Synchronous Machine

1. Armature
   - It is a three phase winding placed in the stator.
   - The three phase armature windings are placed 120° (electrical) apart in space.
   - Stator iron is built up of thin laminations to reduce eddy current loss.
2. Field
   - Field winding is placed in the rotor and requires DC.

There are two types of rotor.

1. Cylindrical Poles
2. Salient Poles

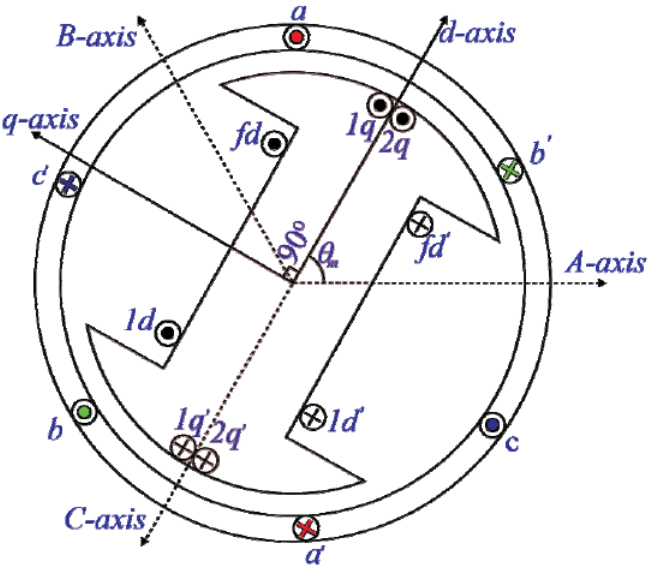


Figure: Schematic diagram of a $3-\phi$ synchronous machine

Source: NPTEL

The synchronous speed is

$$N_s = \frac{120f}{P}$$

where N_s is in rpm, f is the frequency in Hz and P is the number of poles.

The relationship between the electrical angle (θ) and the mechanical angle (θ_m) is

$$\theta = \frac{P}{2}\theta_m$$

In a 2-pole machine,

$$\theta = \theta_m$$

Assumptions made in modelling.

1. Stator windings are distributed sinusoidally along the air gap.
2. Stator slots do not cause any variation in the rotor inductances.
3. Magnetic hysteresis is negligible.
4. Magnetic circuit is considered as linear.

The following convention is used.

1. q-axis is leading d-axis.
2. Two amortisseur in q-axis and one in d-axis are used. Hence 2 windings (including the filed winding) in d-axis and 2 windings in q-axis.

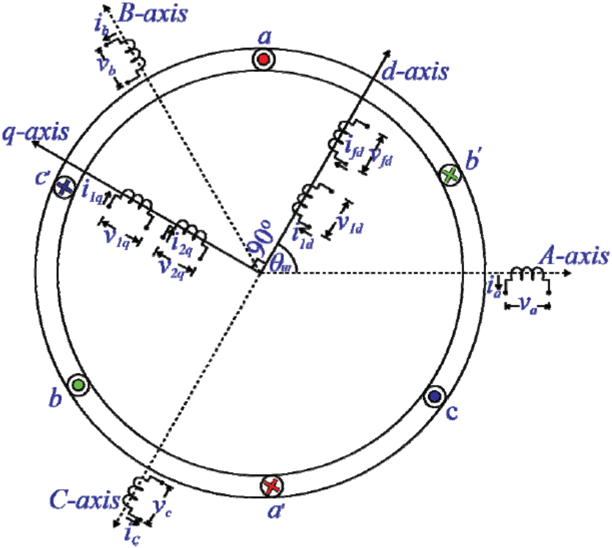


Figure: Stator and Rotor windings of a synchronous machine

Source: NPTEL

Voltage Equations

The stator voltage equations are

$$v_a = -i_a r_s + \frac{d\lambda_a}{dt} \tag{1}$$

$$v_b = -i_b r_s + \frac{d\lambda_b}{dt} \tag{2}$$

$$v_c = -i_c r_s + \frac{d\lambda_c}{dt} \tag{3}$$

where

$$\begin{aligned} v_a, v_b, v_c &= \text{instantaneous stator phase to neutral voltage} \\ i_a, i_b, i_c &= \text{instantaneous stator currents in phases a,b,c} \\ r_s &= \text{stator resistance per phase} \\ \lambda_a, \lambda_b, \lambda_c &= \text{flux linkages in phases a,b,c} \end{aligned}$$

The rotor circuit voltage equations are

$$v_{fd} = i_{fd} r_{fd} + \frac{d\lambda_{fd}}{dt} \tag{4}$$

$$v_{1d} = i_{1d} r_{1d} + \frac{d\lambda_{1d}}{dt} \tag{5}$$

$$v_{1q} = i_{1q} r_{1q} + \frac{d\lambda_{1q}}{dt} \tag{6}$$

$$v_{2q} = i_{2q} r_{2q} + \frac{d\lambda_{2q}}{dt} \tag{7}$$

$v_{fd}, v_{1d}, v_{1q}, v_{2q} =$ rotor field, 1d, 1q, 2q voltages

$i_{fd}, i_{1d}, i_{1q}, i_{2q} =$ rotor field, 1d, 1q, 2q currents

$r_{fd}, r_{1d}, r_{1q}, r_{2q} =$ rotor field, 1d, 1q, 2q resistances

$\lambda_{fd}, \lambda_{1d}, \lambda_{1q}, \lambda_{2q} =$ rotor field, 1d, 1q, 2q flux linkages

Flux Linkage Equations

The flux linkages in the phase a, b and c windings are

$$\lambda_a = -l_{aa} i_a - l_{ab} i_b - l_{ac} i_c + l_{afd} i_{fd} + l_{a1d} i_{id} + l_{a1q} i_{1q} + l_{a2q} i_{2q}$$
$$\lambda_b = -l_{ba} i_a - l_{bb} i_b - l_{bc} i_c + l_{bfd} i_{fd} + l_{b1d} i_{id} + l_{b1q} i_{1q} + l_{b2q} i_{2q}$$
$$\lambda_c = -l_{ca} i_a - l_{cb} i_b - l_{cc} i_c + l_{cfd} i_{fd} + l_{c1d} i_{id} + l_{c1q} i_{1q} + l_{c2q} i_{2q}$$

The negative sign in the stator currents is due to their direction.
In matrix form,

$$\begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix} = -\begin{bmatrix} l_{aa} & l_{ab} & l_{ac} \\ l_{ab} & l_{bb} & l_{bc} \\ l_{ac} & l_{bc} & l_{cc} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} l_{afd} & l_{a1d} & l_{a1q} & l_{a2q} \\ l_{bfd} & l_{b1d} & l_{b1q} & l_{b2q} \\ l_{cfd} & l_{c1d} & l_{c1q} & l_{c2q} \end{bmatrix} \begin{bmatrix} i_{fd} \\ i_{1d} \\ i_{1q} \\ i_{2q} \end{bmatrix} \tag{8}$$

$$\lambda_{abc} = -\mathrm{L_{ss}} \mathrm{i}_{abc} + \mathrm{L_{sr}} \mathrm{i}_r$$

The rotor circuit flux linkages can be written as

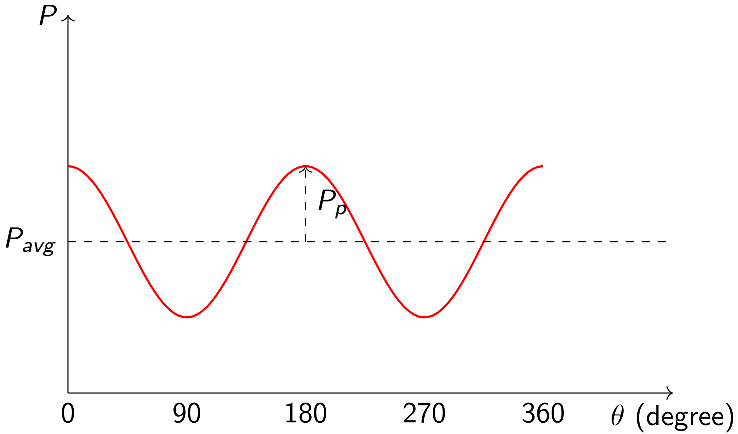
$$\begin{bmatrix}\lambda_{fd}\\ \lambda_{1d}\\ \lambda_{1q}\\ \lambda_{2q}\end{bmatrix} = -\begin{bmatrix} l_{fda} & l_{fdb} & l_{fdc}\\ l_{1da} & l_{1db} & l_{1dc}\\ l_{1qa} & l_{1qb} & l_{1qc}\\ l_{2qa} & l_{2qb} & l_{2qc}\end{bmatrix}\begin{bmatrix} i_a\\ i_b\\ i_c\end{bmatrix} + \begin{bmatrix} l_{fdfd} & l_{fd1d} & 0 & 0\\ l_{1dfd} & l_{1d1d} & 0 & 0\\ 0 & 0 & l_{1q1q} & l_{1q2q}\\ 0 & 0 & l_{2q1q} & l_{2q2q}\end{bmatrix}\begin{bmatrix} i_{fd}\\ i_{1d}\\ i_{1q}\\ i_{2q}\end{bmatrix} \tag{9}$$

The mutual inductance between the d-axis windings and the q-axis windings is zero as their fluxes are in quadrature.

$$\lambda_r = -\mathrm{L_{rs}}\mathrm{i}_{abc} + \mathrm{L_{rr}}\mathrm{i}_r$$

$\mathrm{L_{ss}}$, $\mathrm{L_{sr}}$ and $\mathrm{L_{rs}}$ depend on θ.

The permeance varies with θ.



$$P = P_{avg} + P_p \cos(2\theta)$$

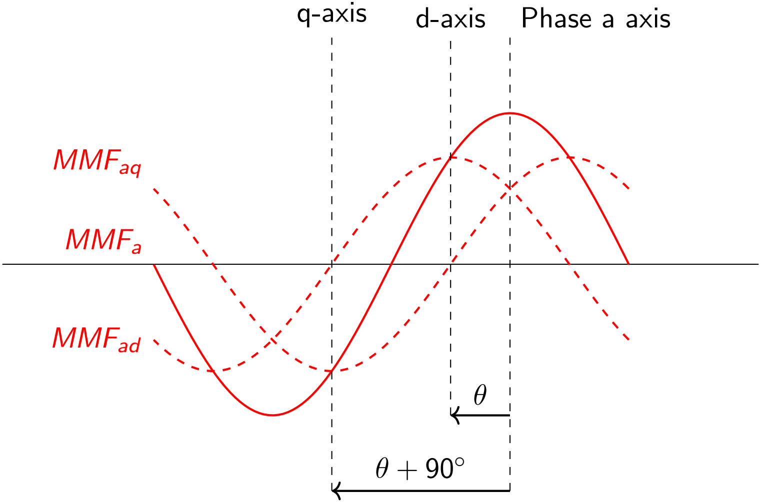
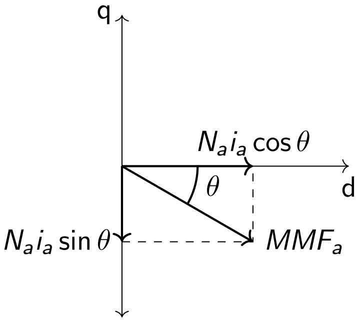


Figure: Phase a mmf

Stator Self Inductances

$$\text{peak} \quad MMF_a = N_a i_a$$
$$\text{peak} \quad MMF_{ad} = N_a i_a \cos\theta$$
$$\text{peak} \quad MMF_{aq} = N_a i_a \cos(\theta + 90^\circ) = -N_a i_a \sin\theta$$



The fluxes along d and q axes are

$$\phi_{ad} = N_a i_a \cos\theta P_d$$

$$\phi_{aq} = -N_a i_a \sin\theta P_q$$

The flux along the axis a is given by

$$\phi_{aa} = \phi_{ad}\cos\theta - \phi_{aq}\sin\theta$$

$$\phi_{aa} = N_a i_a P_d \cos^2\theta + N_a i_a P_q \sin^2\theta$$

$$\phi_{aa} = N_a i_a \left(P_d\left(\frac{1+\cos 2\theta}{2}\right) + P_q\left(\frac{1-\cos 2\theta}{2}\right)\right)$$

$$\phi_{aa} = N_a i_a \left(\frac{P_d + P_q}{2} + \frac{P_d - P_q}{2}\cos 2\theta \right)$$

The self inductance l_{aa} is

$$l_{aa} = \frac{N_a \phi_{aa}}{i_a} = N_a^2 \left(\frac{P_d + P_q}{2} \right) + N_a^2 \left(\frac{P_d - P_q}{2} \cos 2\theta \right)$$

This can be written as

$$l_{aa} = l_{aa0} + l_{aap} \cos 2\theta \tag{10}$$

where $l_{aa0} = N_a^2 \left(\frac{P_d + P_q}{2} \right)$ and $l_{aap} = N_a^2 \left(\frac{P_d - P_q}{2} \right)$ Since the windings of phases b and c are identical to a and are displaced from it by 120° and 240°, respectively,

$$l_{bb} = l_{aa0} + l_{aap} \cos 2(\theta - 120^\circ) = l_{aa0} + l_{aap} \cos(2\theta + 120^\circ) \tag{11}$$

$$l_{cc} = l_{aa0} + l_{aap} \cos 2(\theta - 240^\circ) = l_{aa0} + l_{aap} \cos(2\theta - 120^\circ) \tag{12}$$

Stator Mutual Inductances

The flux linking phase a due to mmf of phase b is

$$\phi_{ba} = \phi_{bd}\cos\theta - \phi_{bq}\sin\theta$$

$$\phi_{ba} = N_b i_b P_d \cos(\theta - 120^\circ)\cos\theta + N_b i_b P_q \sin(\theta - 120^\circ)\sin\theta$$

$$\begin{aligned}\phi_{ba} = N_b i_b [(\frac{P_d}{2}(\cos(2\theta - 120^\circ) + \cos(120^\circ)) + \\ \frac{P_q}{2}(\cos(120^\circ) - \cos(2\theta - 120^\circ))]\end{aligned}$$

$$\phi_{ba} = N_b i_b \left(-\frac{P_d + P_q}{4} + \frac{P_d - P_q}{2}(\cos(2\theta - 120^\circ))\right)$$

The flux linkage of phase a due to current i_b is

$$\lambda_{ba} = N_a \phi_{ba} = N_a N_b i_b \left(-\frac{P_d + P_q}{4} + \frac{P_d - P_q}{2} (\cos(2\theta - 120^\circ)) \right)$$

Since $N_a = N_b$, the mutual inductance between a and b is

$$l_{ba} = \frac{N_a \phi_{ba}}{i_b} = N_a^2 \left(-\frac{P_d + P_q}{4} + \frac{P_d - P_q}{2} (\cos(2\theta - 120^\circ)) \right)$$

$$\begin{aligned} l_{ba} = l_{ab} &= -\frac{1}{2} l_{aa0} + l_{aap} \cos(2\theta - 120^\circ) \\ &= -\frac{1}{2} l_{aa0} - l_{aap} \cos(2\theta + 60^\circ) \end{aligned} \tag{13}$$

Similarly,

$$l_{ca} = l_{ac} = -\frac{1}{2} l_{aa0} - l_{aap} \cos(2\theta - 60^\circ) \tag{14}$$

$$l_{bc} = l_{cb} = -\frac{1}{2} l_{aa0} - l_{aap} \cos(2\theta - 180^\circ) \tag{15}$$

$$\mathrm{L}_{ss} = \begin{bmatrix} l_{aa0} + l_{aap}\cos 2\theta & -\frac{1}{2}l_{aa0} - l_{aap}\cos(2\theta + 60^\circ) & -\frac{1}{2}l_{aa0} - l_{aap}\cos(2\theta - 60^\circ) \\ -\frac{1}{2}l_{aa0} - l_{aap}\cos(2\theta + 60^\circ) & l_{aa0} + l_{aap}\cos(2\theta + 120^\circ) & -\frac{1}{2}l_{aa0} - l_{aap}\cos(2\theta - 180^\circ) \\ -\frac{1}{2}l_{aa0} - l_{aap}\cos(2\theta - 60^\circ) & -\frac{1}{2}l_{aa0} - l_{aap}\cos(2\theta - 180^\circ) & l_{aa0} + l_{aap}\cos(2\theta - 120^\circ) \end{bmatrix} \tag{16}$$

Mutual inductances between stator and rotor

Since the rotor windings see a constant permeance,

$$\mathrm{L}_{sr} = \begin{bmatrix} l_{afd}\cos\theta & l_{a1d}\cos\theta & -l_{a1q}\sin\theta & -l_{a2q}\sin\theta \\ l_{bfd}\cos(\theta-120^\circ) & l_{b1d}\cos(\theta-120^\circ) & -l_{b1q}\sin(\theta-120^\circ) & -l_{b2q}\sin(\theta-120^\circ) \\ l_{cfd}\cos(\theta+120^\circ) & l_{c1d}\cos(\theta+120^\circ) & -l_{c1q}\sin(\theta+120^\circ) & -l_{c2q}\sin(\theta+120^\circ) \end{bmatrix} \tag{17}$$

Let

$$l_{afd} = l_{bfd} = l_{cfd} = l_{sfd}$$
$$l_{a1d} = l_{b1d} = l_{c1d} = l_{s1d}$$
$$l_{a1q} = l_{b1q} = l_{c1q} = l_{s2q}$$
$$l_{a2q} = l_{b2q} = l_{c2q} = l_{s2q}$$

$$\mathrm{L}_{sr} = \begin{bmatrix} l_{sfd}\cos\theta & l_{s1d}\cos\theta & -l_{s1q}\sin\theta & -l_{s2q}\sin\theta \\ l_{sfd}\cos(\theta-120^\circ) & l_{s1d}\cos(\theta-120^\circ) & -l_{s1q}\sin(\theta-120^\circ) & -l_{s2q}\sin(\theta-120^\circ) \\ l_{sfd}\cos(\theta+120^\circ) & l_{s1d}\cos(\theta+120^\circ) & -l_{s1q}\sin(\theta+120^\circ) & -l_{s2q}\sin(\theta+120^\circ) \end{bmatrix} \tag{18}$$

- L_{ss}, L_{sr} and L_{rs} vary with angle θ which in turn varies with time.
- This introduces complexity in solving problems.
- Therefore, a suitable transformation is required.
- $dq0$ transformation is used to transform the stator variables into new variables.

$dq0$ Transformation

The transformation matrix is

$$\mathrm{T}_{dq0} = k_1 \begin{bmatrix} \cos\theta & \cos(\theta - 120^\circ) & \cos(\theta + 120^\circ) \\ -\sin\theta & -\sin(\theta - 120^\circ) & -\sin(\theta + 120^\circ) \\ k_2 & k_2 & k_2 \end{bmatrix} \tag{19}$$

- It is also called Park's Transformation.
- The constants k_1 and k_2 are arbitrary.
- In most of the literature, $k_1 = \frac{2}{3}$ and $k_2 = \frac{1}{2}$ are used.
- $k_1 = \sqrt{\frac{2}{3}}$ and $k_2 = \frac{1}{\sqrt{2}}$ are also used in some literature.
- In this course, $k_1 = \frac{2}{3}$ and $k_2 = \frac{1}{2}$ are used.

Let us apply the $dq0$ transformation to stator currents.

$$
\begin{aligned}
i_a &= I_m \cos(\omega t) \\
i_b &= I_m \cos(\omega t - 120^\circ) \\
i_c &= I_m \cos(\omega t + 120^\circ)
\end{aligned}
$$

$$
\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \mathrm{T}_{dq0} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{20}
$$

$$
\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta - 120^\circ) & \cos(\theta + 120^\circ) \\ -\sin\theta & -\sin(\theta - 120^\circ) & -\sin(\theta + 120^\circ) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} I_m \cos\omega t \\ I_m \cos(\omega t - 120^\circ) \\ I_m \cos(\omega t + 120^\circ) \end{bmatrix}
$$

$$
\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \begin{bmatrix} I_m \cos(\omega t - \theta) \\ I_m \sin(\omega t - \theta) \\ 0 \end{bmatrix} \tag{21}
$$

If θ is measured with respect to the synchronously rotating reference,

$$\theta = \omega t + \delta$$

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \begin{bmatrix} I_m \cos\delta \\ -I_m \sin\delta \\ 0 \end{bmatrix} \tag{22}$$

For balanced steady state operation, i_d and i_q are constant.

$$i_0 = \frac{1}{3}(i_a + i_b + i_c) = 0$$

- Since $k_1 = \frac{2}{3}$ and $k_2 = \frac{1}{2}$, the peak value of i_d and i_q is the same as the peak value of the stator current.
- Two fictitious windings carrying current i_d and i_q rotating at synchronous speed produce the same effect as that of three phase stator windings.

The inverse transformation is

$$\mathrm{T}_{dq0}^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 1 \\ \cos(\theta - 120^\circ) & -\sin(\theta - 120^\circ) & 1 \\ \cos(\theta + 120^\circ) & -\sin(\theta + 120^\circ) & 1 \end{bmatrix} \tag{23}$$

$$\mathrm{v}_{dq0} = \mathrm{T}_{dq0}\mathrm{v}_{abc};\ \mathrm{i}_{dq0} = \mathrm{T}_{dq0}\mathrm{i}_{abc}$$

$$\mathrm{v}_{abc} = \mathrm{T}_{dq0}^{-1}\mathrm{v}_{dq0};\ \mathrm{i}_{abc} = \mathrm{T}_{dq0}^{-1}\mathrm{i}_{dq0}$$

Let us apply the $dq0$ transformation to the instantaneous power.

$$s = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}^T \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \left(\mathrm{T}_{dq0}^{-1} \begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix}\right)^T \left(\mathrm{T}_{dq0}^{-1} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix}\right) \tag{24}$$

$$s = \begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix}^T (\mathrm{T}_{dq0}^{-1})^T \mathrm{T}_{dq0}^{-1} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix}$$

$$s = \begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix}^T \begin{bmatrix} \frac{3}{2} & 0 & 0 \\ 0 & \frac{3}{2} & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix}$$

$$s = v_a i_a + v_b i_b + v_c i_c = \frac{3}{2}(v_d i_d + v_q i_q + 2 v_0 i_0) \tag{25}$$

The transformation is not power invariant.

$$v_a i_a + v_b i_b + v_c i_c \neq v_d i_d + v_q i_q + v_0 i_0$$

If $k_1 = \sqrt{\frac{2}{3}}$ and $k_2 = \frac{1}{\sqrt{2}}$, the transformation will be power invariant.

Let us apply the $dq0$ transformation to stator voltages.

$$\begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix} = -\mathrm{T}_{dq0} \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix} \mathrm{T}_{dq0}^{-1} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} + \mathrm{T}_{dq0} \frac{d}{dt} \left(\mathrm{T}_{dq0}^{-1} \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_0 \end{bmatrix} \right)$$

$$\begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix} = -\mathrm{T}_{dq0} \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix} \mathrm{T}_{dq0}^{-1} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} + \mathrm{T}_{dq0} \mathrm{T}_{dq0}^{-1} \frac{d}{dt} \left(\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_0 \end{bmatrix} \right) + \mathrm{T}_{dq0} \frac{d}{dt} \left(\mathrm{T}_{dq0}^{-1} \right) \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_0 \end{bmatrix}$$

$$\begin{bmatrix} v_d \\ v_q \\ v_0 \end{bmatrix} = -\begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} + \begin{bmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_0 \end{bmatrix} + \frac{d}{dt}\left(\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_0 \end{bmatrix} \right)$$

The stator and rotor voltage equations are as follows:

$$v_d = -r_s i_d - \omega\lambda_q + \frac{d\lambda_d}{dt} \tag{26}$$

$$v_q = -r_s i_q + \omega\lambda_d + \frac{d\lambda_q}{dt} \tag{27}$$

$$v_0 = -r_s i_0 + \frac{d\lambda_0}{dt} \tag{28}$$

$$v_{fd} = r_{fd} i_{fd} + \frac{d\lambda_{fd}}{dt} \tag{29}$$

$$v_{1d} = r_{1d} i_{1d} + \frac{d\lambda_{1d}}{dt} \tag{30}$$

$$v_{1q} = r_{1q} i_{1q} + \frac{d\lambda_{1q}}{dt} \tag{31}$$

$$v_{2q} = r_{2q} i_{2q} + \frac{d\lambda_{2q}}{dt} \tag{32}$$

Applying the $dq0$ transformation to the stator flux equations.

$$\begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_0 \end{bmatrix} = -\mathrm{T}_{dq0}\mathrm{L}_{ss}\mathrm{T}_{dq0}^{-1} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} + \mathrm{T}_{dq0}\mathrm{L}_{sr} \begin{bmatrix} i_{fd} \\ i_{1d} \\ i_{1q} \\ i_{2q} \end{bmatrix} \tag{34}$$

After simplification,

$$\begin{aligned} \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_0 \end{bmatrix} = &- \begin{bmatrix} \frac{3}{2}(l_{aa0} + l_{aap}) & 0 & 0 \\ 0 & \frac{3}{2}(l_{aa0} - l_{aap}) & 0 \\ 0 & 0 & l_{aa0} \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} \\ &+ \begin{bmatrix} l_{sfd} & l_{s1d} & 0 & 0 \\ 0 & 0 & l_{s1q} & l_{s2q} \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} i_{fd} \\ i_{1d} \\ i_{1q} \\ i_{2q} \end{bmatrix} \end{aligned} \tag{35}$$

Let us define the following.

$$l_d = \frac{3}{2}(l_{aa0} + l_{aap}); \quad l_q = \frac{3}{2}(l_{aa0} - l_{aap})$$

where l_d, l_q are the inductances along d, q-axis corresponding to the total flux linkage.

$$l_d = l_{md} + l_{ls}; \quad l_q = l_{mq} + l_{ls}$$

where

$$\begin{aligned} l_{md} &= \text{the mutual inductance along } d \\ l_{mq} &= \text{the mutual inductance along } q \\ l_{ls} &= \text{leakage inductance} \end{aligned}$$

$$
\begin{aligned}
\lambda_d &= -(l_{md} + l_{ls})i_d + l_{sfd}i_{fd} + l_{s1d}i_{1d} && (36)\\
\lambda_q &= -(l_{mq} + l_{ls})i_q + l_{s1q}i_{1q} + l_{s2q}i_{2q} && (37)\\
\lambda_0 &= -l_{ls}i_0 && (38)\\
\lambda_{fd} &= -\frac{3}{2}l_{sfd}i_d + l_{fdfd}i_{fd} + l_{fd1d}i_{1d} && (39)\\
\lambda_{1d} &= -\frac{3}{2}l_{s1d}i_d + l_{fd1d}i_{fd} + l_{1d1d}i_{1d} && (40)\\
\lambda_{1q} &= -\frac{3}{2}l_{s1q}i_q + l_{1q1q}i_{1q} + l_{1q2q}i_{2q} && (41)\\
\lambda_{2q} &= -\frac{3}{2}l_{s2q}i_q + l_{1q2q}i_{1q} + l_{2q2q}i_{2q} && (42)\\
& && (43)
\end{aligned}
$$

Advantages of $dq0$ transformation

1. The dynamic performance equations have constant inductances.
2. For balanced conditions, zero sequence quantities disappear.
3. For balanced steady state operation, the stator quantities have constant values. For other modes of operation, they vary with time. Stability studies involve slow variations having frequencies below 2 to 3 Hz.
4. The d and q-axed parameters may be directly measured from tests.

Per unit representation

- The $dq0$ transformation converts inductances into constants irrespective of angle θ.
- However, the mutual inductances between the stator windings and the rotor windings are not equal.
- To make them equal, a proper per unit system should be chosen.
- This will minimize computational complexity.

Base Quantities for Stator Variables

Let us define the following.

$$
\begin{aligned}
V_{base} &= V_{rms}(l-n) \text{ kV} \\
S_{base} &= 3V_{base}I_{base} = \text{three phase rated MVA} \\
I_{base} &= \frac{S_{base}}{3V_{base}} \text{ kA} \\
Z_{base} &= \frac{V_{base}}{I_{base}} \ \Omega \\
\omega_{base} &= \omega_s = 2\pi f \text{ elect. rad/sec} \\
\omega_{m\ base} &= \frac{2}{P}\omega_{base} \text{ mech. rad/sec} \\
L_{base} &= \frac{Z_{base}}{\omega_{base}} \text{ H} \\
\lambda_{base} &= \frac{V_{base}}{\omega_{base}} \text{ Wb-turn}, \ t_{base} = \frac{1}{\omega_{base}} \text{ s/rad}
\end{aligned}
\tag{44}
$$

$$
\begin{aligned}
V_{dq0,base} &= \sqrt{2}V_{base} \text{ kV} \\
S_{base} &= \frac{3}{2}V_{dq0,base}I_{dq0,base} \text{ MVA} \\
I_{dq0,base} &= \frac{2}{3}\frac{S_{base}}{V_{dq0,base}} = \frac{2}{3}\frac{S_{base}}{\sqrt{2}V_{base}} = \frac{\sqrt{2}S_{base}}{3V_{base}} = \sqrt{2}I_{base} \text{ kA} \\
Z_{dq0,base} &= \frac{V_{dq0,base}}{I_{dq0.base}} \ \Omega \\
L_{dq0,base} &= \frac{Z_{dq0,base}}{\omega_{base}} \text{ H} \\
\lambda_{dq0,base} &= \frac{V_{dq0,base}}{\omega_{base}} \text{ Wb-turn}
\end{aligned}
\tag{45}
$$

Base Quantities for Rotor Variables

$$
\begin{aligned}
S_{base} &= V_{fd,base} I_{fd,base} \\
I_{fd,base} &= \frac{l_{md}}{l_{sfd}} I_{dq0,base} \\
V_{fd,base} &= \frac{S_{base}}{I_{fd,base}} = \frac{3}{2} \frac{l_{sfd}}{l_{md}} V_{dq0,base} \\
\lambda_{fd,base} &= \frac{V_{fd,base}}{\omega_{base}} \\
Z_{fd,base} &= \frac{V_{fd,base}}{I_{fd,base}} \\
L_{fd,base} &= \frac{V_{fd,base}}{\omega_{base} I_{fd,base}} = \frac{Z_{fd,base}}{\omega_{base}}
\end{aligned}
\tag{46}
$$

Let us define them for damper windings.

1. d-axis winding

$$
\begin{aligned}
S_{base} &= V_{1d,base} I_{1d,base} \\
I_{1d,base} &= \frac{l_{md}}{l_{s1d}} I_{dq0,base} \\
V_{1d,base} &= \frac{S_{base}}{I_{1d,base}} = \frac{3}{2} \frac{l_{s1d}}{l_{md}} V_{dq0,base} \\
\lambda_{1d,base} &= \frac{V_{1d,base}}{\omega_{base}} \\
Z_{1d,base} &= \frac{V_{1d,base}}{I_{1d,base}} \\
L_{1d,base} &= \frac{V_{1d,base}}{\omega_{base} I_{1d,base}} = \frac{Z_{1d,base}}{\omega_{base}}
\end{aligned}
\tag{47}
$$

2. q-axis winding

$$
\begin{aligned}
S_{base} &= V_{1q,base} I_{1q,base} \\
I_{1q,base} &= \frac{l_{mq}}{l_{s1q}} I_{dq0,base} \\
V_{1q,base} &= \frac{S_{base}}{I_{1q,base}} = \frac{3}{2} \frac{l_{s1q}}{l_{mq}} V_{dq0,base} \\
\lambda_{1q,base} &= \frac{V_{1q,base}}{\omega_{base}} \\
Z_{1q,base} &= \frac{V_{1q,base}}{I_{1q,base}} \\
L_{1q,base} &= \frac{V_{1q,base}}{\omega_{base} I_{1q,base}} = \frac{Z_{1q,base}}{\omega_{base}}
\end{aligned}
\tag{48}
$$

$$
\begin{aligned}
S_{base} &= V_{2q,base} I_{2q,base} \\
I_{2q,base} &= \frac{l_{mq}}{l_{s2q}} I_{dq0,base} \\
V_{2q,base} &= \frac{S_{base}}{I_{2q,base}} = \frac{3}{2} \frac{l_{s2q}}{l_{mq}} V_{dq0,base} \\
\lambda_{2q,base} &= \frac{V_{2q,base}}{\omega_{base}} \\
Z_{2q,base} &= \frac{V_{2q,base}}{I_{2q,base}} \\
L_{2q,base} &= \frac{V_{2q,base}}{\omega_{base} I_{2q,base}} = \frac{Z_{2q,base}}{\omega_{base}}
\end{aligned}
\tag{49}
$$

Let us define the following.

$$
\begin{aligned}
V_d &= \frac{v_d}{V_{dq0,base}},\; V_q = \frac{v_q}{V_{dq0,base}},\; V_0 = \frac{v_0}{V_{dq0,base}} \\
I_d &= \frac{i_d}{I_{dq0,base}},\; I_q = \frac{i_q}{I_{dq0,base}},\; I_0 = \frac{i_0}{I_{dq0,base}} \\
V_{fd} &= \frac{v_{fd}}{V_{fd,base}},\; V_{1d} = \frac{v_{1d}}{V_{1d,base}},\; V_{1q} = \frac{v_{1q}}{V_{1q,base}},\; V_{2q} = \frac{v_{2q}}{V_{2q,base}} \\
I_{fd} &= \frac{i_{fd}}{I_{fd,base}},\; I_{1d} = \frac{i_{1d}}{I_{1d,base}},\; I_{1q} = \frac{i_{1q}}{I_{1q,base}},\; I_{2q} = \frac{i_{2q}}{I_{2q,base}} \\
\psi_d &= \frac{\lambda_d}{\lambda_{dq0,base}},\; \psi_q = \frac{\lambda_q}{\lambda_{dq0,base}} \\
\psi_{fd} &= \frac{\lambda_{fd}}{\lambda_{fd,base}},\; \psi_{1d} = \frac{\lambda_{1d}}{\lambda_{1d,base}},\; \psi_{1q} = \frac{\lambda_{1q}}{\lambda_{1q,base}},\; \psi_{2q} = \frac{\lambda_{2q}}{\lambda_{2q,base}} \\
R_s &= \frac{r_s}{Z_{dq0,base}} \\
R_{fd} &= \frac{r_{fd}}{Z_{fd,base}},\; R_{1d} = \frac{r_{1d}}{Z_{1d,base}},\; R_{1q} = \frac{r_{1q}}{Z_{1q,base}},\; R_{2q} = \frac{r_{2q}}{Z_{2q,base}},
\end{aligned}
\tag{50}
$$

Voltage Equations in per unit

The stator and rotor voltage equations given in equations (26)-(32) are converted to per unit form as follows:

$$V_d = -R_s I_d - \frac{\omega}{\omega_{base}}\psi_q + \frac{1}{\omega_{base}}\frac{d\psi_d}{dt} \tag{51}$$

$$V_q = -R_s I_q + \frac{\omega}{\omega_{base}}\psi_d + \frac{1}{\omega_{base}}\frac{d\psi_q}{dt} \tag{52}$$

$$V_0 = -R_s I_0 + \frac{1}{\omega_{base}}\frac{d\psi_0}{dt} \tag{53}$$

$$V_{fd} = R_{fd} I_{fd} + \frac{1}{\omega_{base}}\frac{d\psi_{fd}}{dt} \tag{54}$$

$$V_{1d} = R_{1d} I_{1d} + \frac{1}{\omega_{base}}\frac{d\psi_{1d}}{dt} \tag{55}$$

$$V_{1q} = R_{1q} I_{1q} + \frac{1}{\omega_{base}}\frac{d\psi_{1q}}{dt} \tag{56}$$

$$V_{2q} = R_{2q} I_{2q} + \frac{1}{\omega_{base}}\frac{d\psi_{2q}}{dt} \tag{57}$$

Instantaneous Power

Before converting it into per unit, let us substitute voltage equations given in (26)-(28) in (25).

$$
\begin{aligned}
s &= \frac{3}{2}\left\{(-r_s i_d - \omega\lambda_q + \frac{d\lambda_d}{dt})i_d + (-r_s i_q + \omega\lambda_d + \frac{d\lambda_q}{dt})i_q + 2(-r_s i_0 + \frac{d\lambda_0}{dt})i_0\right\} \\
&= \left\{\frac{3}{2}\left(\frac{d\lambda_d}{dt}i_d + \frac{d\lambda_q}{dt}i_q + \frac{d\lambda_0}{dt}i_0\right) + \frac{3}{2}\omega\left(\lambda_d i_q - \lambda_q i_d\right) - \frac{3}{2}\left(r_s i_d^2 + r_s i_q^2 + 2r_s i_0^2\right)\right\}
\end{aligned}
\tag{59}
$$

For a balanced system, v_0 and i_0 are zero.

$$
s = \left\{\underbrace{\frac{3}{2}\left(\frac{d\lambda_d}{dt}i_d + \frac{d\lambda_q}{dt}i_q\right)}_{I} + \underbrace{\frac{3}{2}\omega\left(\lambda_d i_q - \lambda_q i_d\right)}_{II} - \underbrace{\frac{3}{2}\left(r_s i_d^2 + r_s i_q^2\right)}_{III}\right\} \tag{60}
$$

$$I = \text{rate of change of stored magnetic energy in the stator windings}$$
$$II = \text{electrical power crossing the air gap}$$
$$III = \text{stator copper losses}$$

The electrical torque T_e in N-m can be found

$$\begin{aligned} T_e &= \frac{P_e}{\omega_m} = \frac{P_e}{\frac{2}{P}\omega} = \frac{\frac{3}{2}\omega\left(\lambda_d i_q - \lambda_q i_d\right)}{\frac{2}{P}\omega} \\ T_e &= \frac{3}{2}\frac{P}{2}\left(\lambda_d i_q - \lambda_q i_d\right) \text{ N-m} \end{aligned} \tag{61}$$

To convert it into per unit, let us define the following.

$$\begin{aligned} T_{base} &= \frac{S_{base}}{\omega_{m,base}} = \frac{\frac{3}{2}V_{dq0,base}I_{dq0,base}}{\frac{2}{P}\omega_{base}} \\ &= \frac{3}{2}\frac{P}{2}\lambda_{dq0,base}I_{dq0,base} \end{aligned} \tag{62}$$

The electrical torque (T_e) in per unit is given by

$$T_e = \frac{\frac{3}{2}\frac{P}{2}\left(\lambda_d i_q - \lambda_q i_d\right)}{\frac{3}{2}\frac{P}{2}\lambda_{dq0,base} I_{dq0,base}}$$

$$\boxed{T_e = \psi_d I_q - \psi_q I_d} \tag{63}$$

In per unit calculations, the torque and power are the same.

Flux Linkage Equations in per unit

1. Flux linkage equations of windings in d-axis

$$\lambda_d = -(l_{md} + l_{ls})i_d + l_{sfd}i_{fd} + l_{s1d}i_{1d} \tag{64}$$

$$\lambda_{fd} = -\frac{3}{2}l_{sfd}i_d + l_{fdfd}i_{fd} + l_{fd1d}i_{1d} \tag{65}$$

$$\lambda_{1d} = -\frac{3}{2}l_{s1d}i_d + l_{fd1d}i_{fd} + l_{1d1d}i_{1d} \tag{66}$$

From (51),

$$i_d = I_d I_{dq0,base};\ i_{fd} = I_{fd} I_{fd,base};\ i_{1d} = I_{1d} I_{1d,base}$$

Equations (64) -(66) are divided by their respective bases to convert them into per unit form.

$$\frac{\lambda_d}{\lambda_{dq0,base}} = -\frac{(l_{md} + l_{ls})I_d I_{dq0,base}}{\lambda_{dq0,base}} + \frac{l_{sfd} I_{fd} I_{fd,base}}{\lambda_{dq0,base}} + \frac{l_{s1d} I_{1d} I_{1d,base}}{\lambda_{dq0,base}} \tag{67}$$

$$\frac{\lambda_{fd}}{\lambda_{fd,base}} = -\frac{3}{2}\frac{l_{sfd} I_d I_{dq0,base}}{\lambda_{fd,base}} + \frac{l_{fdfd} I_{fd} I_{fd,base}}{\lambda_{fd,base}} + \frac{l_{fd1d} I_{1d} I_{1d,base}}{\lambda_{fd,base}} \quad (68)$$

$$\frac{\lambda_{1d}}{\lambda_{1d,base}} = -\frac{3}{2}\frac{l_{s1d} I_d I_{dq0,base}}{\lambda_{1d,base}} + \frac{l_{fd1d} I_{fd} I_{fd,base}}{\lambda_{1d,base}} + \frac{l_{1d1d} I_{1d} I_{1d,base}}{\lambda_{1d,base}} \quad (69)$$

From (46) - (48),

$$\lambda_{dq0,base} = \frac{V_{dq0.base}}{\omega_{base}};\ \lambda_{fd,base} = \frac{V_{fd,base}}{\omega_{base}};\ \lambda_{1d,base} = \frac{V_{1d,base}}{\omega_{base}}$$

By using the above terms, the equations (67) - (69) can be written as follows:

$$\psi_d = -\frac{\omega_{base}(l_{md} + l_{ls}) I_d I_{dq0,base}}{V_{dq0,base}} + \frac{\omega_{base} l_{sfd} I_{fd} I_{fd,base}}{V_{dq0,base}} + \frac{\omega_{base} l_{s1d} I_{1d} I_{1d,base}}{V_{dq0,base}} \quad (70)$$

$$\psi_{fd} = -\frac{3}{2}\frac{\omega_{base} l_{sfd} I_d I_{dq0,base}}{V_{fd,base}} + \frac{\omega_{base} l_{fdfd} I_{fd} I_{fd,base}}{V_{fd,base}} + \frac{\omega_{base} l_{fd1d} I_{1d} I_{1d,base}}{V_{fd,base}} \tag{71}$$

$$\psi_{1d} = -\frac{3}{2}\frac{\omega_{base} l_{s1d} I_d I_{dq0,base}}{V_{1d,base}} + \frac{\omega_{base} l_{fd1d} I_{fd} I_{fd,base}}{V_{1d,base}} + \frac{\omega_{base} l_{1d1d} I_{1d} I_{1d,base}}{V_{1d,base}} \tag{72}$$

From (46) and (47),

$$I_{fd,base} = \frac{l_{md}}{l_{sfd}} I_{dq0,base}; \quad V_{fd,base} = \frac{3}{2}\frac{l_{sfd}}{l_{md}} V_{dq0,base}$$

$$I_{1d,base} = \frac{l_{md}}{l_{s1d}} I_{dq0,base}; \quad V_{1d,base} = \frac{3}{2}\frac{l_{s1d}}{l_{md}} V_{dq0,base}$$

By using the above terms, the equations (70) - (72) can be written as follows:

$$\psi_d = -\frac{\omega_{base}(l_{md} + l_{ls}) I_d I_{dq0,base}}{V_{dq0,base}} + \frac{\omega_{base} l_{md} I_{fd} I_{dq0,base}}{V_{dq0,base}} + \frac{\omega_{base} l_{md} I_{1d} I_{dq0,b}}{V_{dq0,base}} \tag{73}$$

$$\psi_{fd} = -\frac{\omega_{base} l_{md} I_d I_{dq0,base}}{V_{dq0,base}} + \frac{\omega_{base} l_{fdfd} I_{fd} I_{fd,base}}{V_{fd,base}} + \frac{\omega_{base} l_{fd1d} I_{1d} I_{1d,base}}{V_{fd,base}} \tag{74}$$

$$\psi_{1d} = -\frac{\omega_{base} l_{md} I_d I_{dq0,base}}{V_{dq0,base}} + \frac{\omega_{base} l_{fd1d} I_{fd} I_{fd,base}}{V_{1d,base}} + \frac{\omega_{base} l_{1d1d} I_{1d} I_{1d,base}}{V_{1d,base}} \tag{75}$$

The mutual inductances have been made equal.

2. Flux linkage equations of windings in q-axis

After following the same steps used in d-axis,

$$\psi_q = -\frac{\omega_{base}(l_{mq} + l_{ls}) I_q I_{dq0,base}}{V_{dq0,base}} + \frac{\omega_{base} l_{mq} I_{1q} I_{dq0,base}}{V_{dq0,base}} + \frac{\omega_{base} l_{mq} I_{2q} I_{dq0,base}}{V_{dq0,base}} \tag{76}$$

$$\psi_{1q} = -\frac{\omega_{base} l_{mq} I_q I_{dq0,base}}{V_{dq0,base}} + \frac{\omega_{base} l_{1q1q} I_{1q} I_{1q,base}}{V_{1q,base}} + \frac{\omega_{base} l_{1q2q} I_{2q} I_{2q,base}}{V_{1q,base}} \tag{77}$$

$$\psi_{2q} = -\frac{\omega_{base} l_{mq} I_q I_{dq0,base}}{V_{dq0,base}} + \frac{\omega_{base} l_{1q2q} I_{1q} I_{1q,base}}{V_{2q,base}} + \frac{\omega_{base} l_{2q2q} I_{2q} I_{2q,base}}{V_{2q,base}} \tag{78}$$

Let us define the following parameters to simplify the equations (73) -(75) and (76) - (78).

$$
\begin{aligned}
X_{ls} &= \frac{x_{ls}}{Z_{dq0,base}} = \frac{\omega_{base} l_{ls}}{Z_{dq0,base}} = \frac{\omega_{base} l_{ls} I_{dq0,base}}{V_{dq0,base}} \\
X_{md} &= \frac{x_{md}}{Z_{dq0,base}} = \frac{\omega_{base} l_{md}}{Z_{dq0,base}} = \frac{\omega_{base} l_{md} I_{dq0,base}}{V_{dq0,base}} \\
X_{mq} &= \frac{x_{mq}}{Z_{dq0,base}} = \frac{\omega_{base} l_{mq}}{Z_{dq0,base}} = \frac{\omega_{base} l_{mq} I_{dq0,base}}{V_{dq0,base}} \\
X_{fd} &= \frac{x_{fdfd}}{Z_{fd,base}} = \frac{\omega_{base} l_{fdfd}}{Z_{fd,base}} = \frac{\omega_{base} l_{fdfd} I_{fd,base}}{V_{fd,base}} \\
X_{1d} &= \frac{x_{1d1d}}{Z_{1d,base}} = \frac{\omega_{base} l_{1d1d}}{Z_{1d,base}} = \frac{\omega_{base} l_{1d1d} I_{1d,base}}{V_{1d,base}} \\
X_{1q} &= \frac{x_{1q1q}}{Z_{1q,base}} = \frac{\omega_{base} l_{1q1q}}{Z_{1q,base}} = \frac{\omega_{base} l_{1q1q} I_{1q,base}}{V_{1q,base}} \\
X_{2q} &= \frac{x_{2q2q}}{Z_{2q,base}} = \frac{\omega_{base} l_{2q2q}}{Z_{2q,base}} = \frac{\omega_{base} l_{2q2q} I_{2q,base}}{V_{2q,base}}
\end{aligned} \tag{79}
$$

$$
\begin{aligned}
X_{fd1d} &= \frac{\omega_{base} l_{fd1d} I_{1d,base}}{V_{fd,base}} = \frac{\omega_{base} l_{fd1d}}{V_{fd,base}} \frac{l_{sfd}}{l_{s1d}} I_{fd,base} \\
&= \frac{\omega_{base} l_{fd1d}}{Z_{fd,base}} \frac{l_{sfd}}{l_{s1d}} \\
X_{1q2q} &= \frac{\omega_{base} l_{1q2q} I_{2q,base}}{V_{1q,base}} = \frac{\omega_{base} l_{1q2q}}{V_{1q,base}} \frac{l_{s1q}}{l_{s2q}} I_{1q,base} \\
&= \frac{\omega_{base} l_{1q2q}}{Z_{1q,base}} \frac{l_{s1q}}{l_{s2q}}
\end{aligned}
\tag{80}
$$

$$
\begin{gathered}
X_d = X_{ls} + X_{md},\ X_q = X_{ls} + X_{mq},\ X_{lfd} = X_{fd} - X_{md} \\
X_{l1d} = X_{1d} - X_{md},\ X_{l1q} = X_{1q} - X_{mq},\ X_{l2q} = X_{2q} - X_{mq}
\end{gathered}
\tag{81}
$$

By using (79), (80), (81) and defining $K_d = \frac{X_{fd1d}}{X_{md}}$ and $K_q = \frac{X_{1q2q}}{X_{mq}}$, the flux linkage equations can be written as follows:

$$\psi_d = X_d(-I_d) + X_{md}I_{fd} + X_{md}I_{1d} \tag{82}$$

$$\psi_{fd} = X_{md}(-I_d) + X_{fd}I_{fd} + K_dX_{md}I_{1d} \tag{83}$$

$$\psi_{1d} = X_{md}(-I_d) + K_dX_{md}I_{fd} + X_{1d}I_{1d} \tag{84}$$

$$\psi_q = X_q(-I_q) + X_{mq}I_{1q} + X_{mq}I_{2q} \tag{85}$$

$$\psi_{1q} = X_{mq}(-I_q) + X_{1q}I_{1q} + K_qX_{mq}I_{2q} \tag{86}$$

$$\psi_{2q} = X_{mq}(-I_q) + K_qX_{mq}I_{1q} + X_{2q}I_{2q} \tag{87}$$

If K_d and K_q are assumed to be unity,

1. d-axis flux linkage equations

$$\psi_d = X_d(-I_d) + X_{md} I_{fd} + X_{md} I_{1d} \tag{88}$$
$$\psi_{fd} = X_{md}(-I_d) + X_{fd} I_{fd} + X_{md} I_{1d} \tag{89}$$
$$\psi_{1d} = X_{md}(-I_d) + X_{md} I_{fd} + X_{1d} I_{1d} \tag{90}$$

2. q-axis flux linkage equations

$$\psi_q = X_q(-I_q) + X_{mq} I_{1q} + X_{mq} I_{2q} \tag{91}$$
$$\psi_{1q} = X_{mq}(-I_q) + X_{1q} I_{1q} + X_{mq} I_{2q} \tag{92}$$
$$\psi_{2q} = X_{mq}(-I_q) + X_{mq} I_{1q} + X_{2q} I_{2q} \tag{93}$$

Synchronous Machine Parameters

- Synchronous machine parameters are normally specified in terms of sub-transient and transient reactances, open circuit sub-transient and transient time constants.
- Those parameters need to be defined.

1. Sub-transient reactance

$$X_d'' = X_{ls} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{lfd}} + \frac{1}{X_{l1d}}} \tag{94}$$

   - This can be found by applying a voltage at stator terminals such that only I_d flows with all rotor circuits shorted.
   - At $t = 0^+$, ψ_{fd} and ψ_{1d} will be zero.

Hence,

$$\psi_d = X_d(-I_d) + X_{md} I_{fd} + X_{md} I_{1d} \tag{95}$$
$$0 = X_{md}(-I_d) + X_{fd} I_{fd} + X_{md} I_{1d} \tag{96}$$
$$0 = X_{md}(-I_d) + X_{md} I_{fd} + X_{1d} I_{1d} \tag{97}$$

Solving (96) and (97) for I_{fd} and I_{1d} and considering (81),

$$I_{fd} = \frac{X_{md} X_{l1d}}{X_{md} X_{lfd} + X_{md} X_{l1d} + X_{lfd} X_{l1d}} I_d \tag{98}$$
$$I_{1d} = \frac{X_{md} X_{lfd}}{X_{md} X_{lfd} + X_{md} X_{l1d} + X_{lfd} X_{l1d}} I_d \tag{99}$$

Substituting (98) and (99) in (95),

$$\psi_d = -\left(X_d - \frac{X_{md}^2 (X_{lfd} + X_{l1d})}{X_{md} X_{lfd} + X_{md} X_{l1d} + X_{lfd} X_{l1d}} \right) I_d \tag{100}$$

Since $X_d = X_{ls} + X_{md}$,

$$\begin{aligned}\psi_d &= -\left(X_{ls} + \frac{X_{md}X_{lfd}X_{l1d}}{X_{md}X_{lfd} + X_{md}X_{l1d} + X_{lfd}X_{l1d}}\right) I_d \\ &= -\left(X_{ls} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{lfd}} + \frac{1}{X_{l1d}}}\right) I_d = -X_d'' I_d\end{aligned} \tag{101}$$

Similarly, the quadrature axis sub-transient reactance can be expressed as,

$$X_q'' = X_{ls} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{l1q}} + \frac{1}{X_{l2q}}} \tag{102}$$

2. Transient reactance

$$X_d' = X_{ls} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{lfd}}} \tag{103}$$

It is found by the same approach followed for X_d'' with an additional assumption that the damper winding transients settle down faster compared to the field winding transients. Hence I_{1d} =0. With this assumption, by solving (95) and (96) and using (81),

$$\begin{aligned}\psi_d &= -\left(X_d - \frac{X_{md}^2}{X_{md}+X_{lfd}}\right) I_d = -\left(X_{ls} + \frac{X_{md}X_{lfd}}{X_{md}+X_{lfd}}\right) I_d \\ &= -\left(X_{ls} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{lfd}}}\right) I_d = -X_d' I_d\end{aligned} \tag{104}$$

Similarly, the quadrature axis transient reactance can be defined as

$$X_q' = X_{ls} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{l1q}}} \tag{105}$$

3. Open circuit sub-transient time constant
   - A voltage is applied to the field winding with stator terminals open so that $I_d = 0$.
   - At $t = 0^+$, $\psi_{1d} = 0$.

Substituting these assumptions in (90) and (55),

$$I_{fd} = -\frac{X_{1d}}{X_{md}} I_{1d} \tag{106}$$

$$\frac{1}{\omega_{base}} \left(X_{md} \frac{dI_{fd}}{dt} + X_{1d} \frac{dI_{1d}}{dt} \right) + R_{1d} I_{1d} = V_{1d} = 0 \tag{107}$$

$$\frac{1}{\omega_{base}} \frac{dI_{fd}}{dt} = \left(-\frac{1}{\omega_{base}} \frac{X_{1d}}{X_{md}} \frac{dI_{1d}}{dt} - \frac{R_{1d}}{X_{md}} I_{1d} \right) \tag{108}$$

Substituting (89) in (54), we get

$$\frac{1}{\omega_{base}} \left(X_{fd} \frac{dI_{fd}}{dt} + X_{md} \frac{dI_{1d}}{dt} \right) + R_{fd} I_{fd} = V_{fd} \tag{109}$$

To get a first order differential equation in terms of I_{1d} with a forcing fucntion V_{fd}, let us substitute (106) and (108) in (109).

$$\frac{1}{\omega_{base}}\left(-\frac{X_{fd}X_{1d}}{X_{md}}\frac{dI_{1d}}{dt}-\frac{X_{fd}R_{1d}}{X_{md}}I_{1d}+X_{md}\frac{dI_{1d}}{dt}\right)-\frac{R_{fd}X_{1d}}{X_{md}}I_{1d}=V_{fd} \tag{110}$$

On simplification, we get

$$\frac{1}{\omega_{base}}\left(\frac{X_{md}^2-X_{fd}X_{1d}}{X_{fd}R_{1d}+X_{1d}R_{fd}}\right)\frac{dI_{1d}}{dt}-I_{1d}=\frac{X_{md}}{X_{fd}R_{1d}+X_{1d}R_{fd}}V_{fd} \tag{111}$$

In a practical generator, $R_{1d} >> R_{fd}$. Therefore, $X_{1d}R_{fd}$ can be neglected. Simplifying (111), we get

$$-\frac{1}{\omega_{base}R_{1d}}\left(X_{l1d}+\frac{X_{md}X_{lfd}}{X_{md}+X_{lfd}}\right)\frac{dI_{1d}}{dt}-I_{1d}=\frac{X_{md}}{X_{fd}R_{1d}}V_{fd} \tag{112}$$

Hence, the open circuit d-axis time sub-transient time constant is

$$\begin{aligned} T''_{d0} &= \frac{1}{\omega_{base} R_{1d}} \left(X_{l1d} + \frac{X_{md} X_{lfd}}{X_{md} + X_{lfd}} \right) \\ &= \frac{1}{\omega_{base} R_{1d}} \left(X_{l1d} + \frac{1}{\frac{1}{X_{md}} + \frac{1}{X_{lfd}}} \right) \end{aligned} \tag{113}$$

Similarly, the open circuit q-axis time sub-transient time constant can be expressed as

$$T''_{q0} = \frac{1}{\omega_{base} R_{2q}} \left(X_{l2q} + \frac{1}{\frac{1}{X_{mq}} + \frac{1}{X_{l1q}}} \right) \tag{114}$$

4. Open circuit transient time constant
In addition to the assumptions made in calculating T''_{d0}, the current in the damper winding I_{1d} is zero. From (89) and (54), we get

$$\psi_{fd} = X_{fd} I_{fd} \tag{115}$$

$$\frac{X_{fd}}{\omega_{base} R_{fd}} \frac{dI_{fd}}{dt} + I_{fd} = \frac{V_{fd}}{R_{fd}} \tag{116}$$

Hence, the open circuit d-axis transient time constant, in seconds, is defined as

$$T'_{d0} = \frac{X_{fd}}{\omega_{base} R_{fd}} \tag{117}$$

Similarly, the open circuit q-axis transient time constant can be expressed as

$$T'_{q0} = \frac{X_{1q}}{\omega_{base} R_{1q}} \tag{118}$$

Let us also define the following new variables.

$$E_q' = \frac{X_{md}}{X_{fd}}\psi_{fd}, \quad E_{fd} = \frac{X_{md}}{R_{fd}}V_{fd}, \quad E_d' = -\frac{X_{mq}}{X_{1q}}\psi_{1q} \qquad (119)$$

Our aim is to express the voltage equations and flux linkage equations using the sub-transient and transient reactances and open circuit time constants along with (119).

From (89) and (90),

$$\begin{bmatrix} I_{fd} \\ I_{1d} \end{bmatrix} = \begin{bmatrix} X_{fd} & X_{md} \\ X_{md} & X_{1d} \end{bmatrix}^{-1} \begin{bmatrix} \psi_{fd} \\ \psi_{1d} \end{bmatrix} + \begin{bmatrix} X_{fd} & X_{md} \\ X_{md} & X_{1d} \end{bmatrix}^{-1} \begin{bmatrix} X_{md} \\ X_{md} \end{bmatrix} I_d \qquad (120)$$

Let $\Delta = X_{fd}X_{1d} - X_{md}^2 = X_{lfd}X_{md} + X_{l1d}X_{md} + X_{lfd}X_{l1d}$ Therefore,

$$
\begin{aligned}
I_{fd} &= \frac{1}{\Delta}\left(X_{1d}\psi_{fd} - X_{md}\psi_{1d}\right) + \frac{1}{\Delta}\left(X_{1d}X_{md} - X_{md}^2\right) I_d \\
I_{1d} &= \frac{1}{\Delta}\left(-X_{md}\psi_{fd} + X_{fd}\psi_{1d}\right) + \frac{1}{\Delta}\left(X_{fd}X_{md} - X_{md}^2\right) I_d
\end{aligned} \tag{121}
$$

Equation (121) can be expressed using X''_d, X'_d , X_d and (121) as given below.

$$
\frac{1}{\Delta}X_{1d}\psi_{fd} = \frac{X_{fd}X_{1d}}{X_{md}\Delta}\frac{X_{md}}{X_{fd}}\psi_{fd} = \frac{1}{X_{md}}\left(1 + \frac{X_{md}^2}{\Delta}\right)\frac{X_{md}}{X_{fd}}\psi_{fd}
$$

Since $\frac{X_{md}^2}{\Delta} = \frac{(X_d - X'_d)(X'_d - X''_d)}{(X'_d - X_{ls})^2}$ and $E'_q = \frac{X_{md}}{X_{fd}}\psi_{fd}$,

$$
\frac{1}{\Delta}X_{1d}\psi_{fd} = \frac{1}{X_{md}}\left(1 + \frac{(X_d - X'_d)(X'_d - X''_d)}{(X'_d - X_{ls})^2}\right)E'_q
$$

$$\frac{1}{\Delta}X_{md}\psi_{1d} = \frac{1}{X_{md}}\frac{X_{md}^2}{\Delta}\psi_{1d} = \frac{1}{X_{md}}\left(\frac{(X_d - X_d')(X_d' - X_d'')}{(X_d' - X_{ls})^2}\right)\psi_{1d}$$

$$\frac{1}{\Delta}\left(X_{1d}X_{md} - X_{md}^2\right) I_d = \frac{1}{X_{lfd}}\frac{X_{lfd}X_{l1d}X_{md}}{\Delta}$$

Since $\frac{1}{X_{lfd}} = \frac{(X_d'-X_{ls})}{(X_d-X_d')}X_{md}$ and $\frac{X_{lfd}X_{l1d}X_{md}}{\Delta} = X_d'' - X_{ls}$,

$$\frac{1}{\Delta}\left(X_{1d}X_{md} - X_{md}^2\right) I_d = \frac{1}{X_{md}}\left(\frac{(X_d - X_d')(X_d'' - X_{ls})}{(X_d' - X_{ls})}\right) I_d$$

$$\frac{1}{\Delta}X_{md}\psi_{fd} = \frac{X_{fd}}{\Delta}\frac{X_{md}}{X_{fd}}\psi_{fd}$$

Since $\frac{X_{fd}}{\Delta} = \frac{(X_d'-X_d'')}{(X_d'-X_{ls})^2}$ and $E_q' = \frac{X_{md}}{X_{fd}}\psi_{fd}$,

$$\frac{1}{\Delta}X_{md}\psi_{fd} = \frac{(X_d'-X_d'')}{(X_d'-X_{ls})^2}E_q'$$

$$\frac{X_{fd}}{\Delta}\psi_{1d} = \frac{(X_d'-X_d'')}{(X_d'-X_{ls})^2}\psi_{1d}$$

$$\frac{1}{\Delta}(X_{fd}X_{md} - X_{md}^2)I_d = \frac{X_{lfd}X_{md}}{\Delta}I_d = \frac{(X_d'-X_d'')}{(X_d'-X_{ls})}I_d$$

By using the above terms, the equation (121) can be written as

$$
\begin{aligned}
I_{fd} &= \frac{1}{X_{md}}\left(E'_q + \frac{(X_d - X'_d)(X'_d - X''_d)}{(X'_d - X_{ls})^2}\left(E'_q + \frac{(X'_d - X_{ls})(X''_d - X_{ls})}{(X'_d - X''_d)} I_d + \psi_{1d}\right)\right) \\
&= \frac{1}{X_{md}}\left(E'_q + (X_d - X'_d)(I_d - I_{1d})\right)
\end{aligned}
\tag{122}
$$

$$
I_{1d} = \frac{(X'_d - X''_d)}{(X'_d - X_{ls})^2}(-E'_q + \psi_{1d} + I_d(X'_d - X_{ls})) \tag{123}
$$

Substituting (122) and (123) in (88),

$$
\begin{aligned}
\psi_d &= X_d(-I_d) + X_{md} I_{fd} + X_{md} I_{1d} \\
&= E'_q - X'_d I_d + (X'_d - X_{ls}) I_{1d} \\
&= -X''_d I_d + \frac{(X''_d - X_{ls})}{(X'_d - X_{ls})} E'_q + \frac{(X'_d - X''_d)}{(X'_d - X_{ls})} \psi_{1d}
\end{aligned}
\tag{124}
$$

Similarly, the q-axis flux linkage and currents can be expressed as

$$\psi_q = -X_q'' I_q - \frac{(X_q'' - X_{ls})}{(X_q' - X_{ls})} E_d' + \frac{(X_q' - X_q'')}{(X_q' - X_{ls})} \psi_{2q} \tag{125}$$

$$I_{1q} = \frac{1}{X_{mq}} \left(-E_d' + (X_q - X_q')(I_q - I_{2q}) \right) \tag{126}$$

$$I_{2q} = \frac{(X_q' - X_q'')}{(X_q' - X_{ls})^2} (E_d' + \psi_{2q} + I_q(X_q' - X_{ls})) \tag{127}$$

$$T''_{d0} = \frac{1}{\omega_{base} R_{1d}} \left(X_{l1d} + \frac{X_{md} X_{lfd}}{X_{md} + X_{lfd}} \right) = \frac{1}{\omega_{base} R_{1d}} \left(\frac{X_{l1d} X_{md} + X_{l1d} X_{lfd} + X_{md} X_{lfd}}{X_{fd}} \right)$$

$$= \frac{1}{\omega_{base} R_{1d}} \left(\frac{\Delta}{X_{fd}} \right) = \frac{1}{\omega_{base} R_{1d}} \left(\frac{(X'_d - X_{ls})^2}{(X'_d - X''_d)} \right)$$

Similarly,

$$T''_{q0} = \frac{1}{\omega_{base} R_{2q}} \left(\frac{(X'_q - X_{ls})^2}{(X'_q - X''_q)} \right)$$

By substituting the equations (122) - (127) in the voltage equations given in (54) - (57), we get

$$\frac{1}{\omega_{base}} \frac{d\psi_d}{dt} = V_d + R_s I_d + \frac{\omega}{\omega_{base}} \psi_q \tag{128}$$

$$\frac{1}{\omega_{base}} \frac{d\psi_q}{dt} = V_q + R_s I_q - \frac{\omega}{\omega_{base}} \psi_d \tag{129}$$

$$\frac{1}{\omega_{base}} \frac{d\psi_0}{dt} = V_0 + R_s I_0 \tag{130}$$

$$T'_{d0}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)[I_d - \frac{(X'_d - X''_d)}{(X'_d - X_{ls})^2}(-E'_q + (X'_d - X_{ls})I_d + \psi_{1d}] + E_{fd} \tag{131}$$

$$T''_{d0}\frac{d\psi_{1d}}{dt} = E'_q - (X'_d - X_{ls})I_d - \psi_{1d} \tag{132}$$

$$T'_{q0}\frac{dE'_d}{dt} = -E'_d + (X_q - X'_q)\left(I_q - \frac{(X'_q - X''_q)}{(X'_q - X_{ls})^2}(E'_d + (X'_q - X_{ls})I_q + \psi_{2q})\right. \tag{133}$$

$$T''_{q0}\frac{d\psi_{2q}}{dt} = -E'_d - (X'_q - X_{ls})I_q - \psi_{2q} \tag{134}$$

$$\psi_d = -X''_d I_d + \frac{(X''_d - X_{ls})}{(X'_d - X_{ls})}E'_q + \frac{(X'_d - X''_d)}{(X'_d - X_{ls})}\psi_{1d} \tag{135}$$

$$\psi_q = -X''_q I_q - \frac{(X''_q - X_{ls})}{(X'_q - X_{ls})}E'_d + \frac{(X'_q - X''_q)}{(X'_q - X_{ls})}\psi_{2q} \tag{136}$$

Apart from the above equations, the swing equation also needs to be considered.

$$\frac{d\delta}{dt} = \omega - \omega_s \tag{137}$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_m - T_e - D(\omega - \omega_{base}) \tag{138}$$

where $T_e = \psi_d I_q - \psi_q I_d$

Example 1

A 555 MVA, 24 kV, 0.9 pf, 60 Hz, 3 phase, 2 pole synchronous generator has the following inductances and resistances associated with the stator and rotor field windings: (P. Kundur Example 3.1)

$$\begin{aligned}
l_{aa} &= 3.2758 + 0.0458\cos(2\theta) \text{ mH} \\
l_{ab} &= -1.6379 - 0.0458\cos(2\theta + 60^\circ) \text{ mH} \\
l_{ls} &= 0.4129 \text{ mH} \\
l_{afd} &= 40\cos\theta \text{ mH}; \quad l_{fd} = 576.92 \text{ mH} \\
r_s &= 0.0031\ \Omega; \quad r_{fd} = 0.0715\ \Omega
\end{aligned}$$

1. Determine l_d and l_q in Henrys.
2. Determine x_{md} and x_{mq} in Ohms.
3. Determine the per unit values all the parameters in $dq0$ frame.

Example 1 (contd.)

1. We know that

$$l_d = \frac{3}{2}(l_{aa0} + l_{aap}); \quad l_q = \frac{3}{2}(l_{aa0} - l_{aap})$$

(refer to slide 28).

$$l_d = \frac{3}{2}(3.2758 + 0.0458) = 4.9824 \text{ mH}$$

$$l_q = \frac{3}{2}(3.2758 - 0.0458) = 4.845 \text{ mH}$$

2.

$$x_{md} = x_d - x_{ls} \quad x_{mq} = x_q - x_{ls}$$

(refer to slide 49 and equation (81)) Let us first find inductances

$$l_{md} = l_d - l_{ls} = 4.9824 - 0.4129 = 4.5695 \text{ mH}$$

$$l_{mq} = l_q - l_{ls} = 4.845 - 0.4129 = 4.4321 \text{ mH}$$

$$x_{md} = \omega_{base} l_{md} = 2 \times \pi \times 60 \times 4.5695 = 1.7227 \ \Omega$$

$$x_{mq} = \omega_{base} l_{mq} = 2 \times \pi \times 60 \times 4.4321 = 1.6709 \ \Omega$$

Example 1 (contd.)

3. Let us find the base quantities (refer to slides 32, 33, & 34).

$$V_{base} = \frac{24}{\sqrt{3}} = 13.8564 \text{ kV} \quad S_{base} = 555 \text{ MVA}$$

$$I_{base} = \frac{S_{base}}{3V_{base}} = \frac{555}{3 \times 13.8564} = 13.3512 \text{ kA}$$

$$V_{dq0,base} = \sqrt{2}V_{base} = \sqrt{2} \times 13.8564 = 19.5959 \text{ kV}$$

$$I_{dq0,base} = \sqrt{2}I_{base} = 18.8814 \text{ kA}$$

$$Z_{dq0,base} = \frac{V_{dq0,base}}{I_{dq0,base}} = \frac{19.56}{18.8814} = 1.0378\ \Omega$$

$$L_{dq0,base} = \frac{Z_{dq0,base}}{\omega_{base}} = \frac{1.0378}{2 \times \pi \times 60} = 2.753 \text{ mH}$$

Example 1 (contd.)

$$I_{fd,base} = \frac{l_{md}}{l_{sfd}} I_{dq0,base} = \frac{4.5695}{40} \times 18.8814 = 2.158 \text{ kA}$$

$(l_{sfd} = l_{afd})$.

$$V_{fd,base} = \frac{S_{base}}{I_{fd,base}} = \frac{3}{2}\frac{l_{md}}{l_{sfd}} V_{dq0,base} = 257.183 \text{ kV}$$

$$Z_{fd,base} = \frac{V_{fd,base}}{I_{fd,base}} = 119.18\ \Omega$$

$$L_{fd,base} = \frac{Z_{fd,base}}{\omega_{base}} = 316.12 \text{ mH}$$

Example 1 (contd.)

The per unit values are

$$X_{md} = \frac{1.7227}{1.0378} = 1.66 \text{ p.u.}$$

$$X_{mq} = \frac{1.6709}{1.0378} = 1.61 \text{ p.u.}$$

$$L_{md} = \frac{4.5695}{2.753} = 1.66 \text{ p.u.}$$

$$L_{mq} = \frac{4.4321}{2.753} = 1.61 \text{ p.u.}$$

In the per unit system, $L = X$.

Example 1 (contd.)

$$X_{fd} = L_{fd} = \frac{l_{fd}}{L_{fd,base}} = \frac{576.92}{316.12} = 1.825 \text{ p.u}$$

$$R_s = \frac{0.0031}{1.0378} = 0.003 \text{ p.u.}$$

$$R_{fd} = \frac{0.0715}{119.18} = 0.0006 \text{ p.u.}$$

Steady State Analysis

1. In the steady state, the rotor rotates at the synchronous speed ($\omega = \omega_{base}$).
2. Damper winding currents (I_{1d}, I_{1q}, I_{2q}) are zero.
3. In steady state, all the differential components vanish.
4. In steady state, the zero sequence component is zero.

In steady state, the equations (128) - (137) become,

$$V_d = -R_s I_d - \psi_q \tag{139}$$
$$V_q = -R_s I_q + \psi_d \tag{140}$$
$$E_{fd} = E'_q + (X_d - X'_d)I_d \tag{141}$$
$$\psi_{1d} = E'_q - (X'_d - X_{ls})I_d \tag{142}$$
$$E'_d = (X_q - X'_q)I_q \tag{143}$$
$$\psi_{2q} = -E'_d - (X'_q - X_{ls})I_q \tag{144}$$

The d-axis flux linkage equations given in (88) - (90) in the steady state are

$$\psi_d = X_d(-I_d) + X_{md} I_{fd} \tag{145}$$
$$\psi_{fd} = X_{md}(-I_d) + X_{fd} I_{fd} \tag{146}$$
$$\psi_{1d} = X_{md}(-I_d) + X_{md} I_{fd} \tag{147}$$

The q-axis flux linkage equations given in (91) - (93) in the steady state are

$$\psi_q = X_q(-I_q) \tag{148}$$
$$\psi_{1q} = X_{mq}(-I_q) \tag{149}$$
$$\psi_{2q} = X_{mq}(-I_q) \tag{150}$$

Substituting I_{fd} from (146) in (145), we get

$$\begin{aligned}\psi_d &= X_d(-I_d) + X_{md}\left(\frac{\psi_{fd} + X_{md}I_d}{X_{fd}}\right)\\ &= -I_d\left(X_d - \frac{X_{md}^2}{X_{fd}}\right) + \frac{X_{md}}{X_{fd}}\psi_{fd}\\ &= -I_dX'_d + E'_q\end{aligned} \tag{151}$$

From (144) and (150),

$$\begin{aligned} -X_{mq} I_q &= -E'_d - (X'_q - X_{ls}) I_q \\ (X_{mq} + X_{ls})(-I_q) &= -E'_d - X'_q I_q \\ X_q(-I_q) &= -E'_d - X'_q I_q \\ \psi_q &= -E'_d - X'_q I_q \end{aligned} \tag{152}$$

Substituting (151) and (152) in (139) and (140),

$$V_d = -R_s I_d + X'_q I_q + E'_d \tag{153}$$

$$V_q = -R_s I_q - X'_d I_d + E'_q \tag{154}$$

Let the three phase terminal voltage in per unit be

$$\begin{aligned} v_{ta} &= V_m \cos(\omega t + \alpha) \\ v_{tb} &= V_m \cos(\omega t + \alpha - 120^\circ) \\ v_{tc} &= V_m \cos(\omega t + \alpha + 120^\circ) \end{aligned} \tag{155}$$

Applying the Park's transformation, we get

$$\begin{aligned} V_d &= V_m \cos(\omega t + \alpha - \theta) \\ V_q &= V_m \sin(\omega t + \alpha - \theta) \end{aligned} \tag{156}$$

θ is the angle between d-axis and the stationary axis a. Let

$$\theta = \omega t + \theta_0$$

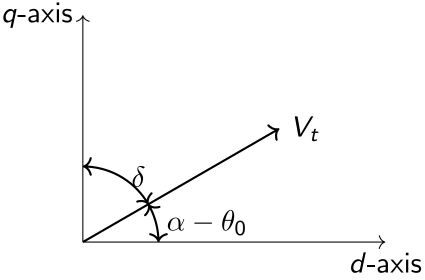
where θ_0 is rotor position at $t = 0$.

In per unit system, peak value and RMS value are the same.

$$\begin{aligned} V_d &= V_t \cos(\alpha - \theta_0) \\ V_q &= V_t \sin(\alpha - \theta_0) \end{aligned} \tag{157}$$

Though V_d adn V_q are scalar quantities, they can be represented as phasors due to their trigonometric relations.

$$V_t \angle \alpha - \theta_0 = V_d + \jmath V_q \tag{158}$$



Let us define δ which is the angle between q-axis and V_t.

$$\delta = 90^\circ - (\alpha - \theta_0)$$

$$V_d + \jmath V_q = V_t(\sin\delta + \jmath\cos\delta) = \overline{V_t} \tag{159}$$

Similarly, the stator current I_t lagging V_t by ϕ can also be expressed as

$$I_d + \jmath I_q = I_t(\sin(\delta+\phi) + \jmath\cos(\delta+\phi)) = \overline{I_t} \tag{160}$$

Substituting (153) and (154) in (159), we get

$$\overline{V_t} = -R_s I_d + X'_q I_q + E'_d + \jmath(-R_s I_q - X'_d I_d + E'_q) \tag{161}$$

Rearranging (161),

$$\begin{aligned}\overline{V_t} &= -R_s I_d + X_q' I_q + E_d' + \jmath(-R_s I_q - X_d' I_d + E_q') \\ &= -R_s(I_d + \jmath I_q) - \jmath X_q(I_d + \jmath I_q) + \jmath X_q(I_d + \jmath I_q) + X_q' I_q + E_d' + \jmath(-X_d' I_d + E_q') \\ &= -(R_s + \jmath X_q)\overline{I_t} + E_d' - (X_q - X_q')I_q + \jmath((X_q - X_d')I_d + E_q' + X_d I_d - X_d I_d) \\ &= -(R_s + \jmath X_q)\overline{I_t} + E_d' - (X_q - X_q')I_q + \jmath((X_q - X_d)I_d + (X_d - X_d')I_d + E_q')\end{aligned}$$

From (143), $E_d' - (X_q - X_q')I_q = 0$. From (141), $E_{fd} = E_q' + (X_d - X_d')I_d$.

$$\overline{V_t} = -(R_s + \jmath X_q)\overline{I_t} + \jmath((X_q - X_d)I_d + E_{fd}) \tag{162}$$

Since $E_{fd} = \frac{X_{md}}{R_{fd}} V_{fd} = X_{md} I_{fd}$,

$$\jmath((X_q - X_d)I_d + X_{md} I_{fd}) = \overline{V_t} + (R_s + \jmath X_q)\overline{I_t} \tag{163}$$

Let the internal voltage of the generator be $E_q = (X_q - X_d)I_d + X_{md}I_{fd}$.

$$\begin{aligned} \jmath E_q &= \overline{V_t} + (R_s + \jmath X_q)\overline{I_t} \\ \overline{E_q} &= \overline{V_t} + (R_s + \jmath X_q)\overline{I_t} \end{aligned} \tag{164}$$

$\overline{E_q}$ is defined as the internal voltage behind the synchronous reactance X_q. Equation (164) can be represented as

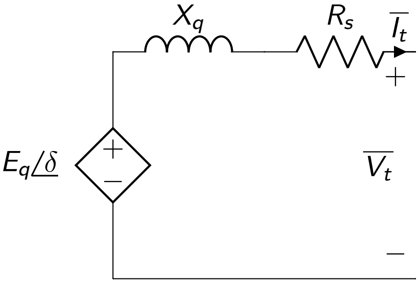


Figure: Equivalent circuit of steady state synchronous generator

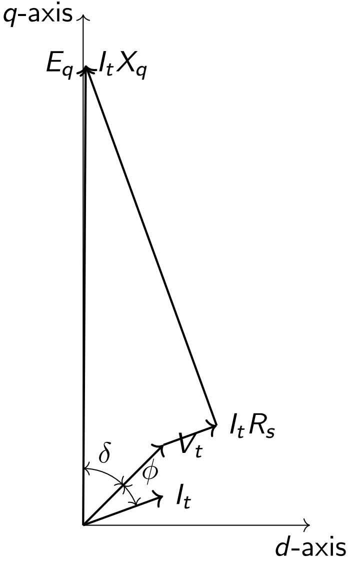


Figure: Phasor diagram

Under no-load or open-circuit conditions, $I_d = I_q = 0$. From (143), (140), (141) , (143), (145), and (148) we get

$$V_q = E_q = E_q' = E_{fd} = V_t = \psi_d = X_{md} I_{fd} \tag{165}$$

$$V_d = E_d' = \psi_q = 0 \tag{166}$$

Therefore

$$\overline{E_q} = \jmath X_{md} I_{fd} \tag{167}$$

The terminal voltage

$$\overline{V_t} = V_d + \jmath V_q = \jmath V_q = \jmath X_{md} I_{fd} \tag{168}$$

Steady state - No Saliency

If saliency is neglected,

$$X_d = X_q = X_s$$

where X_s is the synchronous reactance. Therefore,

$$\overline{E_q} = \overline{V_t} + (R_s + \jmath X_s)\overline{I_t} \tag{169}$$

with $X_d = X_q$, the magnitude of E_q is

$$E_q = X_{md} I_{fd} \tag{170}$$

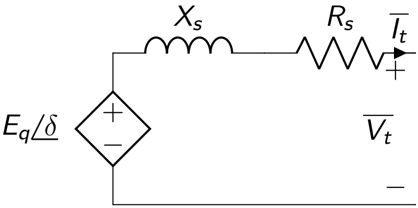


Figure: Steady state equivalent circuit with saliency neglected

Active and Reactive Power

The terminal complex power is

$$\begin{aligned} S &= \overline{V_t}\,\overline{I_t}^* \\ &= (V_d + \jmath V_q)(I_d - \jmath I_q) \\ &= (V_d I_d + V_q I_q) + \jmath(V_q I_d - V_d I_q) \end{aligned}$$

$$P_t = V_d I_d + V_q I_q \tag{171}$$

$$Q_t = V_q I_d - V_d I_q \tag{172}$$

The steady state torque is given by

$$T_e = \psi_d I_q - \psi_q I_d$$

From (139) and (140)

$$\begin{aligned} T_e &= (V_q + R_s I_q) I_q + (V_d + R_s I_d) I_d \\ &= V_d I_d + V_q I_q + (I_d^2 + I_q^2) R_s \qquad (173) \\ &= P_t + I_t^2 R_s = \text{Real}(\overline{E_q}\,\overline{I_t}^*) \end{aligned}$$

Steady state values - Procedure

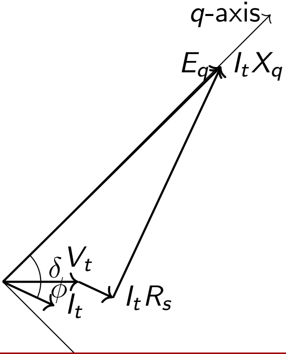
For stability studies, it is necessary to find the initial steady-state values.

1. Normally, P_t, Q_t and V_t are specified. From the terminal conditions,

$$I_t = \frac{\sqrt{P_t^2 + Q_t^2}}{V_t}, \quad \phi = \cos^{-1}\left(\frac{P_t}{V_t I_t}\right)$$

$$\overline{E_q} = V_t\angle 0^\circ + I_t\angle{-\phi}(R_s + \jmath X_q) \tag{174}$$

2. Find δ.



$$\delta = \tan^{-1}\left(\frac{I_t X_q \cos\phi - I_t R_s \sin\phi}{V_t + I_t R_s \cos\phi + I_t X_q \sin\phi}\right) \tag{175}$$

Steady state values - Procedure (contd.)

3. With δ known, the dq components of stator voltage and current are given by

$$V_d = V_t \sin\delta \tag{176}$$

$$V_q = V_t \cos\delta \tag{177}$$

$$I_d = I_t \sin(\delta + \phi) \tag{178}$$

$$I_q = I_t \cos(\delta + \phi) \tag{179}$$

Steady state values - Procedure (contd.)

4. The remaining machine quantities are computed as follows:

$$\psi_d = V_q + R_s I_q$$
$$\psi_q = -(V_d + R_s I_d)$$
$$I_{fd} = \frac{\psi_d + X_d I_d}{X_{md}}$$
$$E_{fd} = I_{fd} R_{fd}$$
$$\psi_{fd} = X_{md}(-I_d) + X_{fd} I_{fd}$$
$$\psi_{1d} = X_{md}(I_{fd} - I_d)$$
$$\psi_{1q} = \psi_{2q} = -X_{mq} I_q$$
$$T_e = P_t + I_t^2 R_s$$

Example 2

Consider a synchronous machine serving a load with

$$\overline{V_t} = 1.0\underline{/10^\circ} \quad \overline{I_t} = 0.5\underline{/-20^\circ}$$

It has $X_d = 1.2$, $X_q = 1.0$, $X_{md} = 1.1$, $X'_d = 0.232$ and $R_s = 0$.
Find δ, V_d, V_q, I_d, I_q, ψ_d, ψ_q, E_q, E_{fd}, T_e, P_t.

1. Let us find δ.

$$\delta = \tan^{-1}\left(\frac{I_t X_q \cos\phi - I_t R_s \sin\phi}{V_t + I_t R_s \cos\phi + I_t X_q \sin\phi}\right) = \tan^{-1}\left(\frac{0.5 \times 1 \times \sqrt{3}/2}{1 + 0.5 \times 1 \times 1/2}\right)$$

$$\delta = 19.1066^\circ$$

Example 2 (contd.)

2. $$V_d = V_t \sin\delta = 1 \times \sin(19.1^\circ) = 0.3273$$
$$V_q = V_t \cos\delta = 1 \times \cos(19.1^\circ) = 0.9449$$
$$I_d = I_t \sin(\delta + \phi) = 0.5 \times \sin(19.1^\circ + 30^\circ) = 0.3780$$
$$I_q = I_t \cos(\delta + \phi) = 0.5 \times \cos(19.1^\circ + 30^\circ) = 0.3273$$

3. $$\psi_d = V_q + R_s I_q = 0.9449$$
$$\psi_q = -(V_d + R_s I_d) = -0.3273$$
$$I_{fd} = \frac{\psi_d + X_d I_d}{X_{md}} = \frac{0.9449 + 1.2 \times 0.3780}{1.1} = 1.2713$$
$$E_q = (X_q - X_d) I_d + X_{md} I_{fd} = (1 - 1.2) \times 0.3780 + 1.1 \times 1.2161 = 1.3229$$

Example 2 (contd.)

4. $$T_e = \psi_d I_q - \psi_q I_d = 0.9449 \times 0.3273 + 0.3273 \times 0.3780 = 0.4330$$

$$P_t = \text{Real}(\overline{V_t}\ \overline{I_t}^*) = \text{Real}(1.0\underline{/10^\circ} \times 0.5\underline{/20^\circ}) = 0.4330$$

$$T_e = P_t + I_t^2 R_s$$

Since $R_s = 0$, $T_e = P_t$.

Saturation Effect

Assumptions

1. Saturation does not affect leakage inductances as the path of leakage fluxes is mainly air.
2. Saturation affects only mutual inductances.
3. The effect of saturation on mutual inductances can be computed through the open circuit characteristics of machines.
4. There is no saturation effect on mutual coupling between d and q axes.

Therefore, the effects of saturation may be represented as

$$X_{md,sat} = K_{sd} X_{md,unsat}, \quad X_{mq,sat} = K_{sq} X_{mq,unsat}$$

where K_{sd} and K_{sq} are the degrees of saturation in the d and q-axis, respectively.

At no load, $I_d = I_q = 0$. From (145) and (148),

$$\psi_d = X_{md} I_{fd}, \quad \psi_q = 0 \tag{180}$$

From stator voltage equations,

$$V_t = \sqrt{V_d^2 + V_q^2} = \psi_d \tag{181}$$

It shows that in per unit the terminal voltage and the stator flux linkage are the same.
At no load, the stator flux linkage is

$$\psi_t = \sqrt{\psi_d^2 + \psi_q^2} = \psi_d = V_t \tag{182}$$

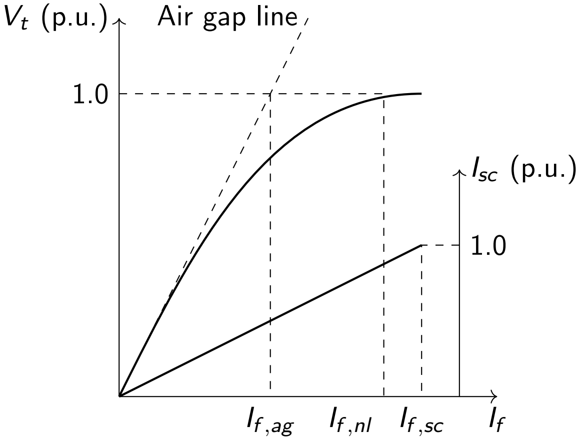


Figure: Open circuit characteristics of a synchronous generator if we neglect R_a,

$$KI_{f,sc} = 1.0X_{s,unsat} \tag{183}$$

where K is the slope of the air gap line.
To produce 1.0 p.u. terminal voltage,

$$KI_{f,ag} = 1.0 \tag{184}$$

Hence,

$$X_{s,unsat} = \frac{I_{f,sc}}{I_{f,ag}} \tag{185}$$

Similarly, the saturated synchronous reactance is

$$X_{s,sat} = \frac{I_{f,sc}}{I_{f,nl}} \tag{186}$$

Let us define the short circuit ratio (SCR).

$$\text{SCR} = \frac{I_{f,nl}}{I_{f,sc}} = \frac{1}{X_{s,sat}} \tag{187}$$

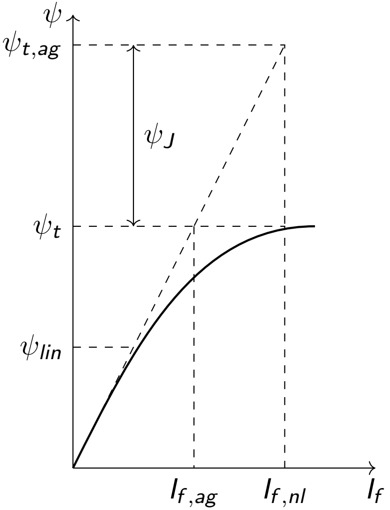


Figure: Open circuit characteristics

$$X_{md,unsat} = \frac{\psi_t}{I_{f,ag}} = \frac{\psi_{t,ag}}{I_{f,nl}} \tag{188}$$

$$X_{md,sat} = \frac{\psi_t}{I_{f,nl}} \tag{189}$$

From (188) and (189),

$$X_{md,sat} = \frac{\psi_t}{\psi_{t,ag}} X_{md,unsat} = \frac{\psi_t}{\psi_t + \psi_J} X_{md,unsat} \tag{190}$$

If $\psi_t \leq \psi_{lin}$,

$$\psi_J = 0; \quad X_{md,sat} = X_{md,unsat}$$

If $\psi_t > \psi_{lin}$,

$$\psi_J = A_{sat} e^{B_{sat}(\psi_t - \psi_{lin})}$$

1. Cylindrical pole rotor
$$X_{mq,sat} = X_{md,sat}$$
2. Salient pole rotor: The effect of saturation on q-axis is neglected as the air gap is more along q-axis.

It can be extended to loaded conditions as given below.

$$\begin{aligned}\psi_d &= (X_{ls} + X_{md})(-I_d) + X_{md}I_{fd} + X_{md}I_{1d} = -X_{ls}I_d + \psi_{md} \\ \psi_q &= (X_{ls} + X_{mq})(-I_q) + X_{mq}I_{1q} + X_{mq}I_{2q} = -X_{ls}I_q + \psi_{mq}\end{aligned} \tag{191}$$

Substituting the stator voltage equations in steady state given in (139) and (140) in (191),

$$\begin{aligned}\psi_{md} &= \psi_d + X_{ls}I_d = V_q + R_sI_q + X_{ls}I_d \\ \psi_{mq} &= \psi_q + X_{ls}I_q = -V_d - R_sI_d + X_{ls}I_q\end{aligned} \tag{192}$$

$$\psi_{md} + \jmath\psi_{mq} = V_q + R_sI_q + X_{ls}I_d + \jmath(-V_d - R_sI_d + X_{ls}I_q) \tag{193}$$

Let E_l be the voltage behind leakage reactance.

$$E_l = V_t + (R_s + \jmath X_{ls}) I_t \tag{194}$$

From (193) and (194),

$$\psi_t = \sqrt{\psi_{md}^2 + \psi_{mq}^2} = |E_l| \tag{195}$$

- ψ_t computed from (195) is used to calculate $X_{md,sat}$ and $X_{mq,sat}$.
- They have to be used for calculating sub-transient and transient reactances and time constants.