EE549 - Power System Dynamics and Control

Synchronous Machine Representation for Stability Studies

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The following equations describe the complete dynamic behavior of a steam turbine generator.

$$\frac{1}{\omega_{base}}\frac{d\psi_d}{dt} = V_d + R_s I_d + \frac{\omega}{\omega_{base}}\psi_q \tag{1}$$

$$\frac{1}{\omega_{base}}\frac{d\psi_q}{dt} = V_q + R_s I_q - \frac{\omega}{\omega_{base}}\psi_d \tag{2}$$

$$\frac{1}{\omega_{base}}\frac{d\psi_0}{dt} = V_0 + R_s I_0 \tag{3}$$

$$\begin{aligned} T'_{d0}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)[I_d - \frac{(X'_d - X''_d)}{(X'_d - X_{ls})^2}(-E'_q \\ + (X'_d - X_{ls})I_d + \psi_{1d}] + E_{fd} \end{aligned} \tag{4}$$

$$T''_{d0}\frac{d\psi_{1d}}{dt} = E'_q - (X'_d - X_{ls})I_d - \psi_{1d} \tag{5}$$

$$T'_{q0}\frac{dE'_d}{dt} = -E'_d + (X_q - X'_q)\left(I_q - \frac{(X'_q - X''_q)}{(X'_q - X_{ls})^2}(E'_d + (X'_q - X_{ls})I_q + \psi_{2q})\right) \tag{6}$$

$$T''_{q0}\frac{d\psi_{2q}}{dt} = -E'_d - (X'_q - X_{ls})I_q - \psi_{2q} \tag{7}$$

$$\psi_d = -X''_d I_d + \frac{(X''_d - X_{ls})}{(X'_d - X_{ls})}E'_q + \frac{(X'_d - X''_d)}{(X'_d - X_{ls})}\psi_{1d} \tag{8}$$

$$\psi_q = -X''_q I_q - \frac{(X''_q - X_{ls})}{(X'_q - X_{ls})}E'_d + \frac{(X'_q - X''_q)}{(X'_q - X_{ls})}\psi_{2q} \tag{9}$$

$$\frac{d\delta}{dt} = \omega - \omega_s \tag{10}$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_m - T_e - D(\omega - \omega_{base}) \tag{11}$$

$$T_E \frac{dE_{fd}}{dt} = -(K_E + S_E(E_{fd}))E_{fd} + V_R \tag{12}$$

$$T_A \frac{dV_R}{dt} = -V_R + K_A R_F - \frac{K_A K_F}{T_F} E_{fd} + K_A(V_{ref} - V_t), \; V_R^{min} < V_R < V_R^{max} \tag{13}$$

$$T_F \frac{dR_F}{dt} = -R_F + \frac{K_F}{T_F} E_{fd} \tag{14}$$

$$T_{RH} \frac{dT_M}{dt} = -\Delta T_M + \left(1 - \frac{K_{HP} T_{RH}}{T_{CH}}\right) P_{CH} + \frac{K_{HP} T_{RH}}{T_{CH}} P_{SV} \tag{15}$$

$$T_{CH} \frac{dP_{CH}}{dt} = -P_{CH} + P_{SV} \tag{16}$$

$$T_{SV} \frac{dP_{SV}}{dt} = -\frac{1}{R_D} \left(\frac{\omega}{\omega_{base}} - 1\right) + P_{ref} - P_{SV}, \quad 0 \leq P_{SV} \leq P_{SV}^{max} \tag{17}$$

Steady State Conditions

1. To find the interaction of a synchronous machine with the network, steady state conditions are used.
2. In the steady state, the rotor rotates at the synchronous speed $(\omega = \omega_{base})$.
3. Damper winding currents (I_{1d}, I_{1q}, I_{2q}) are zero.
4. In steady state, all the differential components vanish.

Therefore, the equations (1) - (7) can be written as follows:

$$V_d = -R_s I_d - \psi_q \tag{18}$$

$$V_q = -R_s I_q + \psi_d \tag{19}$$

$$E_{fd} = E'_q + (X_d - X'_d) I_d \tag{20}$$

$$\psi_{1d} = E'_q - (X'_d - X_{ls}) I_d \tag{21}$$

$$E'_d = (X_q - X'_q) I_q \tag{22}$$

$$\psi_{2q} = -E'_d - (X'_q - X_{ls}) I_q \tag{23}$$

The d-axis flux linkage equations in the steady state

$$\psi_d = X_d(-I_d) + X_{md} I_{fd} \tag{24}$$

$$\psi_{fd} = X_{md}(-I_d) + X_{fd} I_{fd} \tag{25}$$

Substituting I_{fd} from (25) in (24), we get

$$\begin{aligned}\psi_d &= X_d(-I_d) + X_{md}\left(\frac{\psi_{fd} + X_{md} I_d}{X_{fd}}\right) \\ &= -I_d\left(X_d - \frac{X_{md}^2}{X_{fd}}\right) + \frac{X_{md}}{X_{fd}}\psi_{fd} \\ &= -I_d X_d' + E_q'\end{aligned} \tag{26}$$

Similarly,

$$\psi_q = -X_q' I_q - E_d' \tag{27}$$

Substituting (26) and (27) in (18) and (19),

$$V_d = -R_s I_d + X_q' I_q + E_d' \tag{28}$$

$$V_q = -R_s I_q - X_d' I_d + E_q' \tag{29}$$

Let the three phase terminal voltage in per unit be

$$\begin{aligned} v_{ta} &= V_m \cos(\omega t + \alpha) \\ v_{tb} &= V_m \cos(\omega t + \alpha - 120^\circ) \\ v_{tc} &= V_m \cos(\omega t + \alpha + 120^\circ) \end{aligned} \tag{30}$$

Applying the Park's transformation, we get

$$\begin{aligned} V_d &= V_m \cos(\omega t + \alpha - \theta) \\ V_q &= V_m \sin(\omega t + \alpha - \theta) \end{aligned} \tag{31}$$

θ is the angle between d-axis and the stationary axis a. Let

$$\theta = \omega t + \theta_0$$

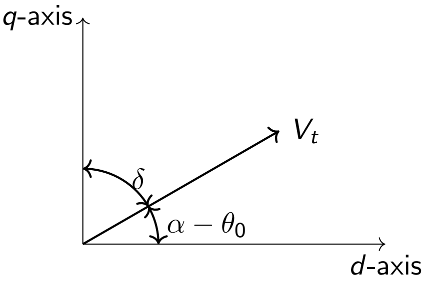
where θ_0 is rotor position at $t = 0$.

In per unit system, peak value and RMS value are the same.

$$\begin{aligned} V_d &= V_t \cos(\alpha - \theta_0) \\ V_q &= V_t \sin(\alpha - \theta_0) \end{aligned} \tag{32}$$

Though V_d adn V_q are scalar quantities, they can be represented as phasors due to their trigonometric relations.

$$V_t \angle \alpha - \theta_0 = V_d + \jmath V_q \tag{33}$$



Let us define δ which is the angle between q-axis and V_t.

$$\delta = 90^\circ - (\alpha - \theta_0)$$

$$V_d + \jmath V_q = V_t(\sin\delta + \jmath\cos\delta) = \overline{V_t} \tag{34}$$

Similarly, the stator current I_t lagging V_t by ϕ can also be expressed as

$$I_d + \jmath I_q = I_t(\sin(\delta+\phi) + \jmath\cos(\delta+\phi)) = \overline{I_t} \tag{35}$$

Substituting (28) and (29) in (34), we get

$$\overline{V_t} = -R_s I_d + X_q' I_q + E_d' + \jmath(-R_s I_q - X_d' I_d + E_q') \tag{36}$$

Rearranging (36),

$$\begin{aligned}\overline{V_t} &= -R_s I_d + X_q' I_q + E_d' + \jmath(-R_s I_q - X_d' I_d + E_q') \\ &= -R_s(I_d + \jmath I_q) - \jmath X_q(I_d + \jmath I_q) + \jmath X_q(I_d + \jmath I_q) + X_q' I_q + E_d' + \jmath(-X_d' I_d + E_q') \\ &= -(R_s + \jmath X_q)\overline{I_t} + E_d' - (X_q - X_q')I_q + \jmath((X_q - X_d')I_d + E_q') \\ &= -(R_s + \jmath X_q)\overline{I_t} + E_d' - (X_q - X_q')I_q + \jmath((X_q - X_d)I_d + (X_d - X_d')I_d + E_q')\end{aligned}$$

From (22), $E_d' - (X_q - X_q')I_q = 0$. From (20), $E_{fd} = E_q' + (X_d - X_d')I_d$.

$$\overline{V_t} = -(R_s + \jmath X_q)\overline{I_t} + \jmath((X_q - X_d)I_d + E_{fd}) \tag{37}$$

Since $E_{fd} = \frac{X_{md}}{R_{fd}} V_{fd} = X_{md} I_{fd}$,

$$\jmath((X_q - X_d)I_d + X_{md} I_{fd}) = \overline{V_t} + (R_s + \jmath X_q)\overline{I_t} \tag{38}$$

Let the internal voltage of the generator be $E_q = (X_q - X_d)I_d + X_{md}I_{fd})$.

$$
\begin{aligned}
\jmath E_q &= \overline{V_t} + (R_s + \jmath X_q)\overline{I_t} \\
\overline{E_q} &= \overline{V_t} + (R_s + \jmath X_q)\overline{I_t}
\end{aligned}
\tag{39}
$$

$\overline{E_q}$ is defined as the internal voltage behind the synchronous reactance X_q. Equation (39) can be represented as

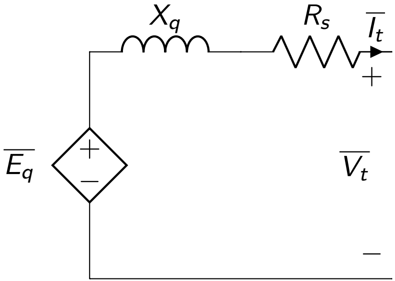


Figure: Equivalent circuit of steady state synchronous generator

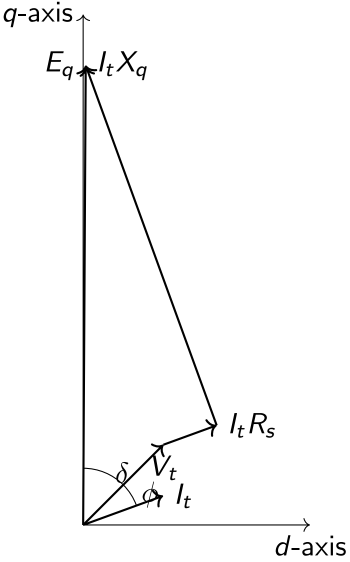
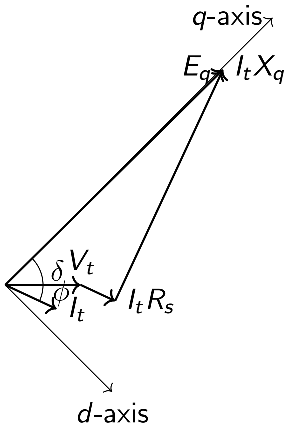


Figure: Phasor diagram

From the terminal conditions,

$$I_t = \frac{\sqrt{P_t^2 + Q_t^2}}{V_t}, \quad \phi = \cos^{-1}\left(\frac{P_t}{V_t I_t}\right)$$

$$\overline{E_q} = V_t\angle 0^\circ + I_t\angle{-\phi}(R_s + \jmath X_q) \tag{40}$$



$$\delta = \tan^{-1}\left(\frac{I_t X_q \cos\phi - I_t R_s \sin\phi}{V_t + I_t R_s \cos\phi + I_t X_q \sin\phi}\right) \tag{41}$$

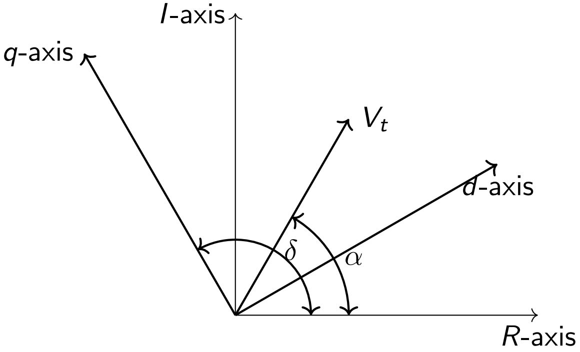
In the steady state at no load, $I_d = I_q = 0$. From (18), (19), (20) , (22), (24), (26) and (27), we get

$$V_q = E_q = E'_q = E_{fd} = V_t = \psi_d = X_{md} I_{fd} \tag{42}$$

$$V_d = E'_d = \psi_q = 0 \tag{43}$$

At no load, $\delta = 0$.

- The load angle in (41) is computed with respect to *dq*-axis of a particular generator.
- In a mutli-machine system, there is a common reference from which all the angles are measured. (slack bus).
- The angle δ should be measured from a common reference.



$$\begin{aligned} V_R &= V_t\cos\alpha = V_d\sin\delta + V_q\cos\delta \\ V_I &= V_t\sin\alpha = -V_d\cos\delta + V_q\sin\delta \end{aligned} \tag{44}$$

$$\begin{aligned} V_t e^{\jmath\alpha} = V_R + \jmath V_I &= (V_d\sin\delta + V_q\cos\delta) + \jmath(-V_d\cos\delta + V_q\sin\delta) \\ &= (V_d + \jmath V_q)(\sin\delta - \jmath\cos\delta) = (V_d + \jmath V_q)e^{\jmath(\delta - \frac{\pi}{2})} \end{aligned} \tag{45}$$

$$(V_d + \jmath V_q) = V_t e^{\jmath(\alpha - \delta + \frac{\pi}{2})} = V_t\sin(\delta - \alpha) + \jmath V_t\cos(\delta - \alpha) \tag{46}$$

To convert from dq-axis frame to a common reference frame, multiply quantities by $e^{\jmath(\delta-\frac{\pi}{2})}$.

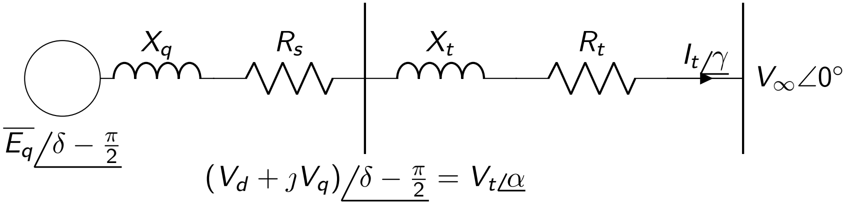


Figure: Single machine connected to an infinite bus

$$\begin{aligned}\jmath E_q e^{\jmath(\delta-\frac{pi}{2})} &= V_t e^{\jmath\alpha} + (R_s+\jmath X_q) I_t e^{\jmath\gamma} \\ E_q e^{\jmath\delta} &= V_t e^{\jmath\alpha} + (R_s+\jmath X_q) I_t e^{\jmath\gamma}\end{aligned} \tag{47}$$

where δ, α and γ are measured with respect to the infinite bus.

The power supplied by the generator at its terminal is

$$\begin{aligned} P_t + \jmath Q_t &= V_t e^{\jmath\alpha} \left(I_t e^{\jmath\gamma}\right)^* = (V_d + \jmath V_q) e^{\jmath(\delta - \frac{\pi}{2})} \left((I_d + \jmath I_q) e^{\jmath(\delta - \frac{\pi}{2})} \right)^* \\ &= (V_d + \jmath V_q)(I_d - \jmath I_q) = (V_d I_d + V_q I_q) + \jmath (V_q I_d - V_d I_q) \end{aligned} \tag{48}$$

The electrical torque output is given as, by substituting (18) and (19),

$$\begin{aligned} T_e &= (\psi_d I_q - \psi_q I_d) = (V_q + R_s I_q) I_q + (V_d + R_s I_d) I_d \\ &= V_d I_d + V_q I_q + I_d^2 R_s + I_q^2 R_s \\ &= P_t + I_t^2 R_s = \text{Real} \left(E_q e^{\jmath\delta} \left(I_t e^{\jmath\gamma}\right)^* \right) \end{aligned} \tag{49}$$

The per unit torque is equal to the real power delivered by the dependent source.

Multi-machine Representation

- In a power system, there are many synchronous generators with their exciter, turbine and speed governor.
- They are connected through a transmission network.
- Loads (static/dynamic) are connected to the transmission network.
- Let there be n_g number of generators in an n bus system.

Assumptions

1. $\frac{d\psi_d}{dt}$ and $\frac{d\psi_q}{dt}$ are neglected.
   - If they are considered, the entire network dynamics will have to be considered. This will increase the computational burden.
   - Stator transients are faster than rotor transients.
2. In the speed induced voltage term, ω is considered as ω_{base}.
   - It does not mean that the speed is constant but the effect is insignificant.
   - This assumption counter balances the first assumption.

The dynamics of an i^{th} generator along with the exciter, steam turbine and speed governor are given as

$$T'_{d0i}\frac{dE'_{qi}}{dt} = -E'_{qi} - (X_{di} - X'_{di})[I_{di} - \frac{(X'_{di} - X''_{di})}{(X'_{di} - X_{lsi})^2}(-E'_{qi} + (X'_{di} - X_{lsi})I_{di} + \psi_{1di}] + E_{fdi} \tag{50}$$

$$T''_{d0i}\frac{d\psi_{1di}}{dt} = E'_{qi} - (X'_{di} - X_{lsi})I_{di} - \psi_{1di} \tag{51}$$

$$T'_{q0i}\frac{dE'_{di}}{dt} = -E'_{di} + (X_{qi} - X'_{qi})\{(I_{qi} - \frac{(X'_{qi} - X''_{qi})}{(X'_{qi} - X_{lsi})^2}(E'_{di} + (X'_{qi} - X_{lsi})I_{qi} + \psi_{2qi})\} \tag{52}$$

$$T''_{q0i}\frac{d\psi_{2qi}}{dt} = -E'_{di} - (X'_{qi} - X_{lsi})I_{qi} - \psi_{2qi} \tag{53}$$

$$\frac{d\delta_i}{dt} = \omega_i - \omega_s \tag{54}$$

$$\frac{2H_i}{\omega_s}\frac{d\omega_i}{dt} = T_{mi} - T_{ei} - D_i(\omega_i - \omega_{base}) \tag{55}$$

$$T_{Ei}\frac{dE_{fdi}}{dt} = -(K_{Ei} + S_{Ei}(E_{fdi}))E_{fdi} + V_{Ri} \tag{56}$$

$$T_A\frac{dV_R}{dt} = -V_R + K_A R_F - \frac{K_A K_F}{T_F}E_{fd} + K_{Ai}(V_{refi} - V_{ti}),\ V_{Ri}^{min} < V_{Ri} < V_{Ri}^{m} \tag{57}$$

$$T_{Fi}\frac{dR_{Fi}}{dt} = -R_{Fi} + \frac{K_{Fi}}{T_{Fi}}E_{fdi} \tag{58}$$

$$T_{RHi}\frac{dT_{Mi}}{dt} = -\Delta T_{Mi} + \left(1 - \frac{K_{HPi}T_{RHi}}{T_{CHi}}\right)P_{CHi} + \frac{K_{HPi}T_{RHi}}{T_{CHi}}P_{SVi} \quad (59)$$

$$T_{CHi}\frac{dP_{CHi}}{dt} = -P_{CHi} + P_{SVi} \quad (60)$$

$$T_{SVi}\frac{dP_{SVi}}{dt} = -\frac{1}{R_{Di}}\left(\frac{\omega_i}{\omega_{base}} - 1\right) + P_{refi} - P_{SVi}, \quad 0 \leq P_{SVi} \leq P_{SVi}^{max} \quad (61)$$

Stator alegraic expressions:

$$-X''_{qi}I_{qi} - \frac{(X''_{qi} - X_{lsi})}{(X'_{qi} - X_{lsi})}E'_{di} + \frac{(X'_{qi} - X''_{qi})}{(X'_{qi} - X_{lsi})}\psi_{2qi} = -V_{di} - R_{si}I_{di} \quad (62)$$

$$-X''_{di}I_{di} + \frac{(X''_{di} - X_{lsi})}{(X'_{di} - X_{lsi})}E'_{qi} + \frac{(X'_{di} - X''_{di})}{(X'_{di} - X_{lsi})}\psi_{1di} = V_{qi} + R_{si}I_{qi} \quad (63)$$

In an n bus system, the power injected into the network can be written as follows:

$$\bar{I}_i = \sum_{j=1}^{n} V_j \underline{/\alpha_j} Y_{ij} \underline{/\theta_{ij}} \tag{64}$$

$$P_i + \jmath Q_i = V_i \underline{/\alpha_i} (\bar{I}_i)^* = \sum_{j=1}^{n} V_i \underline{/\alpha_i} V_j \underline{/-\alpha_j} Y_{ij} \underline{/-\theta_{ij}} \tag{65}$$

The real and reactive power injected at i^{th} bus can be found as

$$P_i = P_{Gi} - P_{Di} = \sum_{j=1}^{n} V_i V_j Y_{ij} \cos(\theta_{ij} + \alpha_j - \alpha_i) \tag{66}$$

$$Q_i = Q_{Gi} - Q_{Di} = -\sum_{j=1}^{n} V_i V_j Y_{ij} \sin(\theta_{ij} + \alpha_j - \alpha_i) \tag{67}$$

The real and reactive power generated at i^{th} generator can be found as

$$\begin{aligned} P_{Gi} &= \text{Real}\left(V_i e^{\jmath\alpha_i}\left((I_{di} + \jmath I_{qi})e^{\jmath(\delta_i - \frac{\pi}{2})}\right)^*\right) \\ &= V_i I_{di}\sin(\delta_i - \alpha_i) + V_i Iqi\cos(\delta_i - \alpha_i) \end{aligned} \tag{68}$$

$$\begin{aligned} Q_{Gi} &= \text{Imag}\left(V_i e^{\jmath\alpha_i}\left((I_{di} + \jmath I_{qi})e^{\jmath(\delta_i - \frac{\pi}{2})}\right)^*\right) \\ &= V_i I_{di}\cos(\delta_i - \alpha_i) - V_i Iqi\sin(\delta_i - \alpha_i) \end{aligned} \tag{69}$$

If the loads are static, they can expressed as a function of voltage. Then the power injected at i^{th} generator bus can be written as

$$\begin{aligned} V_i I_{di} \sin(\delta_i - \alpha_i) + V_i I qi \cos(\delta_i - \alpha_i) - P_{Di}(V_i) \\ = \sum_{j=1}^{n} V_i V_j Y_{ij} \cos(\theta_{ij} + \alpha_j - \alpha_i) \end{aligned} \tag{70}$$

$$\begin{aligned} V_i I_{di} \cos(\delta_i - \alpha_i) - V_i I qi \sin(\delta_i - \alpha_i) - Q_{Di}(V_i) \\ = -\sum_{j=1}^{n} V_i V_j Y_{ij} \sin(\theta_{ij} + \alpha_j - \alpha_i) \end{aligned} \tag{71}$$

$i = 1, 2, \cdots, n_g$.

$$P_{Di} = -\sum_{j=1}^{n} V_i V_j Y_{ij} \cos(\theta_{ij} + \alpha_j - \alpha_i) \qquad (72)$$

$$Q_{Di} = \sum_{j=1}^{n} V_i V_j Y_{ij} \sin(\theta_{ij} + \alpha_j - \alpha_i) \qquad (73)$$

$i = n_g + 1, n_g + 2, \cdots, n.$

- Dynamic equations (50) to (61), stator algebraic equations (62) to (63) and network power balance equations (70) to (73) together are called as differential algebraic equations (DAEs)
- They completely describe the behaviour of a multi-machine power system .
- In case the loads are dynamic loads, the dynamic equations of the loads should also be considered.

Impedance Loads

- If loads are considered as constant impedance type loads, there will be a significant reduction in computational complexity.

Suppose a load at i^{th} bus is represented as constant impedance.

$$P_{Li} + \jmath Q_{Li} = \frac{V_i^2}{Z_{Li}} \tag{74}$$

$$I_{Li} = \frac{P_{Li} - \jmath Q_{Li}}{V_i^*} = \frac{\overline{V_i}}{Z_{Li}^*} \tag{75}$$

The current injected at i^{th} bus is

$$\overline{I_i} = \overline{I_{Gi}} - \overline{I_{Li}} = \overline{I_{Gi}} - \frac{\overline{V_i}}{Z_{Li}^*} = \sum_{j=1}^{n} V_j \underline{/\alpha_j} Y_{ij} \underline{/\theta_{ij}} \tag{76}$$

$$\overline{I_{Gi}} = V_i\underline{/\alpha_i}\, Y_{Li}\underline{/\theta_{Li}} + \sum_{j=1}^{n} V_j\underline{/\alpha_j}\, Y_{ij}\underline{/\theta_{ij}} \quad i = 1, 2, \cdots, n_g \tag{77}$$

$$0 = V_i\underline{/\alpha_i}\, Y_{Li}\underline{/\theta_{Li}} + \sum_{j=1}^{n} V_j\underline{/\alpha_j}\, Y_{ij}\underline{/\theta_{ij}} \quad i = n_g + 1, n_g + 2, \cdots, m \tag{78}$$

The new bus admittance matrix (Y'_{bus}) is formed by adding load impedances as shunt admittances at each bus.

The equations (77) and (78) can be represented as

$$\begin{bmatrix} \overline{I_{G1}} \\ \vdots \\ \overline{I_{Gn_g}} \\ 0_{n_g+1} \\ \vdots \\ 0_n \end{bmatrix} = \mathrm{Y}'_{bus} \begin{bmatrix} \overline{V_1} \\ \vdots \\ \overline{V_{n_g}} \\ \overline{V_{n_g+1}} \\ \vdots \\ \overline{V_n} \end{bmatrix} \tag{79}$$

Let us create n_g additional nodes.

- At i^{th} node, an additional node i' is created with a voltage $\overline{E_i''} = (E_{di}'' + \jmath E_{qi}'')e^{\jmath(\delta_i - \frac{\pi}{2})}$ representing the generator internal voltage.
- The impedance between i and i' is $R_{si} + \jmath X_{di}''$.

The current injected at i' bus is

$$
\begin{aligned}
\overline{I_{i'}} = \overline{I_{Gi}} &= \frac{1}{R_{si} + \jmath X_{di}''}\left((E_{di}'' + \jmath E_{qi}'')e^{\jmath(\delta_i - \frac{\pi}{2})} - V_i e^{\jmath\alpha_i}\right) \\
\overline{I_{i'}} = \overline{I_{Gi}} &= \frac{1}{R_{si} + \jmath X_{di}''}\overline{E_i''} - \frac{1}{R_{si} + \jmath X_{di}''}\overline{V_i}
\end{aligned}
\tag{80}
$$

$i' = 1, 2, \cdots, n_g$.

By combining (79) and (80) , we get

$$\begin{bmatrix} \overline{I_{G1}} \\ \vdots \\ \overline{I_{Gn_g}} \\ 0_1 \\ \vdots \\ 0_{n_g} \\ 0_{n_g+1} \\ \vdots \\ 0_n \end{bmatrix}_{(n_g+n)\times 1} = \begin{bmatrix} Y_{GG} & Y_{GN} \\ Y_{GN}^T & Y'_{bus} \end{bmatrix}_{(n_g+n)\times(n_g+n)} \begin{bmatrix} \overline{E''_1} \\ \vdots \\ \overline{E''_{n_g}} \\ \overline{V_1} \\ \vdots \\ \overline{V_{n_g}} \\ \overline{V_{n_g+1}} \\ \vdots \\ \overline{V_n} \end{bmatrix}_{(n_g+n)\times 1} \tag{81}$$

where

$$Y_{GG} = \begin{bmatrix} \frac{1}{R_{s1}+\jmath X''_{d1}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{1}{R_{sn_g}+\jmath X''_{dn_g}} \end{bmatrix}_{(n_g \times n_g)}$$

$$Y_{GN} = \begin{bmatrix} -\frac{1}{R_{s1}+\jmath X''_{d1}} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -\frac{1}{R_{sn_g}+\jmath X''_{dn_g}} & 0 & \cdots & 0 \end{bmatrix}_{(n_g \times n)}$$

$$\begin{bmatrix} \mathrm{I_G} \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{GG} & Y_{GN} \\ Y_{GN}^T & Y'_{bus} \end{bmatrix} \begin{bmatrix} \mathrm{E}'' \\ \mathrm{V_N} \end{bmatrix} \tag{82}$$

where

$$\mathrm{I_G} = \begin{bmatrix} \overline{I_{G1}} \\ \vdots \\ \overline{I_{Gn_g}} \end{bmatrix}_{n_g \times 1}$$

$$\mathrm{E}'' = \begin{bmatrix} \overline{E''_1} \\ \vdots \\ \overline{E''_{n_g}} \end{bmatrix}_{n_g \times 1}$$

$$\mathrm{V_N} = \begin{bmatrix} \overline{V_1} \\ \vdots \\ \overline{V_{n_g}} \\ \hline \overline{V_{n_g+1}} \\ \vdots \\ \overline{V_n} \end{bmatrix}_{n \times 1}$$

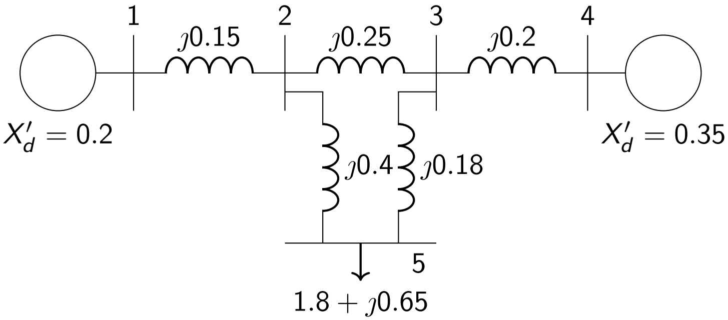
From (82),

$$\mathrm{V_N} = -(\mathrm{Y}'_{bus})^{-1}\mathrm{Y}_{GN}^{T}\mathrm{E}'' \tag{83}$$

$$\mathrm{I_G} = (\mathrm{Y}_{GG} - \mathrm{Y}_{GN}(\mathrm{Y}'_{bus})^{-1}\mathrm{Y}_{GN}^{T})\mathrm{E}'') \tag{84}$$

From (83) and (84), the network voltages and generator currents can be obtained from internal voltages of generators without solving stator algebraic and power balance equations.

Example 1



The voltage at the node 5 is $0.98\underline{/11.53^\circ}$. Assume the load is of constant impedance type. Reduce the bus admittance matrix.

The bus admittance matrix is

$$Y_{bus} = \begin{bmatrix} -j6.67 & j6.67 & 0 & 0 & 0 \\ j6.67 & -j13.16 & j4 & 0 & j2.50 \\ 0 & j4 & -j14.55 & j5 & j5.55 \\ 0 & 0 & j5 & -j5 & 0 \\ 0 & j2.50 & j5.55 & 0 & -j8.05 \end{bmatrix}$$

Since the load is of constant impedance,

$$Y_L = \frac{\overline{I_L}}{\overline{V_5}} = \frac{P_L - jQ_L}{\overline{V_5}^* \overline{V_5}} = 1.8742 - j0.6768$$

The modified bus admittance matrix after adding the load impedance and the generators' internal reactances to the respective nodes is

$$Y'_{bus} = \begin{bmatrix} -j11.67 & j6.67 & 0 & 0 & 0 \\ j6.67 & -j13.16 & j4 & 0 & j2.50 \\ 0 & j4 & -j14.55 & j5 & j5.55 \\ 0 & 0 & j5 & -j7.8571 & 0 \\ 0 & j2.50 & j5.55 & 0 & 1.8742 - j8.7268 \end{bmatrix}$$

With the generator internal nodes, the bus admittance matrix is

$$Y = \begin{bmatrix} Y_{GG} & Y_{GN} \\ Y_{GN}^T & Y'_{bus} \end{bmatrix}$$

where

$$Y_{GG} = \begin{bmatrix} \frac{1}{j0.2} & 0 \\ 0 & \frac{1}{j0.35} \end{bmatrix}$$

$$Y_{GN} = \begin{bmatrix} -\frac{1}{j0.2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{j0.35} & 0 \end{bmatrix}$$

$$Y_{red} = Y_{GG} - Y_{GN}(Y'_{bus})^{-1}Y_{GN}^T$$

$$Y_{red} = \begin{bmatrix} 0.3027 - j1.2608 & 0.2354 + j0.6724 \\ 0.2354 + j0.6724 & 0.1831 - j1.132 \end{bmatrix}$$

Initial Conditions

Initial conditions need to be found before doing any stability analysis. The steps to be followed are

1. Step 1
   - Solve the power balance equations (70) - (73).
   - This is load flow analysis.
   - After load flow analysis, all the network bus complex voltages, real and reactive power injected at a bus are known.
2. Step 2
   Find the internal load angle of each generator.

$$E_i e^{\jmath\delta_i} = V_i e^{\jmath\alpha_i} + (R_{si} + \jmath X_{qi}) I_{Gi} e^{\jmath\gamma_i} \quad i = 1, 2, \cdots, n_g \tag{85}$$

Initial Conditions (contd.)

3. Compute I_d, I_q, V_d and V_q for all the generators.

$$(I_{di} + \jmath I_{qi}) = I_{Gi} e^{\jmath(\gamma_i - \delta_i + \frac{\pi}{2})} \quad (86)$$

$$(V_{di} + \jmath V_{qi}) = V_i e^{\jmath(\alpha_i - \delta_i + \frac{\pi}{2})} \quad (87)$$

$$(88)$$

$i = 1, 2, \cdots, n_g$.

4. Compute E'_{di}, E'_{qi} and E_{fdi}

$$E'_{di} = V_{di} + I_{di}R_{si} - I_{qi}X'_{qi} \quad (89)$$

$$E'_{qi} = V_{qi} + I_{qi}R_{si} + I_{di}X'_{di} \quad (90)$$

$$E_{fdi} = E'_{qi} + (X_{di} - X'_{di})I_{di} \quad (91)$$

$i = 1, 2, \cdots, n_g$.

Initial Conditions (contd.)

5. With E_{fdi} known,

$$V_{Ri} = (K_{Ei} + S_{Ei}(E_{fdi}))E_{fdi} \tag{92}$$

$$R_{Fi} = \frac{K_{Fi}}{T_{Fi}} E_{fdi} \tag{93}$$

$$V_{ref,i} = (\frac{V_{Ri}}{K_{Ai}} + V_i) \tag{94}$$

$$\omega_i = \omega_{base} \tag{95}$$

$$P_{SVi} = P_{ref,i} \tag{96}$$

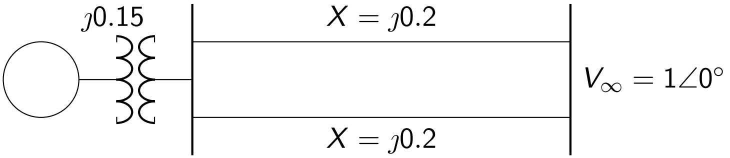
$$P_{CHi} = P_{SVi} \tag{97}$$

$$T_{mi} = P_{CHi} \tag{98}$$

$$T_{ei} = T_{mi} \tag{99}$$

Example 2

A three-phase, 50 Hz, synchronous generator is connected to an infinite bus through a transformer and two parallel transmission lines. The generator is transferring a complex power of $1 + \jmath 0.25$ to the infinite bus.

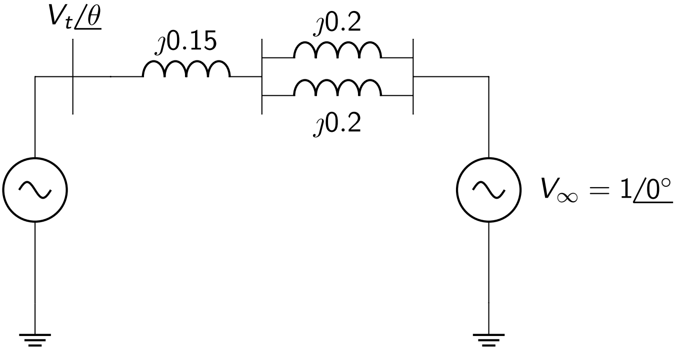


The generator parameters are given below:

$$X_d = 0.8,\ X_q = 0.7,\ X_{ls} = 0.2,\ X_d' = 0.3,\ X_q' = 0.55,\ R_s = 0.0025,$$

$$K_A = 20,\ K_E = 1.0,\ T_E = 0.36,\ K_F = 0.125,\ T_F = 1.8$$

Initialize the synchronous generator.



The current at the infinite bus is

$$I_t\angle\phi = \frac{P - \jmath Q}{1\angle 0^\circ} = 1 - \jmath 0.25 = 1.0308\angle -14.04^\circ$$

The terminal voltage is

$$V_t\angle\theta = 1\angle 0^\circ + \jmath 0.25 \times 1.038\angle -14.04^\circ = 1.0915\angle 13.24^\circ$$

The internal voltage of the synchronous generator is

$$E_q\angle\delta = V_t\angle\theta + (R_s + \jmath X_q)I_t\angle\phi = 1.5617\underline{/37.44^\circ}$$

The other variables are calculated as follows:

$$\begin{aligned}
V_d &= V_t \sin(\delta - \theta) = 0.4474 \\
V_q &= V_t \cos(\delta - \theta) = 0.9950 \\
I_d &= I_t \sin(\delta - \phi) = 0.8064 \\
I_q &= I_t \cos(\delta - \phi) = 0.6420 \\
E'_d &= V_d + I_d R_s - I_q X'_q = 0.0963 \\
E'_q &= V_q + I_q R_s + I_d X'_d = 1.2391 \\
E_{fd} &= E'_q + (X_d - X'_d)I_d = 1.6423
\end{aligned}$$

$$
\begin{aligned}
\psi_{1d} &= E_q' - (X_d' - X_{ls})I_d = 1.158 \\
\psi_{2q} &= -E_d' - (X_q' - X_{ls})I_q = -0.321 \\
R_F &= \frac{K_F}{T_F}E_{fd} = 0.1140 \\
V_R &= (K_E + S_E(E_{fd}))E_{fd} = 1.6467 \\
V_{ref} &= \frac{V_R}{K_A} + V_t = 1.1736 \\
\omega &= \omega_{base} = 314.16 \text{ rad/sec} \\
P_{SV} &= P_{ref} = 1.0 \\
P_{CH} &= P_{SV} = 1.0 \\
T_m &= P_{CH} = 1.0 \\
T_e &= T_m = 1.0
\end{aligned}
$$

Synchronous Machine Models

We can get various models by making some assumptions to reduce computational complexity.

1. Sub-transient model
2. Transient or Two-axis model
3. Flux decay or one-axis model
4. Constant flux linkage or classical model

Sub-transient model

The assumptions made in this model are

1. Stator transients are neglected. $\frac{d\psi_d}{dt} = \frac{d\psi_q}{dt} = 0$.
2. In the speed induced voltage term, ω is considered as ω_{base}.

With these assumptions, the synchronous machine dynamics along with exciter, turbine and governor are given as

$$\psi_q = -V_d - R_s I_d \tag{100}$$

$$\psi_d = V_q + R_s I_q \tag{101}$$

$$V_0 = R_s I_0 = 0 \tag{102}$$

$$T'_{d0}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)[I_d - \frac{(X'_d - X''_d)}{(X'_d - X_{ls})^2}(-E'_q + (X'_d - X_{ls})I_d + \psi_{1d}] + E_{fd} \tag{103}$$

Sub-transient model (contd.)

$$T''_{d0}\frac{d\psi_{1d}}{dt} = E'_q - (X'_d - X_{ls})I_d - \psi_{1d} \tag{104}$$

$$T'_{q0}\frac{dE'_d}{dt} = -E'_d + (X_q - X'_q)\left(I_q - \frac{(X'_q - X''_q)}{(X'_q - X_{ls})^2}(E'_d + (X'_q - X_{ls})I_q + \psi_{2q})\right) \tag{105}$$

$$T''_{q0}\frac{d\psi_{2q}}{dt} = -E'_d - (X'_q - X_{ls})I_q - \psi_{2q} \tag{106}$$

$$\frac{d\delta}{dt} = \omega - \omega_s \tag{107}$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_m - T_e - D(\omega - \omega_{base}) \tag{108}$$

Sub-transient model (contd.)

$$T_E \frac{dE_{fd}}{dt} = -(K_E + S_E(E_{fd}))E_{fd} + V_R \tag{109}$$

$$T_A \frac{dV_R}{dt} = -V_R + K_A R_F - \frac{K_A K_F}{T_F} E_{fd} + K_A(V_{ref} - V_t), \;\; V_R^{min} < V_R < V_R^{max} \tag{110}$$

$$T_F \frac{dR_F}{dt} = -R_F + \frac{K_F}{T_F} E_{fd} \tag{111}$$

$$T_{RH} \frac{dT_M}{dt} = -\Delta T_M + \left(1 - \frac{K_{HP} T_{RH}}{T_{CH}}\right) P_{CH} + \frac{K_{HP} T_{RH}}{T_{CH}} P_{SV} \tag{112}$$

$$T_{CH} \frac{dP_{CH}}{dt} = -P_{CH} + P_{SV} \tag{113}$$

Sub-transient model (contd.)

$$T_{SV}\frac{dP_{SV}}{dt} = -\frac{1}{R_D}\left(\frac{\omega}{\omega_{base}} - 1\right) + P_{ref} - P_{SV}, \quad 0 \le P_{SV} \le P_{SV}^{max} \tag{114}$$

$$\psi_d == -X_d'' I_d + \frac{(X_d'' - X_{ls})}{(X_d' - X_{ls})} E_q' + \frac{(X_d' - X_d'')}{(X_d' - X_{ls})}\psi_{1d} \tag{115}$$

$$\psi_q = -X_q'' I_q + \frac{(X_q'' - X_{ls})}{(X_q' - X_{ls})} E_d' + \frac{(X_q' - X_q'')}{(X_q' - X_{ls})}\psi_{2q} \tag{116}$$

Equations (100) and (101) can be substituted in (115) and (116).

$$V_q + R_s I_q == -X_d'' I_d + \frac{(X_d'' - X_{ls})}{(X_d' - X_{ls})} E_q' + \frac{(X_d' - X_d'')}{(X_d' - X_{ls})}\psi_{1d} \tag{117}$$

$$-V_d - R_s I_d = -X_q'' I_q + \frac{(X_q'' - X_{ls})}{(X_q' - X_{ls})} E_d' + \frac{(X_q' - X_q'')}{(X_q' - X_{ls})}\psi_{2q} \tag{118}$$

Sub-transient model (contd.)

Let us multiply (117) by $\jmath$ and add the resultant equation and (118).

$$V_d + R_s I_d + \jmath(V_q + R_s I_q) = \left(X''_q I_q + \frac{(X''_q + X_{ls})}{(X'_q - X_{ls})} E'_d - \frac{(X'_q - X''_q)}{(X'_q - X_{ls})} \psi_{2q} \right) + \jmath \left(-X''_d I_d + \frac{(X''_d - X_{ls})}{(X'_d - X_{ls})} E'_q + \frac{(X'_d - X''_d)}{(X'_d - X_{ls})} \psi_{1d} \right) \tag{119}$$

Let us add $\jmath X''_d (I_d + \jmath I_q)$ on both sides of (119) and rearrange it.

$$(V_d + \jmath V_q) + (R_s + \jmath X''_d)(I_d + \jmath I_q) = \left((X''_q - X''_d) I_q + \frac{(X''_q + X_{ls})}{(X'_q - X_{ls})} E'_d - \frac{(X'_q - X''_q)}{(X'_q - X_{ls})} \psi_{2q} \right) + \jmath \left(\frac{(X''_d - X_{ls})}{(X'_d - X_{ls})} E'_q + \frac{(X'_d - X''_d)}{(X'_d - X_{ls})} \psi_{1d} \right) \tag{120}$$

Sub-transient model (contd.)

$$(V_d + \jmath V_q) + (R_s + \jmath X_d'')(I_d + \jmath I_q) = \overline{E''} \tag{121}$$

where

$$\overline{E''} = E_d'' + \jmath E_q''$$

$$E_d'' = \left((X_q'' - X_d'')I_q + \frac{(X_q'' + X_{ls})}{(X_q' - X_{ls})} E_d' - \frac{(X_q' - X_q'')}{(X_q' - X_{ls})} \psi_{2q} \right) \tag{122}$$

$$E_q'' = \left(\frac{(X_d'' - X_{ls})}{(X_d' - X_{ls})} E_q' + \frac{(X_d' - X_d'')}{(X_d' - X_{ls})} \psi_{1d} \right) \tag{123}$$

Equation (121) can be further simplified by assuming no saliency effect during sub-transient period. $X_q'' = X_d''$.

Sub-transient model (contd.)

$$(V_d + \jmath V_q) + (R_s + \jmath X_d'')(I_d + \jmath I_q) = E_d'' + \jmath E_q'' \tag{124}$$

where

$$E_d'' = \left(\frac{(X_q'' + X_{ls})}{(X_q' - X_{ls})} E_d' - \frac{(X_q' - X_q'')}{(X_q' - X_{ls})} \psi_{2q} \right) \tag{125}$$

$$E_q'' = \left(\frac{(X_d'' - X_{ls})}{(X_d' - X_{ls})} E_q' + \frac{(X_d' - X_d'')}{(X_d' - X_{ls})} \psi_{1d} \right) \tag{126}$$

Equation (124) can be converted to *RI*-axis (common reference).

$$(V_d + \jmath V_q) e^{\jmath(\delta - \frac{\pi}{2})} + (R_s + \jmath X_d'')(I_d + \jmath I_q) e^{\jmath(\delta - \frac{\pi}{2})} = (E_d'' + \jmath E_q'') e^{\jmath(\delta - \frac{\pi}{2})} \tag{127}$$

Sub-transient model (contd.)

Equation (128) can be represented as the following circuit.

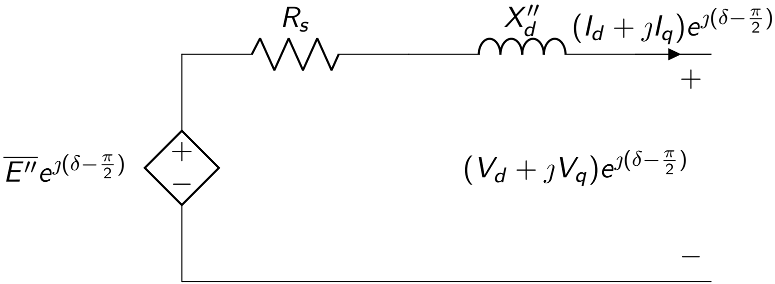


Figure: Dynamic equivalent circuit of a synchronous generator using sub-transient model

Sub-transient model (contd.)

The per unit torque is the real power delivered by the source. (In per unit, $T = P$).

$$
\begin{aligned}
T_e &= \text{real}\left((E''_d + \jmath E''_q) e^{\jmath(\delta - \frac{\pi}{2})} \left((I_d + \jmath I_q) e^{\jmath(\delta - \frac{\pi}{2})} \right)^* \right) \\
&= (E''_d I_d + E''_q I_q)
\end{aligned}
\tag{128}
$$

Transient- Two axis model

In addition to the assumptions made in the sub-transient model, the following assumption is also made here.

1. The sub-transient time constants T''_{d0}, T''_{q0} corresponding to $1d$ and $2q$ are very small compared to transient time constants T'_{d0}, T'_{q0}.
2. Hence, $T''_{d0} = T''_{q0} = 0$.

Therefore, from (104) and (106)

$$\begin{aligned} T''_{d0}\frac{d\psi_{1d}}{dt} &= E'_q - (X'_d - X_{ls})I_d - \psi_{1d} = 0 \\ \psi_{1d} &= E'_q - (X'_d - X_{ls})I_d \end{aligned} \tag{129}$$

$$\begin{aligned} T''_{q0}\frac{d\psi_{2q}}{dt} &= -E'_d - (X'_q - X_{ls})I_q - \psi_{2q} = 0 \\ \psi_{2q} &= -E'_d - (X'_q - X_{ls})I_q \end{aligned} \tag{130}$$

Transient- Two axis model (contd.)

By substituting (129) and (130) in the equations from (103) to (116), we get,

$$T'_{d0}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)I_d + E_{fd} \tag{131}$$

$$T'_{q0}\frac{dE'_d}{dt} = -E'_d + (X_q - X'_q)I_q \tag{132}$$

$$\frac{d\delta}{dt} = \omega - \omega_s \tag{133}$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_m - T_e - D(\omega - \omega_{base}) \tag{134}$$

Transient- Two axis model (contd.)

$$T_E \frac{dE_{fd}}{dt} = -(K_E + S_E(E_{fd}))E_{fd} + V_R \tag{135}$$

$$T_A \frac{dV_R}{dt} = -V_R + K_A R_F - \frac{K_A K_F}{T_F} E_{fd} + K_A(V_{ref} - V_t),\ \ V_R^{min} < V_R < V_R^{max} \tag{136}$$

$$T_F \frac{dR_F}{dt} = -R_F + \frac{K_F}{T_F} E_{fd} \tag{137}$$

$$T_{RH} \frac{dT_M}{dt} = -\Delta T_M + \left(1 - \frac{K_{HP} T_{RH}}{T_{CH}}\right) P_{CH} + \frac{K_{HP} T_{RH}}{T_{CH}} P_{SV} \tag{138}$$

$$T_{CH} \frac{dP_{CH}}{dt} = -P_{CH} + P_{SV} \tag{139}$$

Transient- Two axis model (contd.)

$$T_{SV}\frac{dP_{SV}}{dt} = -\frac{1}{R_D}\left(\frac{\omega}{\omega_{base}} - 1\right) + P_{ref} - P_{SV}, \quad 0 \leq P_{SV} \leq P_{SV}^{max} \tag{140}$$

$$\psi_d = -X_d' I_d + E_q' \tag{141}$$

$$\psi_q = -X_q' I_q - E_d' \tag{142}$$

By substituting (101) and (100) in (141) and (142), respectively, we get

$$V_q + R_s I_q == -X_d' I_d + E_q' \tag{143}$$

$$-V_d - R_s I_d = -X_q' I_q - E_d' \tag{144}$$

Let us multiply (143) by $\jmath$ and add it to (144).

$$V_d + R_s I_d + \jmath(V_q + R_s I_q) = E_d' + X_q' I_q + \jmath(E_q' - X_d' I_d) \tag{145}$$

Transient- Two axis model (contd.)

Let us add $\jmath x'_d(I_d + \jmath I_q)$ to the both side of (145) and rearrange it.

$$(V_d + \jmath V_q) + (R_s + \jmath X'_d)(I_d + \jmath I_q) = E'_d + (X'_q - X'_d)I_q + \jmath E'_q \tag{146}$$

Let us convert (146) to RI-axis with no saliency ($X'_d = X'_q$).

$$(V_d + \jmath V_q)e^{\jmath(\delta - \frac{\pi}{2})} + (R_s + \jmath X'_d)(I_d + \jmath I_q)e^{\jmath(\delta - \frac{\pi}{2})} = (E'_d + \jmath E'_q)e^{\jmath(\delta - \frac{\pi}{2})} \tag{147}$$

Equation (147) can be represented as the following circuit.

Transient- Two axis model (contd.)

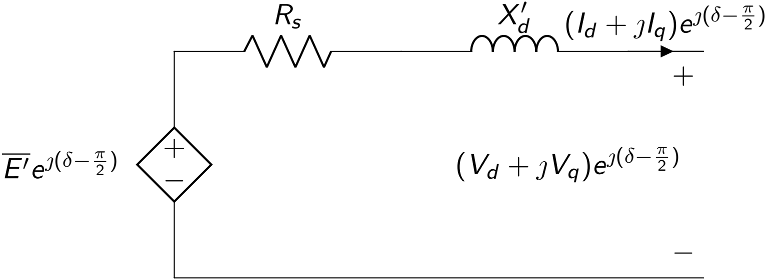


Figure: Dynamic equivalent circuit of a synchronous generator using transient model

Transient- Two axis model (contd.)

The per unit torque is the real power delivered by the source. (In per unit, $T = P$).

$$\begin{aligned} T_e &= \text{real}\left((E'_d + \jmath E'_q) e^{\jmath(\delta - \frac{\pi}{2})} \left((I_d + \jmath I_q) e^{\jmath(\delta - \frac{\pi}{2})} \right)^* \right) \\ &= (E'_d I_d + E'_q I_q) \end{aligned} \tag{148}$$

Flux decay - One axis model

In addition to the assumptions made in the transient model, the following assumption is also made here.

1. The damper winding ($1q$) dynamics are neglected since T'_{q0} is small compared to T'_{d0}.
2. Hence, $T'_{q0} = 0$

Therefore, from (132),

$$\begin{aligned} T'_{q0}\frac{dE'_d}{dt} &= -E'_d + (X_q - X'_q)I_q = 0 \\ E'_d &= (X_q - X'_q)I_q \end{aligned} \tag{149}$$

Flux decay - One axis model (contd.)

By substituting (149) in the equations from (131) to (142), we get

$$T'_{d0}\frac{dE'_q}{dt} = -E'_q - (X_d - X'_d)I_d + E_{fd} \tag{150}$$

$$\frac{d\delta}{dt} = \omega - \omega_s \tag{151}$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_m - T_e - D(\omega - \omega_{base}) \tag{152}$$

$$T_E\frac{dE_{fd}}{dt} = -(K_E + S_E(E_{fd}))E_{fd} + V_R \tag{153}$$

$$T_A\frac{dV_R}{dt} = -V_R + K_A R_F - \frac{K_A K_F}{T_F}E_{fd} + K_A(V_{ref} - V_t),\ V_R^{min} < V_R < V_R^{max} \tag{154}$$

$$T_F\frac{dR_F}{dt} = -R_F + \frac{K_F}{T_F}E_{fd} \tag{155}$$

Flux decay - One axis model (contd.)

$$T_{RH}\frac{dT_M}{dt} = -\Delta T_M + \left(1 - \frac{K_{HP}T_{RH}}{T_{CH}}\right)P_{CH} + \frac{K_{HP}T_{RH}}{T_{CH}}P_{SV} \tag{156}$$

$$T_{CH}\frac{dP_{CH}}{dt} = -P_{CH} + P_{SV} \tag{157}$$

$$T_{SV}\frac{dP_{SV}}{dt} = -\frac{1}{R_D}\left(\frac{\omega}{\omega_{base}} - 1\right) + P_{ref} - P_{SV}, \quad 0 \le P_{SV} \le P_{SV}^{max} \tag{158}$$

$$\psi_d = -X_d' I_d + E_q' \tag{159}$$

$$\psi_q = -X_q I_q \tag{160}$$

Flux decay - One axis model (contd.)

By substituting (101) and (100) in (159) and (160), respectively, we get

$$V_q + R_s I_q = -X_d' I_d + E_q' \tag{161}$$

$$-V_d - R_s I_d = -X_q I_q \tag{162}$$

Let us multiply (162) by $\jmath$, add it to (162), rearrange and express it in *RI*-axis.

$$(V_d + \jmath V_q)e^{\jmath(\delta - \frac{\pi}{2})} + (R_s + \jmath X_d')(I_d + \jmath I_q)e^{\jmath(\delta - \frac{\pi}{2})} = ((X_q - X_d')I_q + \jmath E_q')e^{\jmath(\delta - \frac{\pi}{2})} \tag{163}$$

Flux decay - One axis model (contd.)

Equation (163) can be represented as the following circuit.

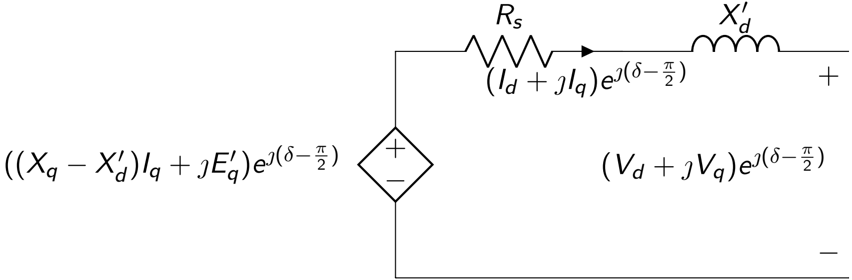


Figure: Dynamic equivalent circuit of a synchronous generator using flux decay model

Flux decay - One axis model (contd.)

The per unit torque is the real power delivered by the source. (In per unit, $T = P$).

$$
\begin{aligned}
T_e &= \text{real}\left(((X_q - X_d')I_q + \jmath E_q')e^{\jmath(\delta - \frac{\pi}{2})}\left((I_d + \jmath I_q)e^{\jmath(\delta - \frac{\pi}{2})}\right)^*\right) \\
&= (X_q - X_d')I_q I_d + E_q' I_q
\end{aligned} \tag{164}
$$

Constant flux linkage - Classical model

- It is the simplest model.
- It is mainly used in large power systems.
- If the time period of study is less that T'_{d0}, the field winding dynamics are neglected.
- $(E'_d + \jmath E'_q)$ is assumed to be constant. It is calculated from the steady state.
- Turbine dynamics are neglected. Hence T_m is constant.

The dynamic equations of this model are

$$\frac{d\delta}{dt} = \omega - \omega_s \tag{165}$$

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = T_m - T_e - D(\omega - \omega_{base}) \tag{166}$$

The dynamic equivalent circuit of this model is shown here.

Constant flux linkage - Classical model (contd.)

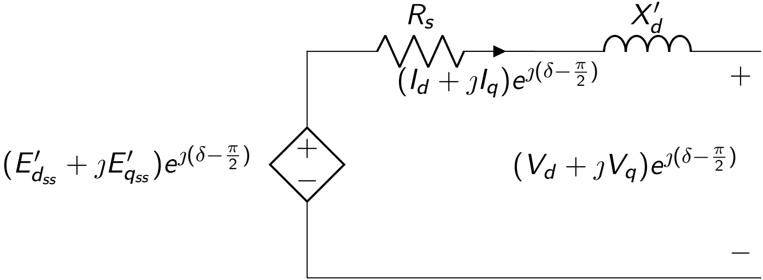


Figure: Dynamic equivalent circuit of a synchronous generator using classical model

Constant flux linkage - Classical model (contd.)

The per unit torque is the real power delivered by the source. (In per unit, $T = P$).

$$\begin{aligned} T_e &= \text{real}\left((E'_{dss} + \jmath E'_{qss})e^{\jmath(\delta - \frac{\pi}{2})} \left((I_d + \jmath I_q)e^{\jmath(\delta - \frac{\pi}{2})} \right)^* \right) \\ &= E'_{dss} I_d + E'_{qss} I_q \end{aligned} \tag{167}$$