Indian Institute of Technology Patna
Department of Electrical Engineering
EE381 - Power Systems
Autumn - 2022
End Semester Exam – Solution.
November 26, 2022

There are 5 questions. They carry equal marks.

$(5 \times 10 = 50)$

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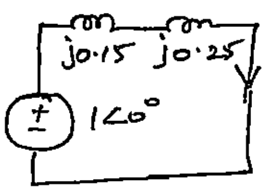
1. (a) A 100 MVA, 20 kV, 60 Hz three phase synchronous generator is connected to a 100 MVA 20/400 kV three phase transformer. The machine has $x''_d = 0.15$ p.u, $x'_d = 0.25$ p.u, and $x_d = 1.25$ p.u. The transformer reactance is 0.25 p.u. The generator is operating at the rated voltage and no load when a three phase short circuit occurs at the secondary terminals of the transformer.

   i. Find the subtransient short circuit current in per unit and Amperes on both sides of the transformer.

   ii. Find the transient short circuit current in per unit and Amperes on both sides of the transformer.

   iii. Find the steady state short circuit current in per unit and Amperes on both sides of the transformer.

(i)



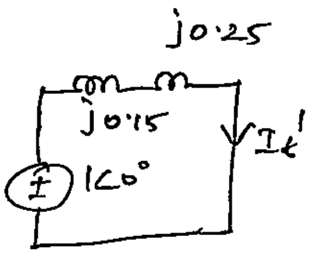
$I_f'' = \frac{1\angle 0°}{j0.4} = 2.5\angle -90°$ p.u

$I_f'' = 2.5 \times \frac{100 \times 10^3}{\sqrt{3} \times 400} = 360.84$ A (HV side)

(2)

$I_f'' = 2.5 \times \frac{100 \times 10^3}{\sqrt{3} \times 20} = 7.22$ kA (LV side)

(ii)



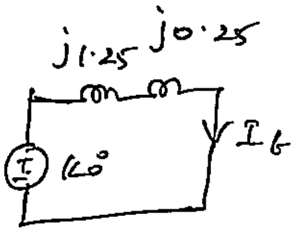
$I_f' = \frac{1\angle 0°}{j0.15 + j0.25} = 2\angle -90°$ p.u

(1.5)

$I_f' = 288.68$ A HV side

$I_f' = 5.77$ kA LV side

(iii)



$I_f = \frac{1\angle 0°}{j1.5} = 0.667\angle -90°$ p.u

(1.5)

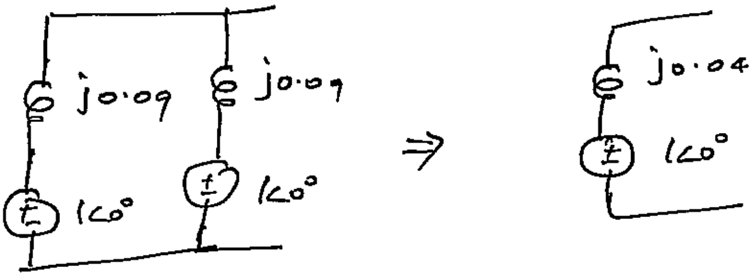
$I_f = 96.23$ A HV side

$I_f = 1.92$ kA LV side

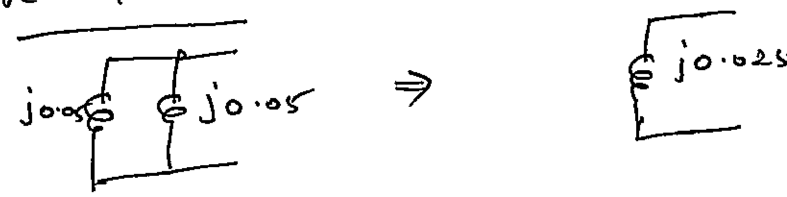
(b) Two 11 kV, 25 MVA three-phase star connected synchronous generators G_1 and G_2 operate in parallel. Each generator has positive, negative and zero sequence reactances of $j0.09$, $j0.05$, $j0.04$ p.u., respectively. A single line to ground fault occurs at the terminals of one of the generators. Find the fault current in Ampere. Also, find the voltage across the grounding resistor R_n in Volts. Assume that the neutral of only generator G_1 is grounded through a resistor $R_n = 1\Omega$.

$R_n = 1 \times \frac{25}{(11)^2} = 0.2066$ p.u (25 → Base MVA; 11 → Base KV)

Positive Sequence N/W:

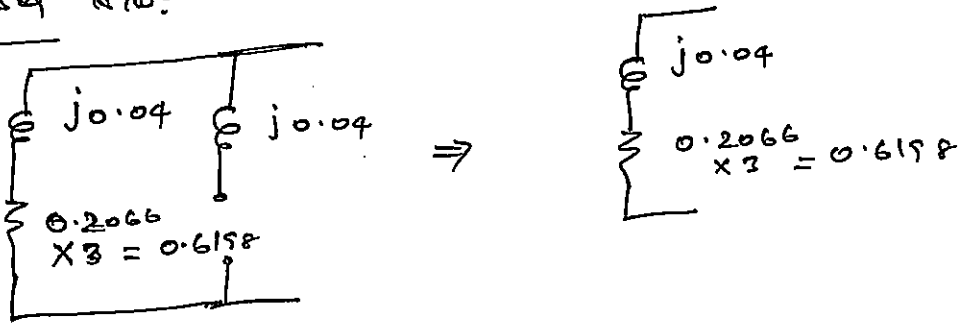


-ve Sequence N/W:

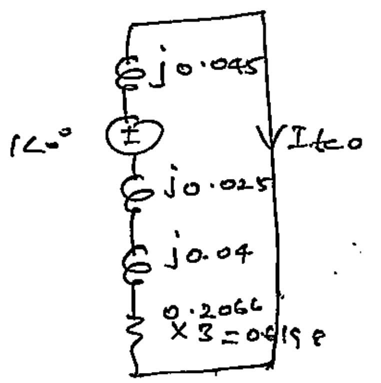


(2)

Zero Seq N/W:



For a SLG fault:



$I_{fa0} = \frac{1\angle 0^\circ}{0.6198 + j0.11}$

$I_{fa0} = 1.56 - j0.28$ p.u

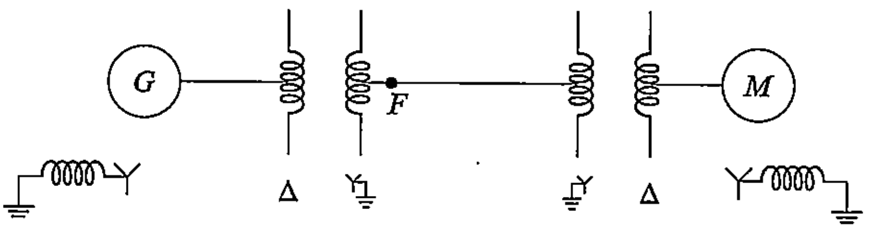
$I_f = 3 \times I_{fa0} = 4.7 - 0.83j$

$I_f = 6.25$ KA (2)

$V_{RN} = I_f \times R_n$

$V_{RN} = 6.25$ KV (1)

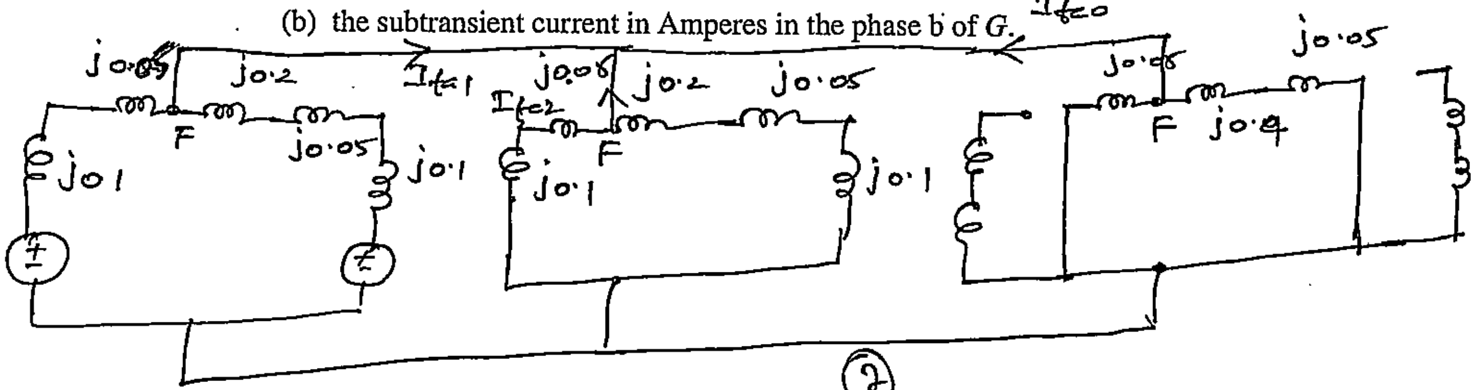
2. A double line to ground fault occurs on lines b and c at point F in the system shown below.



Both machines are rated 1200 kVA, 600 V with reactances of $X_d'' = X_1 = X_2 = 10\%$ and $X_0 = 5\%$. Each three phase transformer is rated 1200 kVA, 600 V - Δ / 3300 V - Y with leakage reactance of 5 %. The reactance of the transmission lines are $X_1 = X_2 = 20\%$ and $X_0 = 40\%$ on a base of 1200 kVA and 3300 V. The reactances of the neutral grounding reactors are 5 % on the base of machines. Assume the prefault current to be zero. Find

(a) the fault current at point F in Amperes.

(b) the subtransient current in Amperes in the phase b of G.



For a DLG fault,

$I_{fa1} + I_{fa2} + I_{fa0} = 0$

By solving the above N/w (3)

$I_{fa1} = -j\,7.33$ pu

$I_{fa2} = j\,2.20$ p.u

$I_{fa0} = j\,5.13$ p.u (5)

$I_f = 3 I_{fa0} = j5.13 \times 3$

a) $I_f = 15.39$ pu $I_f = 3.23$ KA

b) (2)

$I_{fa1G} = -j5.131$ (By current division from the +ve Seq)

$I_{fa2G} = j\,1.54$ (" from the -ve Seq N/w)

$I_{fa0G} = 0$ (No Connection)

$I_b = a^2 I_{fa1G} + a\, I_{fa2G} = -5.78 + j\,1.8$ (3)

$|I_b| = 6.985$ KA

3. Consider a two bus system.

The incremental fuel cost characteristics of plant 1 and plant 2 are given by

$$\frac{dF_1}{dP_1} = 0.025P_1 + 14 \text{ Rs/MWHr}$$
$$\frac{dF_2}{dP_2} = 0.05P_2 + 16 \text{ Rs/MWHr}$$

If 200 MW of power is transmitted from plant 1 to the load, a transmission loss of 20 MW will be incurred.

(a) Find the generation schedule and the load demand if the cost of received power is 20 Rs/MWhr.

(b) Also, evaluate the amount of financial loss that may be incurred between the schedule obtained in the previous step and the schedule to be found when losses are not coordinated but included in the power balance equation.

(1) $P_L = 0.0005 P_1^2$; $L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}}$; $L_2 = 1$ (1)

a) $L_1 \frac{dF_1}{dP_1} = L_2 \frac{dF_2}{dP_2} = \lambda$ (2)

$\left(\frac{1}{1 - 0.001P_1}\right) 0.025 P_1 + 14 = 20$ →

$0.05P_2 + 16 = 20$

$P_1 = 133.33$ MW

$P_2 = 80$ MW

$P_D = 204.45$ MW

b) When losses are not coordinated but included,

$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} \Rightarrow 0.025P_1 + 14 = 0.05P_2 + 16$ — (1)

$P_1 + P_2 = 204.45 + 0.0005 P_1^2$ (2) $(P_1 + P_2 = P_D + P_L)$

On solving (1) and (2),

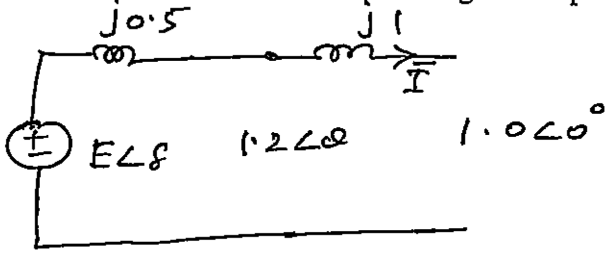
(3) $P_1 = 172.94$ MW
$P_2 = 46.47$ MW

Financial loss $= \int_{172.94}^{133.3} \frac{dF_1}{dP_1} dP_1 + \int_{46.47}^{80} \frac{dF_2}{dP_2} dP_2$

(2)

Financial Loss = ~~[illegible]~~ 63.69 Rs/hr

4. (a) Find the steady state power limit of a power system consisting of a generator with equivalent reactance of 0.5 per unit, connected to an infinite bus through a series reactance of 1 per unit. The terminal voltage of the generator is held at 1.2 per unit. and the infinite bus voltage at 1.0 per unit.



$$I = \left(\frac{1.2\angle\theta - 1.0}{j1}\right)$$

$$E\angle\delta = 1.2\angle\theta + \left(\frac{1.2\angle\theta - 1}{j1}\right) j0.5$$

$E\cos\delta + jE\sin\delta = (1.8\cos\theta - 0.5) + j1.8\sin\theta$

at the steady state power limit, $\delta = 90°$ ✓ (1)

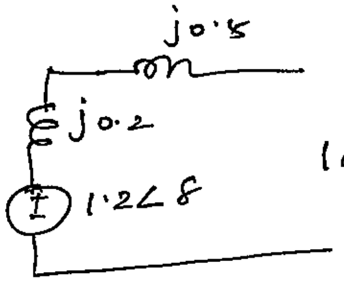
$\therefore E\cos\delta = 0$

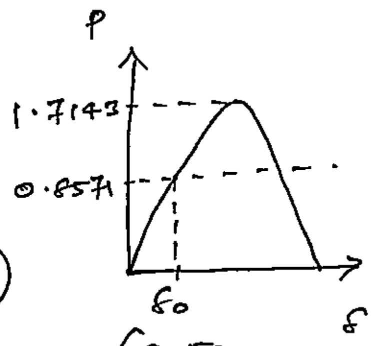
$1.8\cos\theta = 0.5 \Rightarrow \boxed{\theta = 73.87°}$

$E = 1.73 \rightarrow (1.8\sin\theta)$

$$\boxed{P_{max} = \frac{1.73 \times 1}{1.5} = 1.152}$$ ✓ (2)

(b) A synchronous generator of reactance 0.20 p.u. is connected to an infinite bus ($|V| = 1.0$ p.u.) through a transformer and a line of total reactance of 0.50 p.u. The generator no load voltage is 1.20 p.u. and its inertia constant is $H = 4$ sec. The resistance and machine damping may be assumed negligible. The system frequency is 50 Hz. Calculate the frequency of natural oscillations in Hz if the generator is loaded to 50 % of its maximum power limit.





$$P_{max} = \frac{1.2 \times 1}{0.7} = 1.7143$$ ✓ (1)

$P_m = 0.5 \times P_{max} = 0.8571$

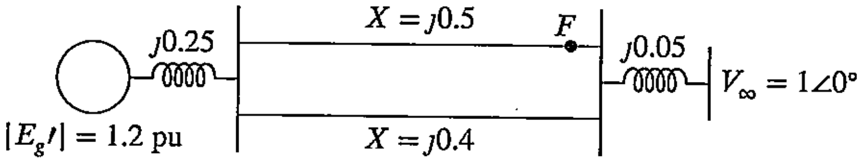
δ_0 (0.5236 rad) ✓ (1) (30°)

$$\omega_n = \sqrt{\frac{2\omega_s P_s}{2 \times H}} = \sqrt{\frac{100 \times \pi \times 1.7143 \times \cos 30°}{2 \times 4}}$$ ✓ (2)

$\boxed{\omega_n = 7.6355 \text{ rad/sec}}$ ✓

$\boxed{f_n = 1.2152 \text{ Hz}}$ ✓ (1)

5. Consider the system given below. A three phase short circuit fault occurs at the point F. The fault is cleared by simultaneously opening the breakers of the line located at either ends of the line.



If the generator was delivering 1.0 pu power prior to the fault. Determine the following.

(a) the power angle δ_0 before the fault.

(b) the maximum permissible angle δ_{max} after clearing the fault.

(c) the critical clearing angle δ_{cr}.

(d) the new operating angle δ_{new}.

Pre fault:

$$P_{max\,I} = \frac{1.2 \times 1}{0.522} = 2.3$$ ✓ (5) (1)

During fault:

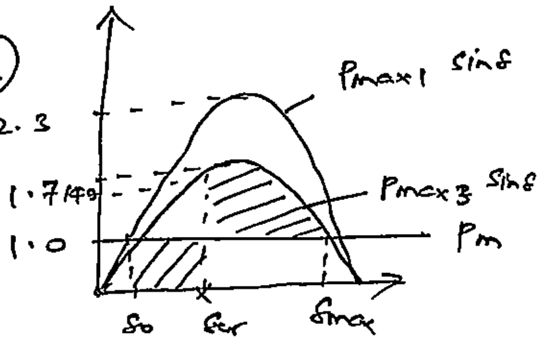
$$P_{max\,II} = 0$$ ✓ (1)

Post fault:

$$P_{max\,3} = \frac{1.2 \times 1}{0.7} = 1.7143$$ ✓ (1)

a) $\delta_0 = 0.4498$ rad ✓

$\delta_0 = 25.77°$



b) $$\delta_{max} = \pi - \sin^{-1}\left(\frac{P_m}{P_{max\,3}}\right) = 2.52 \text{ rad} = 144.315°$$ ✓ (2)

c) $$\delta_{cr} = \cos^{-1}\left(\frac{P_m(\delta_{max} - \delta_0) + P_{max\,3}\cos\delta_{max}}{P_{max\,3}}\right)$$ ✓ (3)

$\delta_{cr} = 1.1651$ rad

$\delta_{cr} = 66.75°$

d) $$\delta_{new} = \sin^{-1}\left(\frac{P_m}{P_{max\,3}}\right) = 0.6228 \text{ rad} = 35.68°$$ ✓ (1)