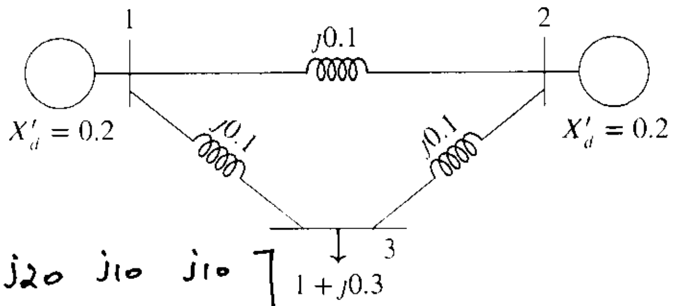
Indian Institute of Technology Patna
Department of Electrical Engineering
EE549 - Power System Dynamics and Control
Spring - 2022
End Semester Exam – Solution.
April 27, 2022

Time : 3 Hour Marks : 50

1. The voltage at the node 3 is $0.98\angle 10^{\circ}$. Reduce the bus admittance matrix. Assume the load is of constant impedance type.



$$Y_{bus} = \begin{bmatrix} -j20 & j10 & j10 \\ j10 & -j20 & j10 \\ j10 & j10 & -j20 \end{bmatrix}$$

Load Admittance is

$$y_L = \frac{1 - j0.3}{(0.98)^2} = 1.0412 - j0.3124$$

After adding the load admittance, and the reciprocal of the generator reactances to the Y_{bus} matrix, we get the modified matrix.

$$Y_{bus}' = \begin{bmatrix} -j25 & j10 & j10 \\ j10 & -j25 & j10 \\ j10 & j10 & 1.0412 - j20.3124 \end{bmatrix}$$

The new Y_{bus} matrix after adding the generator nodes,

$$Y = \begin{bmatrix} Y_{GG} & Y_{GN} \\ Y_{GN}^T & Y_{bus}' \end{bmatrix}$$

$$Y_{GG} = \begin{bmatrix} -5j & 0 \\ 0 & -5j \end{bmatrix}$$

$$Y_{GN} = \begin{bmatrix} 5j & 0 & 0 \\ 0 & 5j & 0 \end{bmatrix}$$

$$Y_{GN}^T = \begin{bmatrix} 5j & 0 \\ 0 & 5j \\ 0 & 0 \end{bmatrix}$$

The reduced Y_{bus} matrix

$$Y_{red} = Y_{GG} - Y_{GN}\left(Y_{bus}'\right)^{-1} Y_{GN}^T = \begin{bmatrix} 0.2324 - j2.2521 & 0.2324 + j2.0324 \\ 0.2324 + j2.0324 & 0.2324 - j2.2521 \end{bmatrix} \quad 1$$

2. Consider a SMIB system.

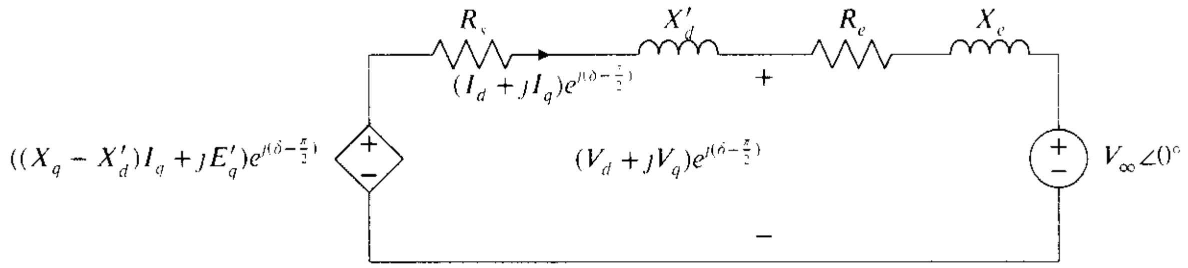


Figure 1: Synchronous Machine Flux Decay Model

(a) Determine T_e.

(b) Express T_e in terms of δ and E_q'.

a) $T_e = \text{Real}\left\{((X_q - X_d')I_q + jE_q')e^{j(\delta-\pi/2)}(I_d + jI_q)^* e^{j(\pi/2-\delta)}\right\}$

$$\boxed{T_e = (X_q - X_d')I_d I_q + E_q' I_q}$$

3

b) $$(I_d + jI_q)e^{j(\delta-\pi/2)} = \frac{(V_d + jV_q)e^{j(\delta-\pi/2)} - V_\infty}{R_e + jX_e}$$

Equating real and imaginary parts.

$$\left.\begin{aligned} I_d R_e - I_q X_e &= V_d - V_\infty \sin\delta \\ I_q R_e + I_d X_e &= V_q - V_\infty \cos\delta \end{aligned}\right\} \quad - (1)$$

2

We know that

$$\left.\begin{aligned} V_d &= X_q I_q - R_s I_d \\ V_q &= -R_s I_q - X_d' I_d + E_q' \end{aligned}\right\} \quad - (2)$$

From (1) and (2)

(A) — $I_d = \frac{1}{\Delta}\left\{(R_s + R_e)V_\infty \sin\delta + (X_e + X_q)(E_q' - V_\infty \cos\delta)\right\}$

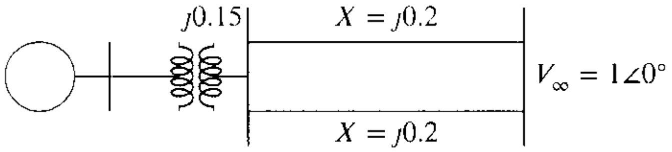
(B) — $I_q = \frac{1}{\Delta}\left\{(X_e + X_d')V_\infty \sin\delta + (R_s + R_e)(E_q' - V_\infty \cos\delta)\right\}$

where $\Delta = (R_e + R_s)^2 + (X_e + X_q)(X_e + X_d')$

$$\boxed{T_e = (X_q - X_d')\ (A) \times (B) + E_q' \times (B)}$$

3

3. Consider a SMIB system. The synchronous generator is represented using the flux decay model. The generator parameters are as follows:



$$X_d = 1.8 \quad X_q = 1.7 \quad X_d' = 0.17 \quad R_a = 0.003 \quad T_{do}' = 0.4\text{ s} \quad H = 4 \quad D = 0$$

If the terminal conditions are

$$P_t = 1.0 \quad |V_t| = 1.0$$

(a) Find the initial conditions.

(b) Prove that $T_e = P_t + I_a^2 R_a$.

$\bar{V}_t = |V_t| \angle \alpha$

$$P_t = \frac{V_t V_\infty}{0.25} \sin \alpha$$

$$1 = \frac{1 \times 1}{0.25} \sin \alpha$$

$\boxed{\alpha = 0.2527 \text{ rad} \quad (14.48^\circ \text{ deg})}$

$$\bar{I}_t = \frac{\bar{V}_t - V_\infty}{j0.25}$$

$$= 1.0081 \angle 0.1264 \text{ (rad)}$$

$\boxed{\gamma = 0.1264}$

$$\bar{E}_q = \bar{V}_t + \bar{I}_t (R_a + j x_q)$$

$$\bar{E}_q = 2.09 \angle 1.2014 \text{ (rad)}$$

$\boxed{\delta_0 = 1.2014 \text{ rad} \quad (68.83^\circ \text{ deg})}$

$$V_{d_0} = V_t \sin(\delta - \alpha) = 0.8126$$

$$V_{q_0} = V_t \cos(\delta - \alpha) = 0.5828$$

$$I_{d_0} = I_t \sin(\delta - \gamma) = 0.8867$$

$$I_{q_0} = I_t \cos(\delta - \gamma) = 0.4796$$

$$E_{q_0}' = V_{q_0} + R_s I_q + x_d' I_{d_0} = 0.7350$$

$$E_{fd_0} = E_{q_0}' + (x_d - x_d') I_{d_0} = 2.1807$$

$$T_{e_0} = (x_q - x_d') I_{d_0} I_{q_0} + E_{q_0}' I_{q_0} = \boxed{1.003 \text{ p.u}}$$

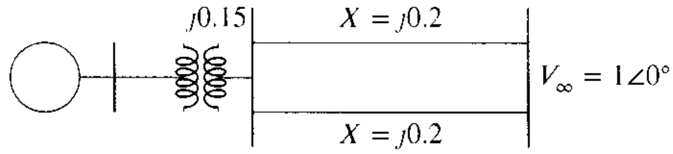
3

$P_t = 1 \; ; \quad |I_t| = 1.008$

$$P_t + I_t^2 R_a$$

$$\boxed{= 1.003 \text{ p.u}}$$

4. Consider a SMIB system. The synchronous generator is represented using the classical model.



The generator parameters are as follows:

$$X_d' = 0.3 \quad H = 3.5 \quad D = 0 \quad f = 60 \text{ Hz}$$

If the prefault system conditions are

$$P_t = 1.0 \quad |V_t| = 1.0$$

$\alpha = \sin^{-1}\left(\frac{1}{4}\right) = 0.2527$

$E_d' = \bar{V}_t + \bar{I}_t \times jX_d'$

$= 1.08\angle 0.53$

Analyze the transient stability of the system for a solid three phase fault at the sending end on one of the transmission lines. The fault is cleared by simultaneously opening the circuit breakers at both ends of the line.

$\boxed{\delta_0 = 0.53 \text{ rad} \quad \delta_0 = 30.6^\circ}$

(a) Determine the critical clearing angle and time using the Equal Area Criterion.

(b) Verify it using the energy function method.

a) $P_{max_1} = 1.9647$; $P_{max_2} = 0$; $P_{max_3} = 1.6625$

$\boxed{\delta_c = 67.58^\circ \quad 1.18 \text{ rad}}$

$t_c = \sqrt{\dfrac{4H(\delta_c - \delta_0)}{\omega_s \times P_m}} = \boxed{0.1554 \text{ sec}}$

b) The critical energy at δ_{UEP}

$$\begin{cases} \delta_{UEP} = \pi - \sin^{-1}\left(\dfrac{P_m}{P_{max_3}}\right) \\ \delta_{SEP} = \sin^{-1}\left(\dfrac{P_m}{P_{max_3}}\right) \end{cases}$$

$V_{cr} = -P_m(\delta_{UEP} - \delta_{SEP}) - P_{max_3}(\cos\delta_{UEP} - \cos\delta_{SEP})$

$\boxed{V_{cr} = 0.8053}$

The transient energy at the clearing angle

$(\delta_c = 67.58^\circ \quad t_c = 0.1548)$

$V_{cl} = \frac{1}{2}M(\omega')^2 - P_m(\delta_c - \delta_{SEP}) - P_{max_3}(\cos\delta_c - \cos\delta_{SEP})$

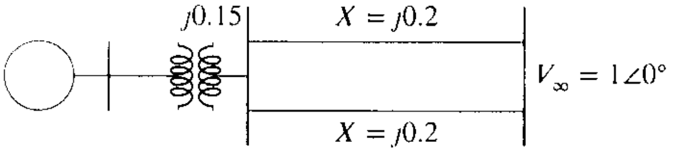
$\omega' = \omega_c - \omega_{syn}$

$\omega_{syn} = 377$ rad/sec

$\omega_c = 385.337$ rad/s

4 $\boxed{V_{cl} = 0.8052}$ $\boxed{V_{cl} < V_{cr}}$

5. Consider a SMIB system. The synchronous generator is represented using the classical model.



The generator parameters are as follows:

$$X_d' = 0.3 \quad H = 3.5 \quad D = 0 \quad f = 60 \text{ Hz}$$

Analyze the small signal characteristics of the system about the steady operating condition following the loss of one line. The post fault system conditions at the machine terminal are

$$P_t = 1.0 \quad |V_t| = 1.0$$

(a) determine the time response of δ if $\Delta\delta = 5°$

(b) determine the time response of ω if $\Delta\omega = 0$

Post Fault:

$$P_{max} = 1.6625 \qquad \underset{(SEP)}{\delta_{new}} = \sin^{-1}\left(\frac{1}{1.6625}\right) = 0.6454 \text{ rad.}$$

$$\begin{bmatrix} \Delta\dot{\delta} \\ \Delta\dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{\omega_{base}}{2H} P_{max}\cos\delta_{SEP} & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix}$$

$$\begin{bmatrix} \Delta\dot{\delta} \\ \Delta\dot{\omega} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -71.53 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta\omega \end{bmatrix}$$

$$\lambda_1, \lambda_2 = \pm j\, 8.46$$

$$\phi = \begin{bmatrix} -j0.1174 & j0.1174 \\ 0.99 & 0.99 \end{bmatrix} \qquad \psi = \begin{bmatrix} j4.26 & 0.5 \\ -j4.26 & 0.5 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} j0.37 \\ -j0.37 \end{bmatrix}$$

$$\delta = \delta_{SEP} + \Delta\delta(t)$$

$$\boxed{\delta(t) = 0.6454 + 0.088\cos(8.46t)}$$

$$\boxed{\omega(t) = 377 - 0.74\sin(8.46t)}$$

5