{AB} = \sqrt[4]{D_{ab} D_{ab'} D_{a'b} D_{a'b'}}$$

$$D_{BC} = \sqrt[4]{D_{bc} D_{bc'} D_{b'c} D_{b'c'}}$$

$$D_{CA} = \sqrt[4]{D_{ca} D_{ca'} D_{c'a} D_{c'a'}}$$

and

$$GMR = \sqrt[3]{GMR_A GMR_B GMR_C}$$

where

$$GMR_A = \sqrt{r' D_{aa'}}$$

$$GMR_B = \sqrt{r' D_{bb'}}$$

$$GMR_C = \sqrt{r' D_{cc'}}$$

Proximity Effect

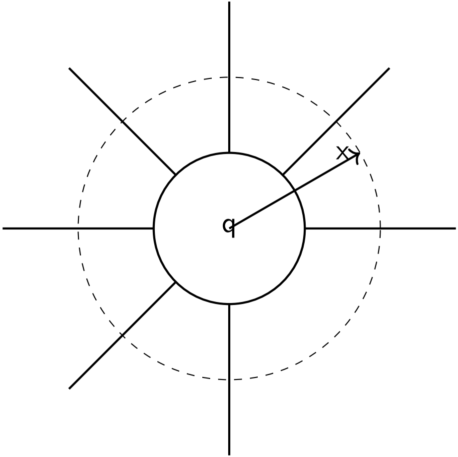
“When an alternating current (AC) flows through a conductor, it creates an associated alternating magnetic field around it. The alternating magnetic field induces eddy currents in adjacent conductors, altering the overall distribution of current flowing through them. The result is that the current is concentrated in the areas of the conductor farthest away from nearby conductors carrying current in the same direction. “

- This increases the resistance.
- The effect will be more as frequency increases.

However, the effect is negligibly small in overhead transmission lines because of spacing between conductors. The effect is dominant in underground cables because of close proximity.

Capacitance of a Single conductor:

Consider a conductor of radius r with a charge of q coulombs.



The capacitance of the conductor is

$$C = \frac{q}{V}$$

Gauss's law :

$$\oiint_A D \cdot da = q_e$$

where

$D =$ electric flux density C/m^2

$da =$ differential area with direction normal to the surface m^2

$A =$ total closed surface area m^2

$q_e =$ net charge enclosed by the surface

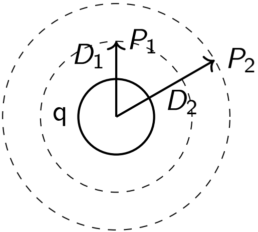
By Gauss's law, the electric flux density at a cylinder of radius x when the conductor has a length of 1 m is

$$D = \frac{q}{A} = \frac{q}{2\pi x} \; C/m^2$$

$$E = \frac{D}{\epsilon_0 \epsilon_r}$$

Where E is the electric field intensity in V/m.
ϵ_0 is the permittivity of free space in F/m (8.854×10^{-12} F/m)
ϵ_r is the relative permittivity of the medium. ($\epsilon_r = 1$ for air)

$$E = \frac{q}{2\pi x \epsilon_0} \text{ V/m}$$



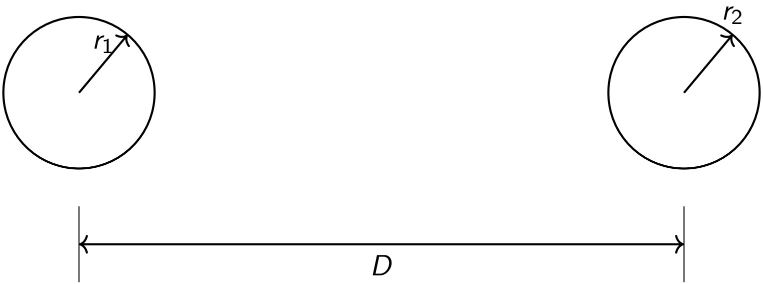
To find the potential difference between the points P_1 and P_2,

$$V_{12} = \int_{D_1}^{D_2} E\,dx = \int_{D_1}^{D_2} \frac{q}{2\pi x \epsilon_0}\,dx$$

$$V_{12} = \frac{q}{2\pi\epsilon_0} \ln \frac{D_2}{D_1} \text{ V}$$

Capacitance of a Single phase line:

Consider two solid round conductors with radii of r1 and r2. Let the conductor 1 carry a charge of q_1 C/m and 2 carry a charge of q_2 C/m.



- The effect of the presence of other conductors on the charge distribution is neglected since $D >> r$.
- The effect of ground on the distribution of flux is neglected since the height of the conductor is much larger than D.

The voltage between the conductors due to q_1 is

$$V_{12}(q_1) = \frac{q_1}{2\pi\epsilon_0} \ln \frac{D}{r_1}$$

The voltage between the conductors due to q_2 is

$$V_{21}(q_2) = \frac{q_2}{2\pi\epsilon_0} \ln \frac{D}{r_2}$$

This can be rewritten as

$$V_{12}(q_2) = \frac{q_2}{2\pi\epsilon_0} \ln \frac{r_2}{D}$$

By superposition,

$$V_{12} = V_{12}(q_1) + V_{12}(q_2)$$

$$V_{12} = \frac{q_1}{2\pi\epsilon_0} \ln \frac{D}{r_1} + \frac{q_2}{2\pi\epsilon_0} \ln \frac{r_2}{D} \text{ V}$$

For a single phase line, $q_1 = -q_2 = q$.

$$V_{12} = \frac{q}{2\pi\epsilon_0} \ln \frac{D}{r_1} - \frac{q}{2\pi\epsilon_0} \ln \frac{r_2}{D} = \frac{q}{2\pi\epsilon_0} \ln \frac{D^2}{r_1 r_2} \text{ V}$$

Assuming $r_1 = r_2 = r$.

$$V_{12} = \frac{q}{2\pi\epsilon_0} \ln \frac{D^2}{r^2} = \frac{q}{\pi\epsilon_0} \ln \frac{D}{r} \text{ V}$$

The capacitance between the conductors is

$$C_{12} = \frac{\pi\epsilon_0}{\ln \frac{D}{r}} \text{ F/m}$$

For the transmission line modeling, the capacitance is defined between the conductor and neutral.

$$C_{1n} = C_{2n} = 2C_{12} = \frac{2\pi\epsilon_0}{\ln \frac{D}{r}} \text{ F/m}$$

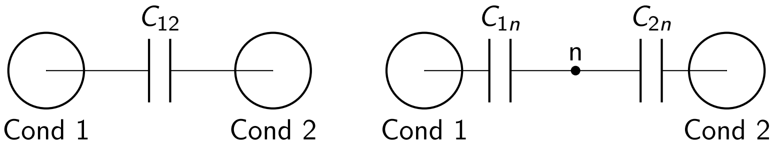
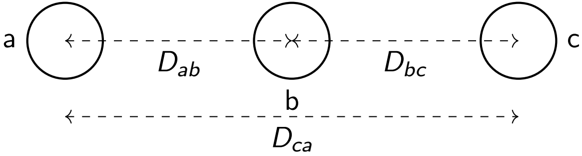


Figure: Capacitance between conductors and Equivalent Capacitance to Ground

The value of capacitance is

$$\boxed{C = \frac{2\pi\epsilon_0}{\ln\frac{D}{r}}\ \text{F/m}}$$

Capacitance of a Three-Phase Transposed Line:



Assume the system is balanced.

$$q_a + q_b + q_c = 0$$

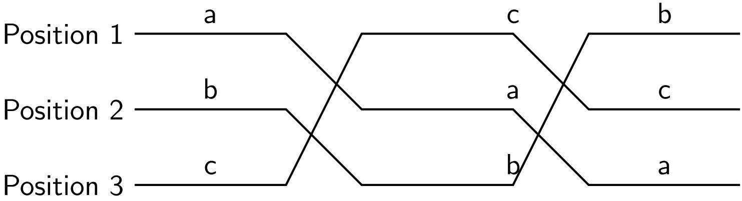


Figure: Transposition

Using superposition, the voltage V_{ab} for the first, second and third sections of the transposition are given respectively as

$$V_{ab}(1) = \frac{q_a}{2\pi\epsilon_0} \ln \frac{D_{ab}}{r} + \frac{q_b}{2\pi\epsilon_0} \ln \frac{r}{D_{ab}} + \frac{q_c}{2\pi\epsilon_0} \ln \frac{D_{bc}}{D_{ca}} \text{ V}$$

$$V_{ab}(2) = \frac{q_a}{2\pi\epsilon_0} \ln \frac{D_{bc}}{r} + \frac{q_b}{2\pi\epsilon_0} \ln \frac{r}{D_{bc}} + \frac{q_c}{2\pi\epsilon_0} \ln \frac{D_{ca}}{D_{ab}} \text{ V}$$

$$V_{ab}(3) = \frac{q_a}{2\pi\epsilon_0} \ln \frac{D_{ab}}{r} + \frac{q_b}{2\pi\epsilon_0} \ln \frac{r}{D_{ab}} + \frac{q_c}{2\pi\epsilon_0} \ln \frac{D_{ab}}{D_{bc}} \text{ V}$$

The average value of the voltage

$$V_{ab} = \frac{V_{ab}(1) + V_{ab}(2) + V_{ab}(3)}{3}$$

$$V_{ab} = \frac{1}{3 \times 2\pi\epsilon_0}(q_a \ln \frac{D_{ab}D_{bc}D_{ca}}{r^3} + q_b \ln \frac{r^3}{D_{ab}D_{bc}D_{ca}} + q_c \ln \frac{D_{ab}D_{bc}D_{ca}}{D_{ab}D_{bc}D_{ca}})$$

$$V_{ab} = \frac{1}{2\pi\epsilon_0}\left(q_a \ln \frac{\sqrt[3]{D_{ab}D_{bc}D_{ca}}}{r} + q_b \ln \frac{r}{\sqrt[3]{D_{ab}D_{bc}D_{ca}}}\right)$$

$$V_{ab} = \frac{1}{2\pi\epsilon_0}\left(q_a \ln \frac{GMD}{r} + q_b \ln \frac{r}{GMD}\right)$$

where $GMD = \sqrt[3]{D_{ab}D_{bc}D_{ca}}$.
Similarly,

$$V_{ac} = \frac{1}{2\pi\epsilon_0}\left(q_a \ln \frac{GMD}{r} + q_c \ln \frac{r}{GMD}\right)$$

$$V_{ab} + V_{ac} = \frac{1}{2\pi\epsilon_0}\left(2q_a \ln \frac{GMD}{r} + (q_b + q_c) \ln \frac{r}{GMD}\right)$$

Since $q_a + q_b + q_c = 0$,

$$V_{ab} + V_{ac} = \frac{1}{2\pi\epsilon_0}\left(2q_a \ln\frac{GMD}{r} - q_a \ln\frac{r}{GMD}\right) = \frac{3}{2\pi\epsilon_0} q_a \ln\frac{GMD}{r}$$

For a set of balanced voltages,

$$V_{ab} = V_{an} - V_{bn} = V_{an}\underline{/0^\circ} - V_{an}\underline{/-120^\circ}$$

$$V_{ac} = V_{an} - V_{cn} = V_{an}\underline{/0^\circ} - V_{an}\underline{/-240^\circ}$$

$$V_{ab} + V_{ac} = 3V_{an}\underline{/0^\circ}$$

$$V_{an} = \frac{V_{ab} + V_{ac}}{3} = \frac{1}{2\pi\epsilon_0} q_a \ln\frac{GMD}{r}\ \text{V}$$

The capacitance to neutral is

$$\boxed{C = \frac{2\pi\epsilon_0}{\ln\frac{GMD}{r}}\ \text{F/m}}$$

In general,

$$\boxed{C = \frac{2\pi\epsilon_0}{\ln \frac{GMD}{GMR}} \text{ F/m}}$$

where

$$GMD = \sqrt[3]{D_{ab}D_{bc}D_{ca}}$$

For a single conductor per phase

$$GMR = r$$

For 2-conductor bundle,

$$GMR = \sqrt{r \times d}$$

Similarly for 3-conductor bundle,

$$GMR = \sqrt[3]{r \times d \times d}$$

For 4-conductor bundle,

$$GMR = \sqrt[4]{r \times d \times d \times \sqrt{2}d}$$

Typical Values

Table: Overhead lines

Nominal Voltage	230 kV	345 kV	500 kV	765 kV	1100 kV
r (Ω/km)	0.050	0.037	0.028	0.012	0.005
x (Ω/km)	0.488	0.367	0.325	0.329	0.292
b ($\mu\mho$/km)	3.371	4.518	5.200	4.978	5.544

Table: Underground cables

Nominal Voltage	115 kV	230 kV	500 kV
r (Ω/km)	0.0590	0.0277	0.0128
x (Ω/km)	0.3026	0.3388	0.2454
b ($\mu\mho$/km)	230.4	245.6	96.5

Source : P. Kundur, "Power System Stability and Control".