Line Models

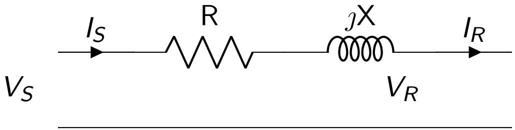
There are three models based on the length of lines.

1. Short line - line length is less than 80 kM (50 miles)
2. Medium line - line length is between 80 kM (50 miles) and 240 kM (150 miles)
3. Long line - line length is more than 240 kM (150 miles)

- Lumped parameters are used in the short and medium line models.
- Distributed parameters are used in the long line model.

Short Line Model:

- Shunt capacitance is neglected.
- Lumped parameters are used.



The total impedance of the line is

$$Z = R + \jmath X \ \Omega$$

where

$$R = r \times l; \quad X = x \times l$$

r and x are resistance and reactance per kM and l is the length of the line in kM.

$$V_S = V_R + I_R Z$$

$$I_S = I_R$$

By comparing with $ABCD$ parameters,

$$A = 1;\; B = Z;\; C = 0;\; D = 1$$

For symmetrical networks, $A = D$.
For reciprocal networks, $AD - BC = 1$.

$$\%\text{Regn} = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100$$

Since $V_{R,NL} = V_S$,

$$\%\text{Regn} = \frac{|V_S| - |V_{R.FL}|}{|V_{R,FL}|} \times 100$$

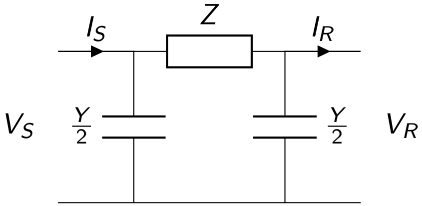
Medium Line Model:

- Shunt capacitance is considered.
- Lumped parameters are used.

There are two different representations.

1. nominal-π network
2. nominal -T network

1. nominal -π network
   This representation is used in load flow studies.



where $Y = \jmath\omega C$ and C is the total capacitance of the line in F.

$$I_S = I_R + V_R\frac{Y}{2} + V_S\frac{Y}{2}$$

$$V_S = V_R + (I_R + V_R\frac{Y}{2})Z = V_R(1 + \frac{YZ}{2}) + ZI_R$$

Substituting V_S in I_S,

$$I_S = V_R Y(1 + \frac{YZ}{4}) + I_R(1 + \frac{YZ}{2})$$

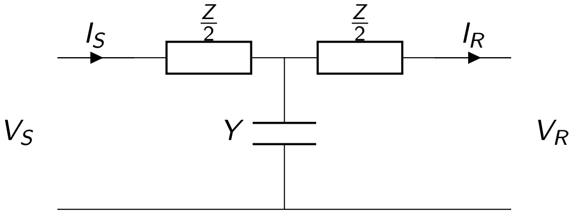
By comparing with $ABCD$ parameters,

$$A = (1 + \frac{YZ}{2}) = D$$

$$B = Z$$

$$C = Y(1 + \frac{YZ}{4})$$

2. nominal -T network



$$I_S = I_R + (V_R + I_R\frac{Z}{2})Y = YV_R + (1 + \frac{YZ}{2})I_R$$

$$V_S = V_R + I_R\frac{Z}{2} + I_S\frac{Z}{2}$$

Substituting I_S in V_S,

$$V_S = (1 + \frac{YZ}{2})V_R + Z(1 + \frac{YZ}{4})I_R$$

By comparing with $ABCD$ parameters,

$$A = (1 + \frac{YZ}{2}) = D$$

$$B = Z(1 + \frac{YZ}{4})$$

$$C = Y$$

$$\%\text{Regn} = \frac{|V_{R,NL}| - |V_{R,FL}|}{|V_{R,FL}|} \times 100$$

Since $V_{R,NL} = \dfrac{V_S}{A}$,

$$\%\text{Regn} = \frac{\dfrac{|V_S|}{|A|} - |V_{R,FL}|}{|V_{R,FL}|} \times 100$$

Example 1 (Grainger and Stevenson 6.12) :
A 60 Hz three-phase transmission line is 100 miles long. It has a total series impedance of $35 + \jmath 140\ \Omega$ and a shunt admittance of $930 \times 10^{-6} \angle 90^{\circ}\ \mho$. It delivers 40 MW at 220 kV with 0.9 power factor lagging. Use nominal -π model.

1. Determine the sending end voltage.
2. Find the voltage regulation.

$$V_S = AV_R + BI_R;\ I_S = CV_R + DI_R$$

In nominal-π model,

$$A = (1 + \frac{YZ}{2}) = (1 + \frac{\jmath 930 \times 10^{-6} \times (35 + \jmath 140)}{2}) = 0.9344 + \jmath 0.01395$$

$$B = Z = (35 + \jmath 140)\ \Omega$$

$$V_{R,Phase} = \frac{220}{\sqrt{3}} = 127.01\ \text{kV}$$

$$|I_R| = \frac{40 \times 10^3}{\sqrt{3} \times 220 \times 0.9} = 116.64 \text{ A}$$

$$I_R = 116.64\angle{-25.84^\circ} \text{ A}$$

On substituting,

$$V_{S,Phase} \approx 130\angle 6.6^\circ \text{ kV}$$

$$V_{S,Line} = \sqrt{3} \times 130 \approx 225 \text{ kV}$$

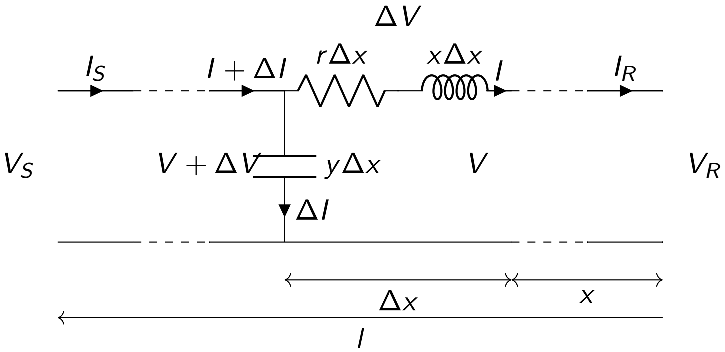
The % voltage regulation

$$= \frac{\frac{|V_S|}{|A|} - |V_{R,FL}|}{|V_{R,FL}|} \times 100$$

$$= \frac{\frac{130}{|0.9344 + \jmath 0.01395|} - 127}{127} \times 100 = 9.53 \%$$

Long Line Model:

- Distributed parameters are used.
- This gives more accuracy.



Let Δx be a very small element of the line. The impedance and admittance of the section are $z\Delta x$ and $y\Delta x$, respectively. From the circuit,

$$\Delta V = Iz\Delta x$$

$$\frac{\Delta V}{\Delta x} = Iz$$

As $\Delta x \to 0$,

$$\frac{dV}{dx} = Iz$$

Similarly,

$$\Delta I = Vy\Delta x$$

Pursuing the same steps,

$$\frac{dI}{dx} = Vy$$

Let us differentiate them w.r.to x,

$$\frac{d^2V}{dx^2} = z\frac{dI}{dx}$$

$$\frac{d^2I}{dx^2} = y\frac{dV}{dx}$$

On substituting the values of dI/dx and dV/dx in the above equations,

$$\frac{d^2V}{dx^2} = yzV$$

$$\frac{d^2I}{dx^2} = yzI$$

On solving the voltage equation,

$$V = A_1 e^{\sqrt{yz}x} + A_2 e^{-\sqrt{yz}x}$$

Since,

$$I = \frac{1}{z} \times \frac{dV}{dx}$$

$$I = \frac{1}{\sqrt{z/y}} A_1 e^{\sqrt{yz}x} - \frac{1}{\sqrt{z/y}} A_2 e^{-\sqrt{yz}x}$$

To find the constants A_1 and A_2, we use the following conditions.

$$V = V_R \quad \text{and} \quad I = I_R \quad \text{when } x{=}0$$

On substitution,

$$V_R = A_1 + A_2$$

$$I_R = \frac{1}{\sqrt{z/y}}(A_1 - A_2)$$

Let $Z_c = \sqrt{z/y}$.

$$A_1 = \frac{V_R + I_R Z_c}{2}$$

$$A_2 = \frac{V_R - I_R Z_c}{2}$$

Let $\gamma = \sqrt{yz}$.

$$V = \frac{V_R + I_R Z_c}{2} e^{\gamma x} + \frac{V_R - I_R Z_c}{2} e^{-\gamma x}$$

$$I = \frac{V_R/Z_c + I_R}{2} e^{\gamma x} - \frac{V_R/Z_c - I_R}{2} e^{-\gamma x}$$

where $Z_c = \sqrt{z/y}$ and is called the **characteristic impedance** of the line and $\gamma = \sqrt{zy}$ and is called the **propagation constant**.

When $x = l$, $V = V_S$ and $I = I_S$.

$$V_S = \frac{V_R + I_R Z_c}{2} e^{\gamma l} + \frac{V_R - I_R Z_c}{2} e^{-\gamma l}$$

$$I_S = \frac{V_R/Z_c + I_R}{2} e^{\gamma l} - \frac{V_R/Z_c - I_R}{2} e^{-\gamma l}$$

Since $\sinh \gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{2}$ and $\cosh \gamma l = \frac{e^{\gamma l} + e^{-\gamma l}}{2}$.

$$V_S = V_R \cosh \gamma l + I_R Z_c \sinh \gamma l$$

$$I_S = V_R \frac{\sinh \gamma l}{Z_c} + I_R \cosh \gamma l$$

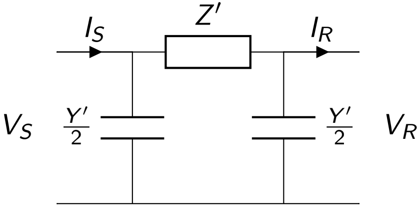
$ABCD$ parameters are,

$$A = \cosh \gamma l = D$$

$$B = Z_c \sinh \gamma l$$

$$C = \frac{\sinh \gamma l}{Z_c}$$

Equivalent-π Representation of a Long Line:



$ABCD$ parameters of the above representation are

$$A = (1 + \frac{Y'Z'}{2}) = D$$

$$B = Z'$$

$$C = Y'(1 + \frac{Y'Z'}{4})$$

By comparing this with the $ABCD$ parameters of a long line.

$$Z' = Z_c \sinh \gamma l = \sqrt{\frac{z}{y}} \sinh \gamma l = zl \frac{\sinh \gamma l}{l\sqrt{yz}} = Z \frac{\sinh \gamma l}{\gamma l} \ \Omega$$

$$\cosh\gamma l = (1 + \frac{Y'Z'}{2}) = 1 + \frac{Y'}{2} Z_c \sinh\gamma l$$

Rearranging the above equation,

$$\frac{Y'}{2} = \frac{1}{Z_c} \frac{\cosh\gamma l - 1}{\sinh\gamma l}$$

By using the following identity,

$$\tanh\frac{\gamma l}{2} = \frac{\cosh\gamma l - 1}{\sinh\gamma l}$$

$$\frac{Y'}{2} = \frac{1}{Z_c}\tanh\frac{\gamma l}{2} = \sqrt{\frac{y}{z}}\tanh\frac{\gamma l}{2}$$

$$\frac{Y'}{2} = \frac{Y}{2}\frac{\tanh\frac{\gamma l}{2}}{\frac{\gamma l}{2}}$$

When l becomes small, $\sinh \gamma l \approx \gamma l$ and $\tanh \frac{\gamma l}{2} \approx \frac{\gamma l}{2}$.

$$Z' \approx Z; \quad Y' \approx Y$$

This means that when the length of the line is small, modeling of lines with lumped parameters is enough. However, if the line length increases, lumped parameters model gives inaccurate results.

How small should it be ?

Interpretation of the Equations:

$$V = \frac{V_R + I_R Z_c}{2} e^{\gamma x} + \frac{V_R - I_R Z_c}{2} e^{-\gamma x}$$

$$I = \frac{V_R/Z_c + I_R}{2} e^{\gamma x} - \frac{V_R/Z_c - I_R}{2} e^{-\gamma x}$$

Z_c and γ are complex quantities. Let

$$\gamma = \alpha + \jmath\beta$$

where α is the attenuation constant in nepers/unit length. β is the phase constant in radians/unit length.

$$V = \frac{V_R + I_R Z_c}{2} e^{\alpha x} e^{\jmath\beta x} + \frac{V_R - I_R Z_c}{2} e^{-\alpha x} e^{-\jmath\beta x}$$

$$I = \frac{V_R/Z_c + I_R}{2} e^{\alpha x} e^{\jmath\beta x} - \frac{V_R/Z_c - I_R}{2} e^{-\alpha x} e^{-\jmath\beta x}$$

- The first term of the above equations increases in magnitude and advances in phase from the receiving end. It is called the *incident wave*.
- The second term of the above equations decreases in magnitude and retards in phase from the receiving end. It is called the *reflected wave*

For a lossless line, $R = 0$. Z_c is a pure real number and is called as the **surge impedance**. γ is a pure imaginary number.

$$Z_c = \sqrt{\frac{L}{C}}\,\Omega$$

$$\gamma = \jmath\beta$$

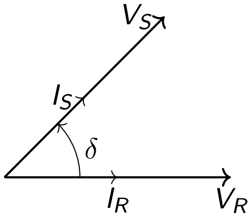
If a line is terminated with the surge impedance, $I_R = \frac{V_R}{Z_c}$.

$$V = V_R e^{\jmath\beta x}$$

$$I = I_R e^{\jmath\beta x}$$

- There is no reflected wave of either voltage or current.
- The magnitudes of voltage and current are constant. But they are phase displaced by βl radian.

This line is called a flat line or an infinite line.



The phase angle between V_S and V_R

$$\delta = \beta l$$

A wavelength λ is the distance along a line between two points of a wave which differ in phase by 360° or 2π radian.

If β is the phase shift in radian per kM, the wavelength in kM is

$$\lambda = \frac{2\pi}{\beta}$$

The velocity of propagation of a wave in km/sec

$$\text{Velocity} = \lambda f = \frac{2\pi f}{\beta}$$

For a lossless line of length l,

$$\beta = \omega\sqrt{LC}$$

$$\lambda = \frac{1}{f\sqrt{LC}} \text{ km}$$

$$\text{Velocity} = \frac{1}{\sqrt{LC}} \text{ km/sec}$$

- The actual velocity of the propagation of a wave along a line will be close to the speed of light.

- If $f = 50$ Hz, the wavelength (λ) will be 6000 km.

$$|\gamma l| = |\beta l| = \frac{2\pi l}{\lambda}$$

γl will be small when l/λ is small. We have already seen that when γl is small, the lumped circuit model is enough.

Therefore, lumped parameter medium line model may be used for lines up to 400 km.

Surge Impedance Loading (SIL)

SIL of a line is the power delivered by a line to a purely resistive load equal to its surge impedance.

The line current will be

$$I_L = \frac{|V_L|}{\sqrt{3} \times \sqrt{\frac{L}{C}}} \quad \text{A}$$

where $|V_L|$ is the line to line voltage at the load. Since the load is pure resistance,

$$\text{SIL} = \sqrt{3} \times |V_L| \times \frac{|V_L|}{\sqrt{3} \times \sqrt{\frac{L}{C}}} \quad \text{W}$$

If $|V_L|$ is in kV,

$$\text{SIL} = \frac{|V_L|^2}{\sqrt{\frac{L}{C}}} \, \text{MW}$$

Example 2 (Kothari Example 5.5) :
A three-phase 50 Hz transmission line is 400 km long. The voltage at the sending end is 220 kV. The line parameters are r =0.125 Ω/km, x=0.4 Ω/km and $y = 2.8 \times 10^{-6}$ ℧/km Find the following:

1. the sending end current and the receiving end voltage when there is no load on the line.
2. the maximum permissible line length if the receiving end no load voltage is not to exceed 235 kV.

1. Let us use both nominal-π (lumped) and long line models.
   1.1 nominal - π Model :

$$Z = z \times l = (0.125 + \jmath 0.4) \times 400 = (50 + \jmath 160)\ \Omega$$

$$Y = y \times l = \jmath 2.8 \times 10^{-6} \times 400 = \jmath 1.12 \times 10^{-3}\ ℧$$

When I_R =0,

$$V_S = AV_R; \quad I_S = CV_R$$

$$A = (1 + \frac{YZ}{2}) = 0.91 + \jmath 0.028$$

$$C = Y(1 + \frac{YZ}{4}) = \jmath 0.011$$

$$|V_R| = \frac{|V_S|}{|A|} = \frac{220/\sqrt{3}}{0.91} = 139.45 \text{ kV (phase)}$$

$$\boxed{|V_R| = 241.54 \text{ kV (line)}}$$

$$|I_S| = |C||V_R| = 149.2 \text{ A}$$

1.2 Long Line Model :

$$\gamma = \sqrt{zy} = \sqrt{(0.125 + \jmath 0.4) \times \jmath 2.8 \times 10^{-6}} = 0.0002 + \jmath 0.0011$$

$$Z_c = \sqrt{\frac{z}{y}} = 382.44 - \jmath 58.365 \ \Omega$$

$$V_S = \cosh \gamma l V_R \qquad I_S = V_R \frac{\sinh \gamma l}{Z_c}$$

$$V_R = 139.27 \text{ kV (phase)}$$

$$\boxed{V_R = 241.23 \text{ kV (line)}}$$

$$I_S = 151.4 \text{ A}$$

- Even the lumped model gives almost the same results.
- But the receiving end voltage is higher than the sending end voltage. Why?

2. To find the maximum permissible length, let us use medium line model.

$$|A| = \frac{220/\sqrt{3}}{235/\sqrt{3}}$$

$$|A| = 0.9362$$

Since l is unknown,

$$A = (1 + zy \times l^2/2) = 1 + ((0.125 + \jmath 0.4) \times \jmath 2.8 \times 10^{-6} \times l^2)/2$$

$$A = (1 - 0.56 \times 10^{-6} l^2) + \jmath 0.75 \times 10^{-6} l^2$$

Neglecting the imaginary part (it is very small),

$$|A| \approx 1 - 0.56 \times 10^{-6} l^2$$

$$l = 338 \text{ km}$$

Ferranti Effect:

When a long line is operating under no load or light load condition, the receiving end voltage is greater than the sending end voltage. This is known as Ferranti-effect.

Assume No load condition. When $x = l$ and $I_R = 0$,

$$V_S = \frac{V_R}{2} e^{\alpha l} e^{\jmath \beta l} + \frac{V_R}{2} e^{-\alpha l} e^{-\jmath \beta l}$$

At $l = 0$, $V = V_R$.

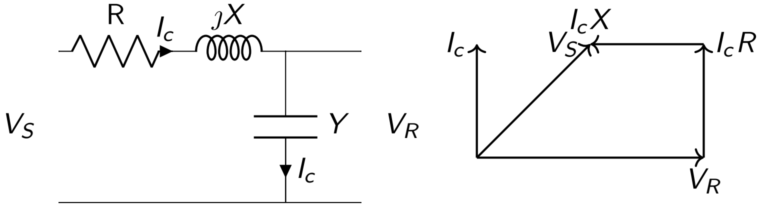
$$V_R = \frac{V_R}{2} + \frac{V_R}{2}$$

As l increases

- the incident voltage increases in magnitude and advances in phase by βl.
- the reflected voltage decreases in magnitude and retards in phase by the same angle.

The sum of these two components of sending end voltage gives a voltage which is smaller than V_r.

Let us consider the lumped parameter model to study this.



If the reactive power generated at a point is more than the reactive power absorbed, the voltage at that point becomes higher than the normal value and vice versa.

- At no-load or light loaded conditions, the capacitance is predominant.

- Since the capacitors produce reactive power, the line generates more reactive power than consumption (Inductors consume).
- To avoid this problem, inductors are connected in parallel. When load on the line increases, they will be disconnected. Otherwise, consumption of reactive power increases and the voltage will drop.

This can also be explained as follows:

- If the transmission line is loaded with the surge impedance, no net reactive power and the voltage is flat.
- If it is loaded above the surge impedance loading, it needs reactive power and hence the voltage drops at the receiving end.
- If it is loaded below the surge impedance loading, it produces more reactive power and hence the voltage increases at the receiving end.

Typical Parameters

Table: Overhead lines

Nominal Voltage	230 kV	345 kV	500 kV	765 kV	1100 kV
r (Ω/km)	0.050	0.037	0.028	0.012	0.005
x (Ω/km)	0.488	0.367	0.325	0.329	0.292
b ($\mu\mho$/km)	3.371	4.518	5.200	4.978	5.544
Z_c (Ω)	380	285	250	257	230
SIL (MW)	140	420	1000	2280	5260

Table: Underground cables

Nominal Voltage	115 kV	230 kV	500 kV
r (Ω/km)	0.0590	0.0277	0.0128
x (Ω/km)	0.3026	0.3388	0.2454
b ($\mu\mho$/km)	230.4	245.6	96.5
Z_c (Ω)	36.2	37.1	50.4
SIL (MW)	365	1426	4960

Source : P. Kundur, "Power System Stability and Control".

Power Flow Through a Transmission Line:

The power flow through a line can be found using $ABCD$ parameters.

$$V_S = AV_R + BI_R$$

Solving for I_R,

$$I_R = \frac{V_S - AV_R}{B}$$

Substituting this in the following equation,

$$I_S = CV_R + DI_R$$

We get

$$I_S = \frac{D}{B}V_S - \frac{1}{B}V_R$$

Let

$$V_S = |V_S|\angle\delta \quad V_R = |V_R|\angle 0^\circ$$

$$A = |A|\angle\alpha \quad B = |B|\angle\beta$$

On Substitution,

$$I_R = \frac{|V_S|\angle\delta - \beta}{|B|} - \frac{|A||V_R|\angle\alpha - \beta}{|B|}$$

$$I_S = \frac{|D||V_S|\angle\alpha + \delta - \beta}{|B|} - \frac{|V_R|\angle-\beta}{|B|}$$

The per phase complex power at the sending end and the receiving end are

$$S_S = V_S I_S^*$$

$$S_R = V_R I_R^*$$

$$P_S + \jmath Q_S = \frac{|D||V_S|^2\angle\beta - \alpha}{|B|} - \frac{|V_S||V_R|\angle\beta + \delta}{|B|}$$

$$P_R + \jmath Q_R = \frac{|V_R||V_S|\angle\beta - \delta}{|B|} - \frac{|A||V_R|^2\angle\beta - \alpha}{|B|}$$

The per phase real and reactive power at the sending end are

$$P_S = \frac{|D||V_S|^2\cos(\beta-\alpha)}{|B|} - \frac{|V_S||V_R|\cos(\beta+\delta)}{|B|}$$

$$Q_S = \frac{|D||V_S|^2\sin(\beta-\alpha)}{|B|} - \frac{|V_S||V_R|\sin(\beta+\delta)}{|B|}$$

The per phase real and reactive power at the receiving end are

$$P_R = \frac{|V_R||V_S|\cos(\beta-\delta)}{|B|} - \frac{|A||V_R|^2\cos(\beta-\alpha)}{|B|}$$

$$Q_R = \frac{|V_R||V_S|\sin(\beta-\delta)}{|B|} - \frac{|A||V_R|^2\sin(\beta-\alpha)}{|B|}$$

Let us find P_S, Q_S, P_R and Q_R for short lines with $R = 0$.
In short lines,

$$A = 1\angle 0^\circ \quad B = |Z|\angle\theta$$

Since $R = 0$,

$$B = |X|\angle 90^\circ$$

Substituting A and B,

$$\boxed{P_S = \frac{|V_S||V_R|}{|X|}\sin\delta}$$

$$\boxed{P_R = \frac{|V_R||V_S|}{|X|}\sin\delta}$$

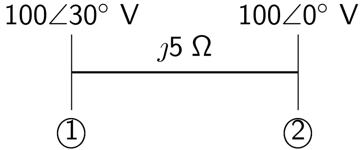
$$\boxed{Q_S = \frac{|V_S|^2}{|X|} - \frac{|V_S||V_R|\cos\delta}{|X|}}$$

$$\boxed{Q_R = \frac{|V_R||V_S|\cos\delta}{|X|} - \frac{|V_R|^2}{|X|}}$$

where $|V_S|$ and $|V_R|$ are per phase sending end and receiving end voltages.

Do you remember where we saw these equations?

Example 3 :



The real and reactive power supplied by bus 1 are

$$P_1 = \frac{|V_1||V_2|}{|X|}\sin\delta = \frac{100 \times 100}{5}\sin(30^\circ) = 1000 \text{ Watts}$$

$$Q_1 = \frac{|V_1|^2}{|X|} - \frac{|V_1||V_2|\cos\delta}{|X|} = \frac{100^2}{5} - \frac{100 \times 100\cos(30^\circ)}{5} = 268 \text{ VAr}$$

The real and reactive power received by bus 2 are

$$P_2 = \frac{|V_1||V_2|}{|X|}\sin\delta = \frac{100 \times 100}{5}\sin(30^\circ) = 1000 \text{ Watts}$$

$$Q_2 = \frac{|V_1||V_2|\cos\delta}{|X|} - \frac{|V_2|^2}{|X|} = \frac{100 \times 100\cos(30^\circ)}{5} - \frac{100^2}{5} = -268 \text{ VAr}$$

Load Modelling

Loads can be categorized as

1. Static Loads
   - Heating Loads
   - Lighting Loads
2. Dynamic Loads
   - Synchronous Motors
   - Induction Motors

We represent loads using the constant power type in this course.