Faults

A fault is a failure which interferes with the normal flow of current.

Causes

- Lightning
- Insulation failure
- Faulty Operation

Fault Analysis

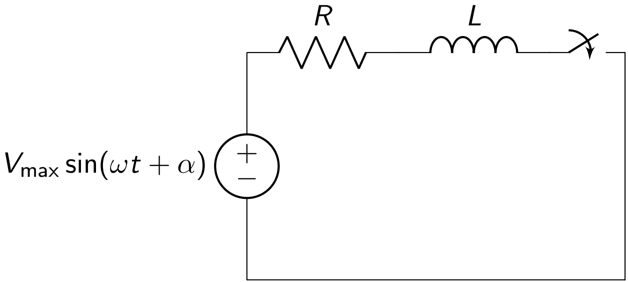
Faults lead to heavy currents in the system which needs to be protected. It is all about finding the magnitude of fault currents under fault conditions.

Types of Faults

1. Symmetrical Faults
   1.1 Three pahse short circuit
2. Unsymmetrical Faults
   2.1 Single line to ground fault
   2.2 Line to line fault
   2.3 Double line to ground fault
   2.4 Open conductor faults

- Symmetrical faults are very rare but severe.
- Unsymmetrical faults are common.
- Analyzing symmetrical faults is easy.
- Analyzing unsymmetrical faluts needs a special tool.

Transients in RL Circuits



After the switch is closed,

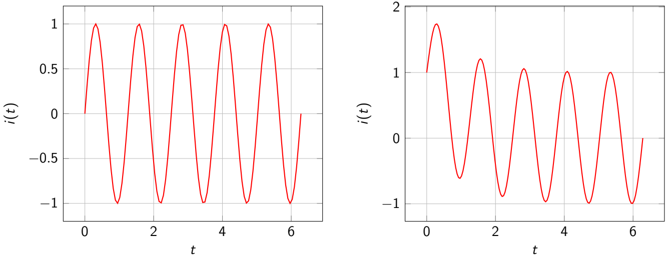
$$V_{\max}\sin(\omega t+\alpha) = Ri + L\frac{di}{dt}$$

On solving it for i

$$i = \frac{V_{\max}}{|Z|}\left[\sin(\omega t+\alpha-\theta) - \exp(\frac{-Rt}{L})\sin(\alpha-\theta)\right]$$

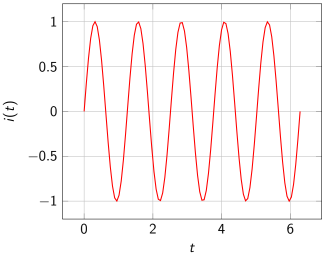
Transients in RL Circuits - Contd...

where $|Z| = \sqrt{R^2 + (\omega L)^2}$ and $\theta = \tan^{-1}(\omega L/R)$. If the voltage is applied at $t = 0$,

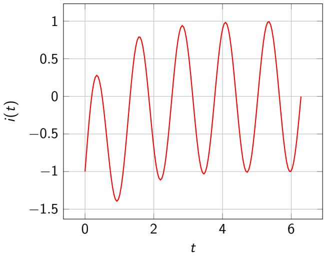


(a) $\alpha - \theta = 0$ (b) $\alpha - \theta = -\pi/2$

Transients in RL Circuits - Contd...



(c) $\alpha - \theta = \pi$

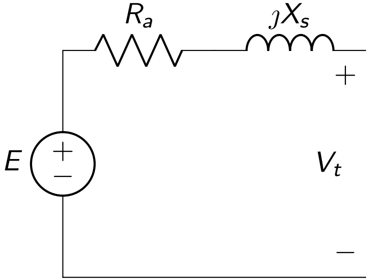


(d) $\alpha - \theta = \pi/2$

1. DC component is zero when $\alpha - \theta = 0$ or π
2. DC component is maximum when $\alpha - \theta = \pm\pi/2$
3. DC component may have any value from 0 to $V_{\mathsf{max}}/|Z|$ depending on the instantaneous value when the switch is closed and on the power factor of the circuit.

Synchronous Machine

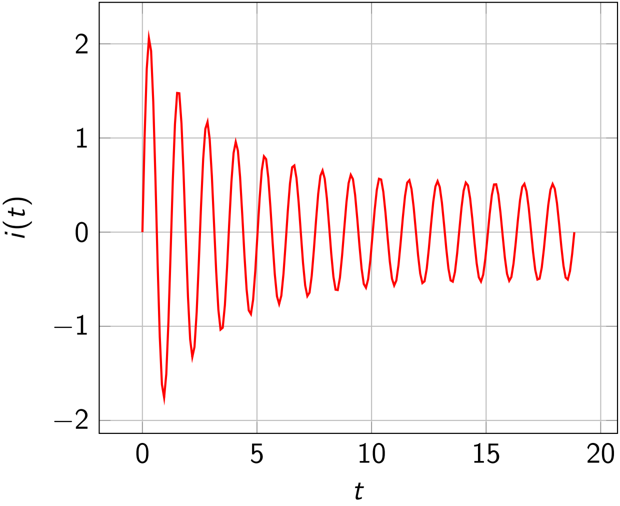
The per phase equivalent circuit is



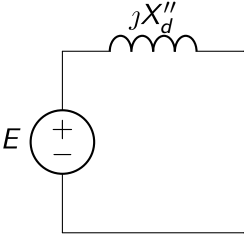
When a short circuit occurs,

- Since the instantaneous voltage is different in each phase, DC component will be different in each phase.
- Unlike RL circuits, the phase current in each phase will follow a different pattern.

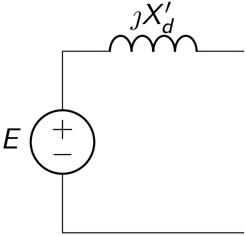
Synchronous Machine - Short Circuit Current



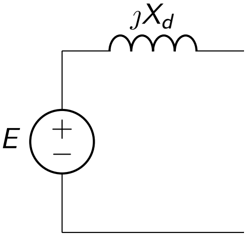
Synchronous Machine - Under Short Circuit Conditions



(e) Subtransient



(f) Transient



(g) Steady State

Synchronous Machine on Load

Let us consider a synchronous generator that supplies a balanced load.

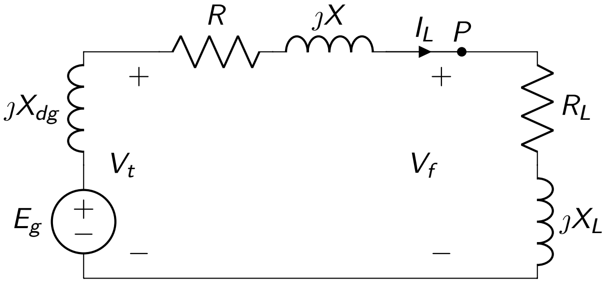


Figure: Steady state Condition

Synchronous Machine on Load- Prefault Condition

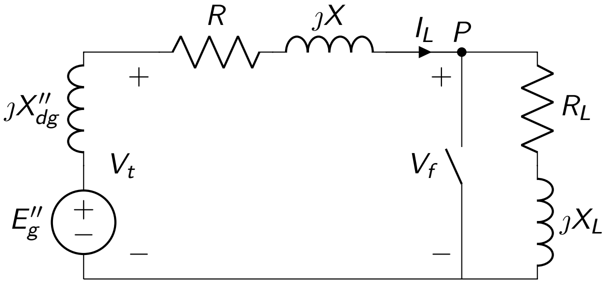


Figure: Prefault Condition

$$E''_g = V_t + \jmath X''_{dg} I_L = V_f + (R + \jmath X) I_L + \jmath X''_{dg} I_L$$

Synchronous Machine on Load- Prefault Condition

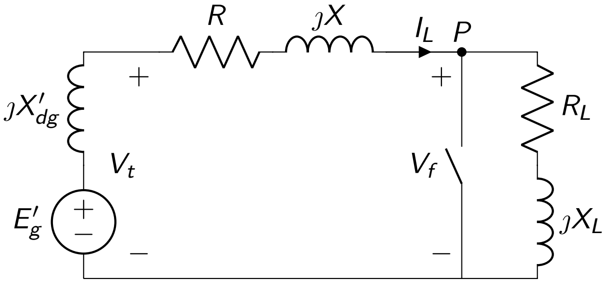


Figure: Prefault Condition

$$E'_g = V_t + \jmath X'_{dg} I_L = V_f + (R + \jmath X) I_L + \jmath X'_{dg} I_L$$

Synchronous Machine on Load - During Fault

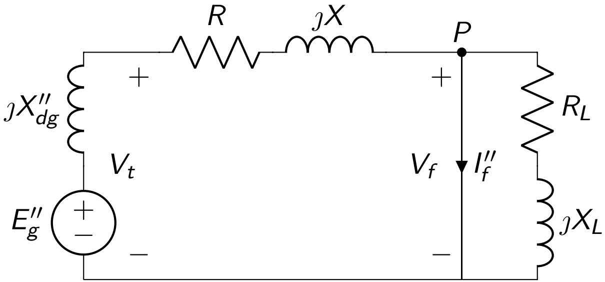


Figure: During Fault

$$I''_f = \frac{E''_g}{(R + \jmath X) + \jmath X''_{dg}} = \frac{V_f + (R + \jmath X)I_L + \jmath X''_{dg} I_L}{(R + \jmath X) + \jmath X''_{dg}}$$

Since the static load does not supply any fault current, it can be removed,

$$I''_f = \frac{V_f}{(R + \jmath X) + \jmath X''_{dg}}$$

Synchronous Generator supplying Synchronous Motor

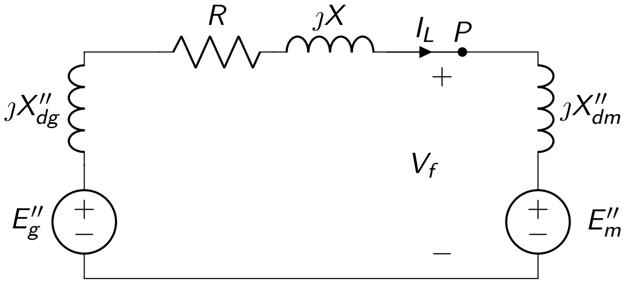


Figure: Prefault Condition

$$E''_g = V_f + (R + \jmath X + \jmath X''_{dg}) I_L$$
$$E''_m = V_f - (\jmath X''_{dm}) I_L$$

Synchronous Generator supplying Synchronous Motor

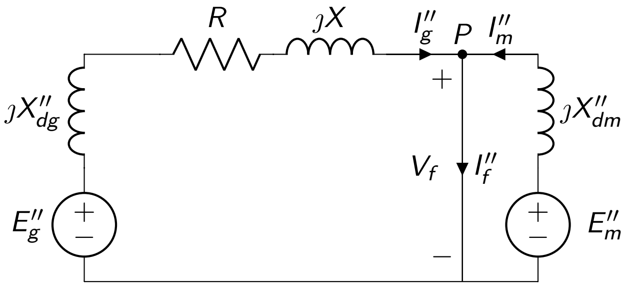


Figure: Postfault Condition

$$I''_g = \frac{E''_g}{(R + \jmath X) + \jmath X''_{dg}} = \frac{V_f}{(R + \jmath X) + \jmath X''_{dg}} + I_L$$

$$I''_m = \frac{E''_m}{\jmath X''_{dm}} = \frac{V_f}{\jmath X''_{dm}} - I_L$$

Fault Current Calculation

$$I''_f = I''_g + I''_m = \frac{V_f}{(R + \jmath X) + \jmath X''_{dg}} + \frac{V_f}{\jmath X''_{dm}}$$

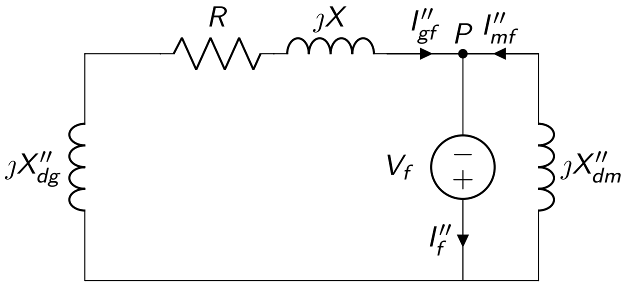


Figure: Applying V_f to the dead network

Fault Current Calculation using Thevenin's Theorem

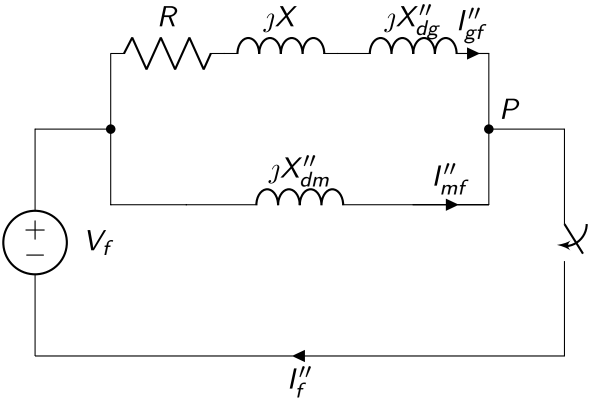
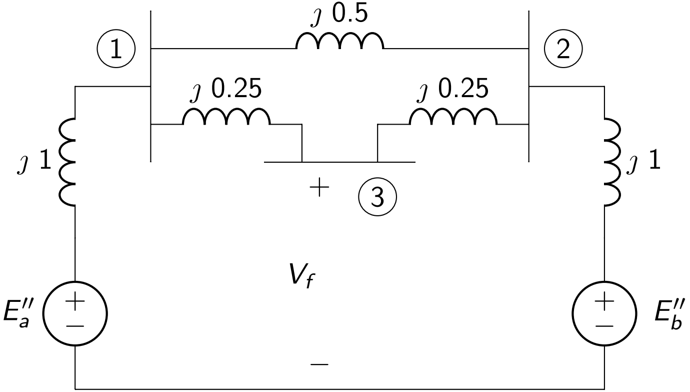


Figure: Thevenin Equivalent

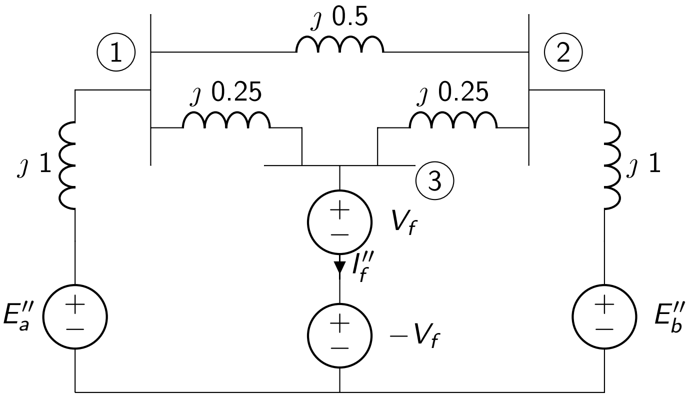
$$I_f'' = \frac{V_f}{Z_{\text{Th}}} \text{ where } Z_{\text{th}} = \frac{\jmath X_{dm}''(R + \jmath X + \jmath X_{dg}'')}{(R + \jmath X + \jmath X_{dg}'' + \jmath X_{dm}'')}$$

Fault Calculations using Z_{Bus}



Fault Calculations - Contd...

If a three phase fault occurs at bus 3,



To find the changes in voltages,

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ -V_f \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -I_f \end{bmatrix}$$

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ -V_f \end{bmatrix} = -I_f'' \begin{bmatrix} Z_{13} \\ Z_{23} \\ Z_{33} \end{bmatrix}$$

$$I_f'' = \frac{V_f}{Z_{33}}$$

$$\begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix} = \begin{bmatrix} -\frac{Z_{13}}{Z_{33}} V_f \\ -\frac{Z_{23}}{Z_{33}} V_f \\ -V_f \end{bmatrix}$$

If the faulted network is assumed to be without load before the fault occurs,

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} V_f \\ V_f \\ V_f \end{bmatrix} + \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} V_f \\ V_f \\ V_f \end{bmatrix} + \begin{bmatrix} -\frac{Z_{13}}{Z_{33}} V_f \\ -\frac{Z_{23}}{Z_{33}} V_f \\ -V_f \end{bmatrix} = V_f \begin{bmatrix} 1 - \frac{Z_{13}}{Z_{33}} \\ 1 - \frac{Z_{23}}{Z_{33}} \\ 0 \end{bmatrix}$$

When a three phase fault occurs on bus k,

$$I_f'' = \frac{V_f}{Z_{kk}}$$

By neglecting prefault load currents, the voltage at any bus j during the fault

$$V_j = V_f - Z_{jk} I_f'' = V_f - \frac{Z_{jk}}{Z_{kk}} V_f$$

The subtransient current I_{ij}'' from bus i to j in the line of impedance Z_b connecting those two buses

$$I_{ij}'' = \frac{V_i - V_j}{Z_b} = -I_f'' \left(\frac{Z_{ik} - Z_{jk}}{Z_b} \right) = -\frac{V_f}{Z_b} \left(\frac{Z_{ik} - Z_{jk}}{Z_{kk}} \right)$$

Selection of Circuit Breakers

$$\text{Short-circuit MVA} = \sqrt{3} \times \text{nominal kV} \times I_{\text{SC}} \times 10^{-3}$$

where I_{SC} is the three phase short circuit current in A.

$$\text{Base MVA} = \sqrt{3} \times \text{base kV} \times I_{\text{base}} \times 10^{-3}$$

If base kV equals nominal kV,

$$\text{Short-circuit MVA in per unit} = I_{\text{SC}} \text{in per unit}$$

At nominal voltage, the Thevenin impedance is

$$Z_{\text{Th}} = \frac{1}{I_{\text{SC}}} \text{per unit} = \frac{1}{\text{short circuit MVA}} \text{per unit}$$

Two factors to be considered are

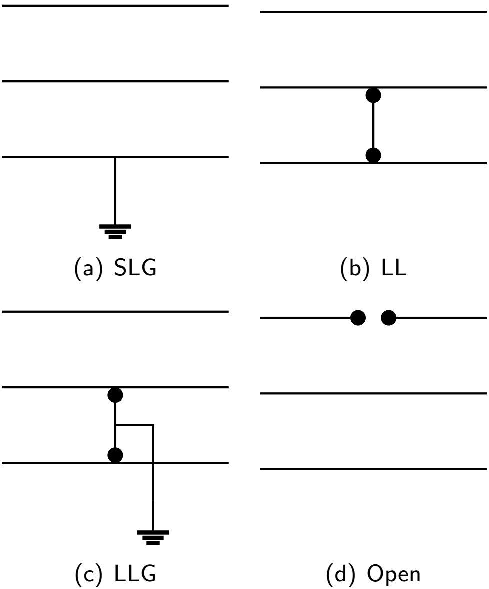
1. The maximum instantaneous current which the breaker must carry
2. The total current when the breaker contacts part to interrupt the circuit

The momentary current which occurs immediately after a fault is higher than the initial symmetrical RMS current.

$$\text{momentary current} = 1.6 \times \text{symmetrical current}$$

- The factor 1.6 is to account for the DC component.
- The interrupting current is lower than the momentary current and depends on the speed of the breaker such as 8, 5,3 or 2 cycles.

Unsymmetrical Faults



(a) SLG (b) LL

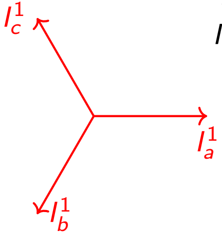
(c) LLG (d) Open

Figure:

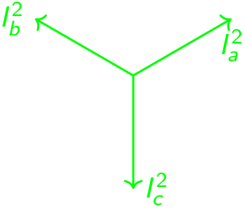
Symmetrical Components

Any arbitrary set of three phasors can be written as

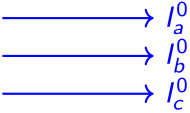
$$I_a = I_a^0 + I_a^1 + I_a^2$$
$$I_b = I_b^0 + I_b^1 + I_b^2$$
$$I_c = I_c^0 + I_c^1 + I_c^2$$



(a) Positive Sequence



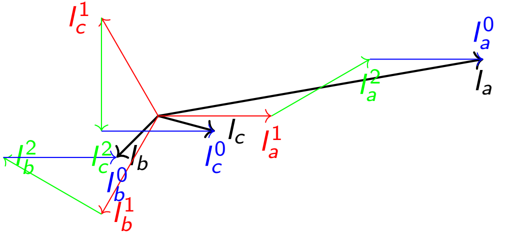
(b) Negative Sequence



(c) Zero Sequence

Figure: Balanced sets

Unbalanced Currents



$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} I_a^0 \\ I_b^0 \\ I_c^0 \end{bmatrix} + \begin{bmatrix} I_a^1 \\ I_b^1 \\ I_c^1 \end{bmatrix} + \begin{bmatrix} I_a^2 \\ I_b^2 \\ I_c^2 \end{bmatrix}$$

Let us introduce $\alpha = 1\angle 120^\circ$.

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = I_a^0 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + I_a^1 \begin{bmatrix} 1 \\ \alpha^2 \\ \alpha \end{bmatrix} + I_a^2 \begin{bmatrix} 1 \\ \alpha \\ \alpha^2 \end{bmatrix}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix}$$

$$\mathsf{A} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

$$\mathsf{I} = \mathsf{A}\mathsf{I}_s$$

where I is the phase current vector and I_s is the symmetrical components vector.

$$\mathsf{I}_s = \mathsf{A}^{-1}\mathsf{I}$$

$$\mathsf{A}^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}$$

$$\begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

Once I_s is known, the remaining six components can be calculated by using the properties of the positive, negative and zero sequence sets.

Similarly for unbalanced voltages,

$$V_a = V_a^0 + V_a^1 + V_a^2$$
$$V_b = V_b^0 + V_b^1 + V_b^2$$
$$V_c = V_c^0 + V_c^1 + V_c^2$$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} V_a^0 \\ V_a^1 \\ V_a^2 \end{bmatrix}$$

$$V_a^0 = \frac{1}{3}(V_a + V_b + V_c)$$
$$V_a^1 = \frac{1}{3}(V_a + \alpha V_b + \alpha^2 V_c)$$
$$V_a^2 = \frac{1}{3}(V_a + \alpha^2 V_b + \alpha V_c)$$

Observations

1. Line currents into a Δ connected circuit have no zero sequence current.
   Let I_a, I_b and I_c be a set of unbalanced line-line currents.

$$I_a^0 = \frac{1}{3}(I_a + I_b + I_c)$$

$$I_a^0 = \frac{1}{3}(I_{ab} - I_{ca} + I_{bc} - I_{ab} + I_{ca} - I_{bc}) = 0 \qquad (1)$$

2. Zero sequence line-line voltage is always zero.
   Let V_{ab}, V_{bc} and V_{ca} be a set of unbalanced line-line voltages.

$$V_{ab}^0 = \frac{1}{3}(V_{ab} + V_{bc} + V_{ca})$$

$$V_{ab}^0 = \frac{1}{3}(V_{an} - V_{bn} + V_{bn} - V_{cn} + V_{cn} - V_{an}) = 0$$

Power Calculation using Symmetrical Components

The total complex power flowing into a three phase circuit is

$$S_{3\phi} = P + \jmath Q = V_a I_a^* + V_b I_b^* + V_c I_c^*$$

where V_a, V_b and V_c are the voltages to reference at the terminals and I_a, I_b and I_c are the currents flowing into the circuit.
In matrix notation,

$$S_{3\phi} = \begin{bmatrix} V_a & V_b & V_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

$$S_{3\phi} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

$$S_{3\phi} = [\mathrm{AV}_s]^T[\mathrm{AI}_s]^*$$

where $\mathrm{V}_s = \begin{bmatrix} V_a^0 \\ V_a^1 \\ V_a^2 \end{bmatrix}$ and $\mathrm{I}_s = \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix}$.

Since $(\mathrm{AB})^T = \mathrm{B}^T\mathrm{A}^T$,

$$S_{3\phi} = \mathrm{V}_s^T\mathrm{A}^T\mathrm{A}^*\mathrm{I}_s^*$$

$\mathrm{A}^T = \mathrm{A}$ and $\alpha^* = \alpha^2$.

$$S_{3\phi} = \begin{bmatrix} V_a^0 & V_a^1 & V_a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix}^*$$

$$S_{3\phi} = \begin{bmatrix} V_a^0 & V_a^1 & V_a^2 \end{bmatrix} 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix}^*$$

$$S_{3\phi} = 3 \begin{bmatrix} V_a^0 & V_a^1 & V_a^2 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix}^*$$

So, the complex power is

$$S_{3\phi} = V_a I_a^* + V_b I_b^* + V_c I_c^* = 3V_a^0 (I_a^0)^* + 3V_a^1 (I_a^1)^* + 3V_a^2 (I_a^2)^*$$

Transformation is power invariant.

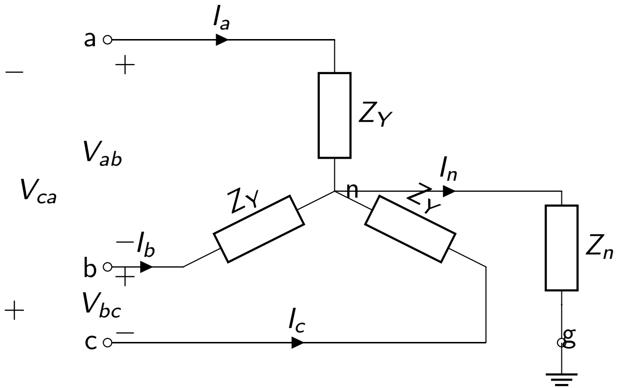
Unbalanced Fault Analysis

1. An unbalanced network is divided into three balanced networks.
2. The three balanced networks are solved on per phase basis.
3. Using the superposition principle, the three responses are added.

General Procedure

1. Find the per phase positive sequence, negative sequence and zero sequence networks of the given system.
2. Connect them as per the fault conditions at the fault point.
3. Determine the sequence currents and voltages.
4. Find the unbalanced fault currents using the sequence currents.

Sequence Circuits of Y Impedance



$$
\begin{aligned}
I_n &= I_a + I_b + I_c = (I_a^0 + I_a^1 + I_a^2) + (I_b^0 + I_b^1 + I_b^2) + (I_c^0 + I_c^1 + I_c^2) \\
&= (I_a^0 + I_b^0 + I_c^0) + (I_a^1 + I_b^1 + I_c^1) + (I_a^2 + I_b^2 + I_c^2) \\
&= 3I_a^0
\end{aligned}
$$

The voltage of phase a with respect to ground as

$$V_a = V_{an} + V_{ng}$$

where $V_{ng} = 3I_a^0 Z_n$. Similarly

$$V_b = V_{bn} + V_{ng}$$
$$V_c = V_{cn} + V_{ng}$$

In matrix form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} + \begin{bmatrix} V_{ng} \\ V_{ng} \\ V_{ng} \end{bmatrix}$$

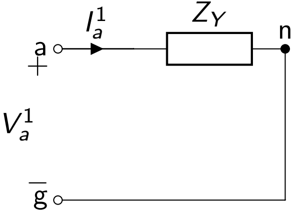
$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_Y I_a \\ Z_Y I_b \\ Z_Y I_c \end{bmatrix} + \begin{bmatrix} 3I_a^0 Z_n \\ 3I_a^0 Z_n \\ 3I_a^0 Z_n \end{bmatrix} = Z_Y \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + 3I_a^0 Z_n \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\mathsf{A}\begin{bmatrix} V_a^0 \\ V_a^1 \\ V_a^2 \end{bmatrix} = Z_Y \mathsf{A} \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} + 3I_a^0 Z_n \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

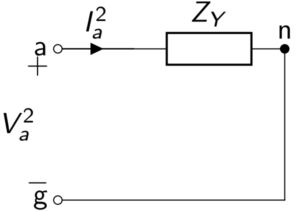
Premultiplying both sides by A^{-1},

$$\begin{bmatrix} V_a^0 \\ V_a^1 \\ V_a^2 \end{bmatrix} = Z_Y \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} + 3I_a^0 Z_n \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

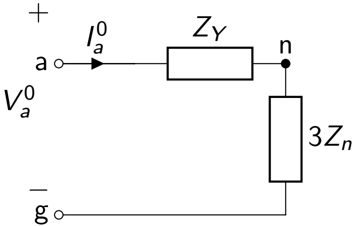
$$\begin{aligned} V_a^0 &= (Z_Y + 3Z_n) I_a^0 && = Z^0 I_a^0 \\ V_a^1 &= Z_Y I_a^1 && = Z^1 I_a^1 \\ V_a^2 &= Z_Y I_a^2 && = Z^2 I_a^2 \end{aligned}$$



(a) Positive Sequence Circuit



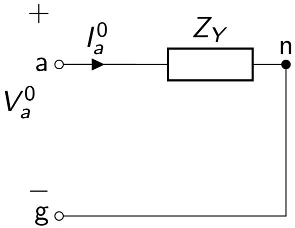
(b) Negative Sequence Circuit



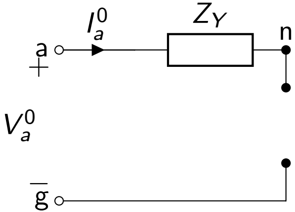
(c) Zero Sequence Circuit

Figure: Sequence Circuits of Y Impedance

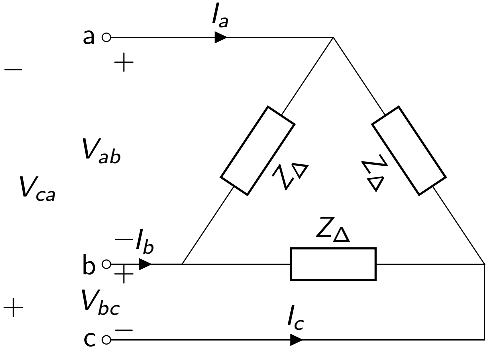
If the neutral of the Y connected circuit is grounded through zero impedance,



If there is no connection between neutral and ground there can not be any zero sequence current flow.



Sequence Circuits of Δ Impedance



Since a Δ connected circuit can not provide a path through neutral, the line currents flowing into it or its equivalent Y circuit can not contain any zero sequence components.

$$V_{ab} = Z_\Delta I_{ab} \quad V_{bc} = Z_\Delta I_{bc} \quad V_{ca} = Z_\Delta I_{ca}$$

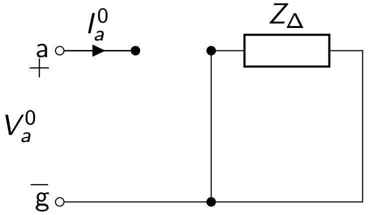
$$V_{ab} + V_{bc} + V_{ca} = (V_{ab}^0 + V_{ab}^1 + V_{ab}^2) + (V_{bc}^0 + V_{bc}^1 + V_{bc}^2) + (V_{ca}^0 + V_{ca}^1 + V_{ca}^2)$$

$$V_{ab} + V_{bc} + V_{ca} = 3V_{ab}^0 = 3Z_\Delta I_{ab}^0$$

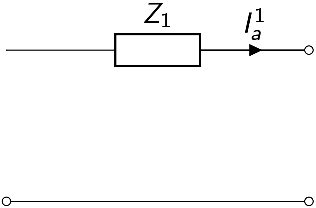
Since the sum of line-line voltages is always zero,

$$V_{ab}^0 = I_{ab}^0 = 0$$

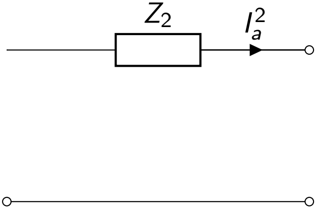
The zero sequence circuit a Δ connected circuit is,



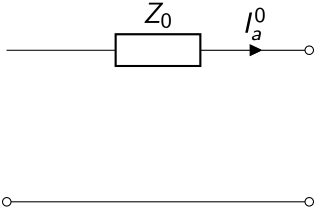
Sequence Circuits of a symmetrical transmission line



(a) Positive Sequence Circuit



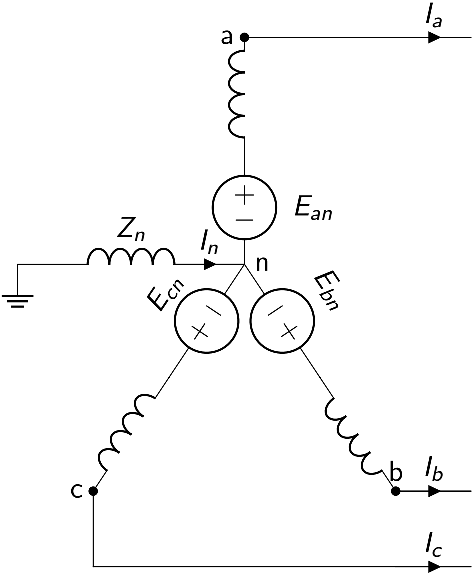
(b) Negative Sequence Circuit



(c) Zero Sequence Circuit

$Z_1 = Z_2 \neq Z_0$. But $Z_0 > Z_1$. Z_0 is 2 to 3.5 times Z_1.

Sequence Circuits of a synchronous machine



The terminal voltages are as follows:

$$V_{an} = E_{an} - I_a(R + \jmath\omega L_s) + \jmath\omega M_s(I_b + I_c)$$
$$V_{bn} = E_{bn} - I_b(R + \jmath\omega L_s) + \jmath\omega M_s(I_c + I_a)$$
$$V_{cn} = E_{cn} - I_c(R + \jmath\omega L_s) + \jmath\omega M_s(I_a + I_b)$$

In Matrix form

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = -[R+\jmath\omega(L_s+M_s)] \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + \jmath\omega M_s \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + \begin{bmatrix} E_{an} \\ E_{bn} \\ E_{cn} \end{bmatrix}$$

By using $\mathrm{V} = \mathrm{AV}_s$ and $\mathrm{I} = \mathrm{AI}_s$

$$\begin{bmatrix} V_{an}^0 \\ V_{an}^1 \\ V_{an}^2 \end{bmatrix} = -[R+\jmath\omega(L_s+M_s)] \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} + \jmath\omega M_s \mathrm{A}^{-1} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \mathrm{A} \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} + \mathrm{A}^{-1} \begin{bmatrix} E_{an} \\ E_{bn} \\ E_{cn} \end{bmatrix}$$

Since the synchronous generator is designed to supply balanced three phase voltages, E_{an}, E_{bn} and E_{cn} are positive sequence phasors.

$$\begin{bmatrix} V_{an}^0 \\ V_{an}^1 \\ V_{an}^2 \end{bmatrix} = -[R + \jmath\omega(L_s + M_s)] \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} + \jmath\omega M_s \mathsf{A}^{-1} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \mathsf{A} \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} + \mathsf{A}^{-1} \begin{bmatrix} E_{an} \\ \alpha^2 E_{an} \\ \alpha E_{an} \end{bmatrix}$$

$$\begin{bmatrix} V_{an}^0 \\ V_{an}^1 \\ V_{an}^2 \end{bmatrix} = -[R + \jmath\omega(L_s + M_s)] \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} + \jmath\omega M_s \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} + \begin{bmatrix} 0 \\ E_{an} \\ 0 \end{bmatrix}$$

$$
\begin{aligned}
V_{an}^0 &= -RI_a^0 - \jmath\omega(L_s - 2M)I_a^0 \\
V_{an}^1 &= E_{an} - RI_a^1 - \jmath\omega(L_s + M_s)I_a^1 \\
V_{an}^2 &= -RI_a^2 - \jmath\omega(L_s + M_s)I_a^2
\end{aligned}
$$

$$
\begin{aligned}
V_{an}^0 &= -I_a^0[R + \jmath\omega(L_s - 2M)] && = -I_a^0 Z_{g0} \\
V_{an}^1 &= E_{an} - I_a^1[R_a + \jmath\omega(L_s + M_s)] && = E_{an} - I_a^1 Z_1 \\
V_{an}^2 &= -I_a^2[R + \jmath\omega(L_s + M_s)] && = -I_a^2 Z_2
\end{aligned}
$$

where Z_{g0} ,Z_1 and Z_2 are the zero, positive and negative sequence impedances respectively.

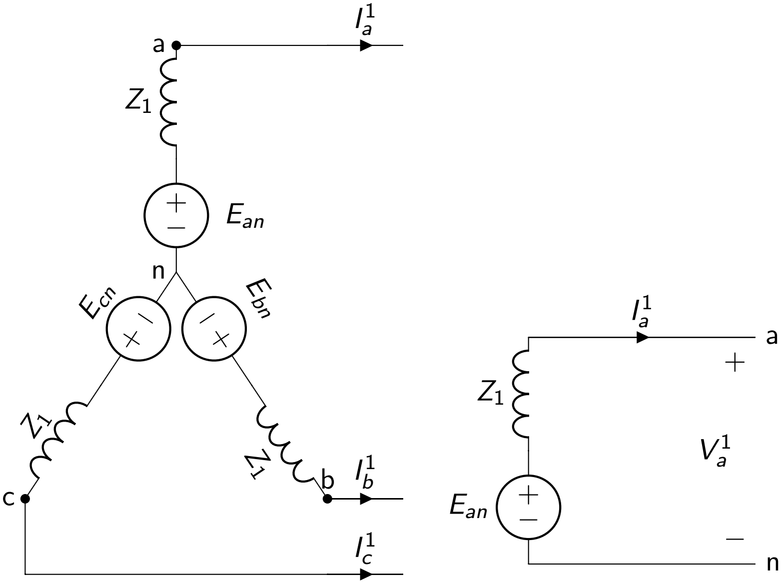


Figure: Postive sequence network

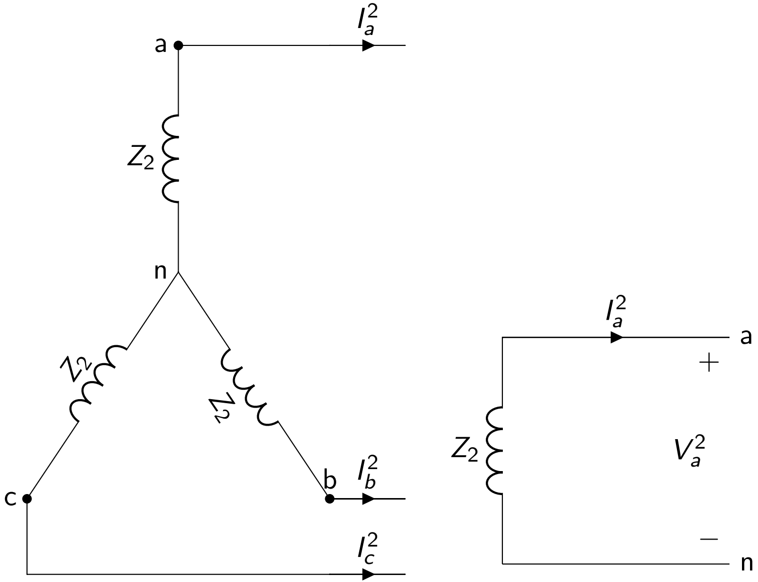


Figure: Negative sequence network

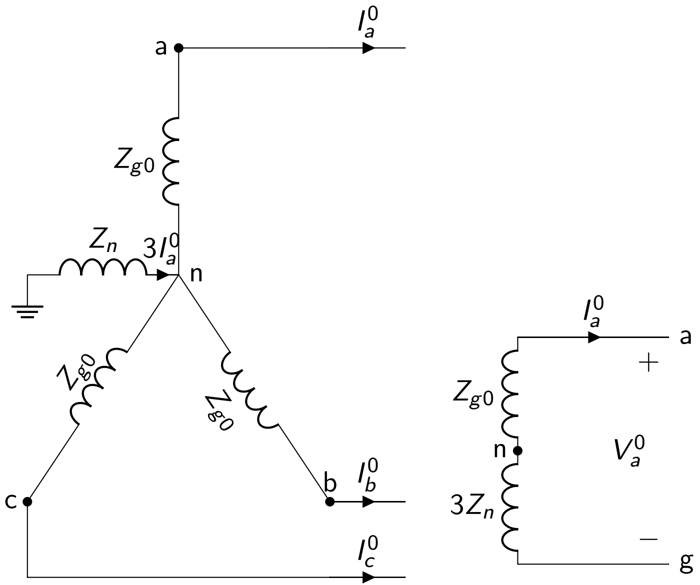


Figure: Zero sequence network

Sequence Circuits of Transformers

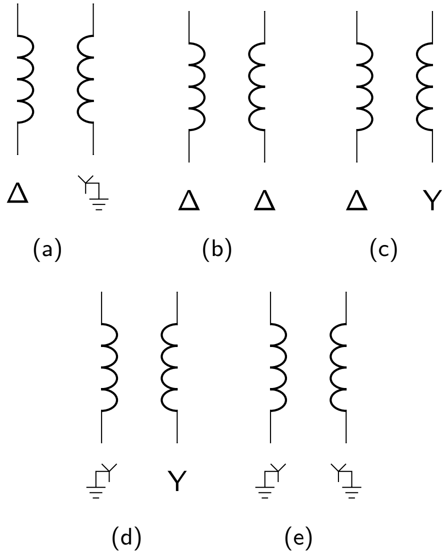
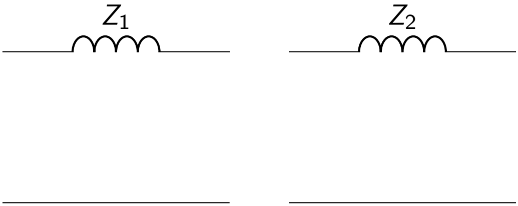


Figure: Different Connections

Positive and negative sequence currents do not need a connection between the neutral and ground.



(a) Positive sequence circuit

(b) Negative sequence circuit

Figure:

$Z_1 = Z_2$. It is the same irrespective of transformer connections.

Zero sequence circuits of Transformers

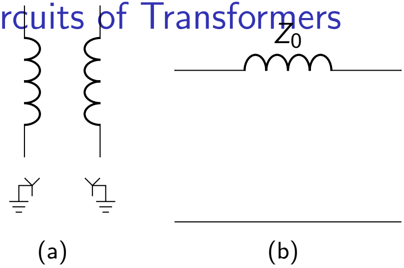


Figure: Y-Y Bank. Both neutrals grounded

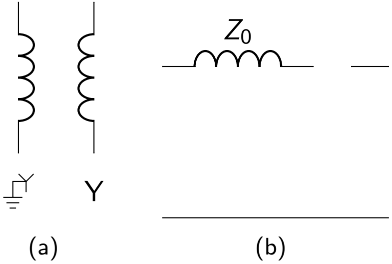


Figure: Y-Y Bank. One grounded

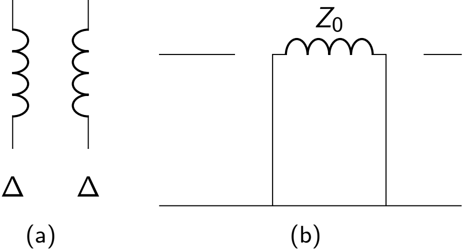


Figure: Δ – Δ Bank

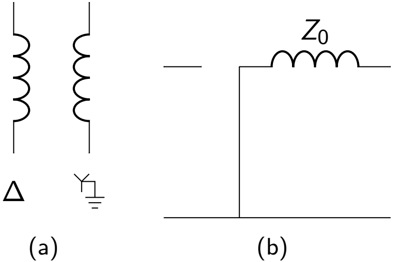


Figure: Δ-Y Bank. Y grounded

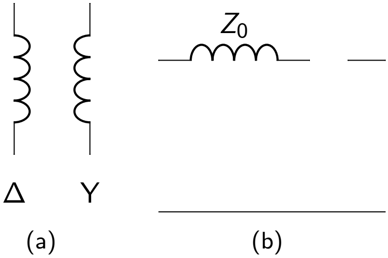


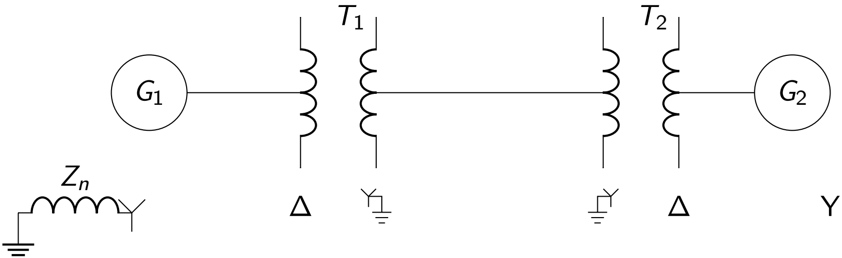
Figure: Δ - Y Bank

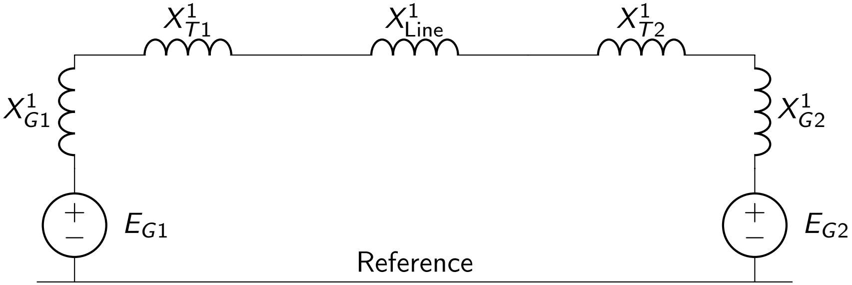
$Z_0 = Z + 3Z_n$ where Z is the zero sequence leakage impedance and Z_n is the impedance connected between the neutral and ground.

Sequence Networks

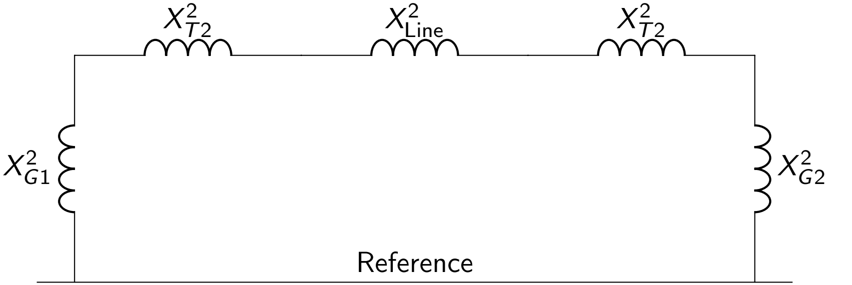
- In any part of the network, the voltage drop caused by current of a certain sequence depends on only the impedance of that part of the network to current flow of that sequence.
- The impedances to positive and negative sequence currents Z_1 and Z_2 are equal in any static circuit and may be considered approximately equal in synchronous machines under subtransient conditions.
- In any part of the network, impedance to zero-sequence current Z_0 is generally different from Z_1 and Z_2.
- Only positive sequence circuits of rotating machines contain sources which are of positive sequence voltages.
- Neutral is the reference for voltages in positive and negative sequence circuits.
- No positive and negative sequence currents flow between neutral points and ground.
- Impedance Z_n between neutral and ground is not included in positive and negative sequence circuits but is represented by $3Z_n$ in the zero sequence circuits only.

Power System - Example





(a) Positive sequence circuit



(b) Negative sequence circuit

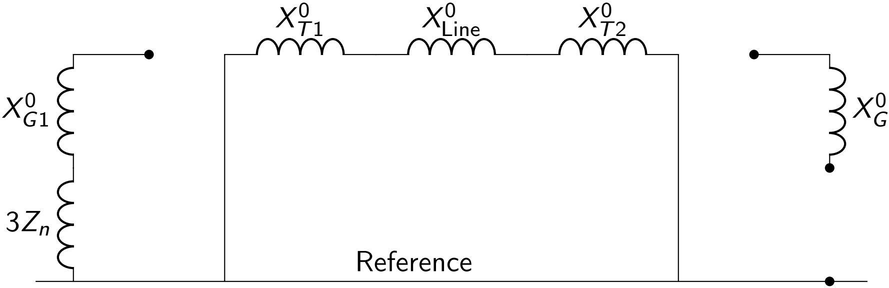


Figure: Zero sequence circuit

Single Line to Ground Faults

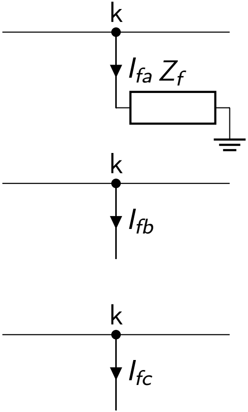


Figure: Single line to ground fault

The conditons at the fault bus k are

$$I_{fb} = 0 \quad I_{fc} = 0 \quad V_{ka} = Z_f I_{fa}$$

With $I_{fb} = I_{fc} = 0$,

$$\begin{bmatrix} I_{fa}^0 \\ I_{fa}^1 \\ I_{fa}^2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} I_{fa} \\ 0 \\ 0 \end{bmatrix}$$

$$I_{fa}^0 = I_{fa}^1 = I_{fa}^2 = \frac{I_{fa}}{3}$$

Assume that the prefault voltage at all the buses is V_f.

$$\begin{aligned} V_{ka}^0 &= -Z_{kk}^0 I_{fa}^0 \\ V_{ka}^1 &= V_f - Z_{kk}^1 I_{fa}^1 \\ V_{ka}^2 &= -Z_{kk}^2 I_{fa}^2 \end{aligned}$$

Since $V_{ka} = Z_f \times 3I_{fa}^0$,

$$V_{ka} = V_{ka}^0 + V_{ka}^1 + V_{ka}^2 = V_f - (Z_{kk}^0 + Z_{kk}^1 + Z_{kk}^2)I_{fa}^0$$

$$3Z_f I_{fa}^0 = V_f - (Z_{kk}^0 + Z_{kk}^1 + Z_{kk}^2)I_{fa}^0$$

$$I_{fa}^0 = I_{fa}^1 = I_{fa}^2 = \frac{V_f}{(Z_{kk}^0 + Z_{kk}^1 + Z_{kk}^2) + 3Z_f}$$

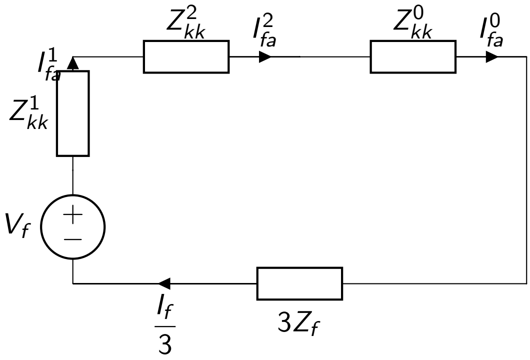
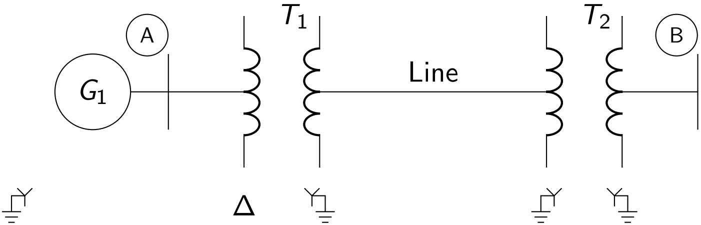


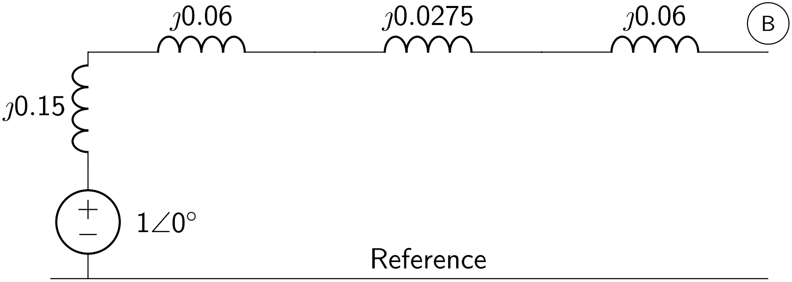
Figure: Connection of the Thevenin equivalents for a LG fault

Example

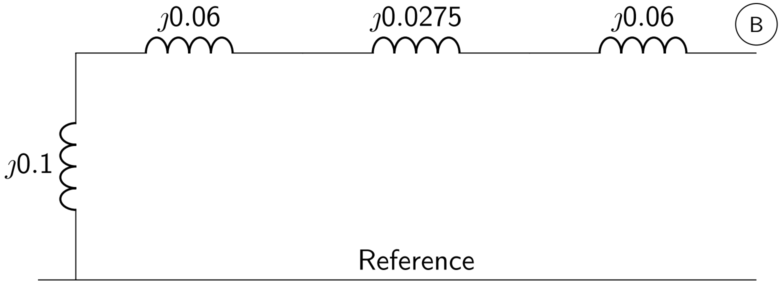
A 3-phase power system is represented by one-line diagram as shown in the figure below:



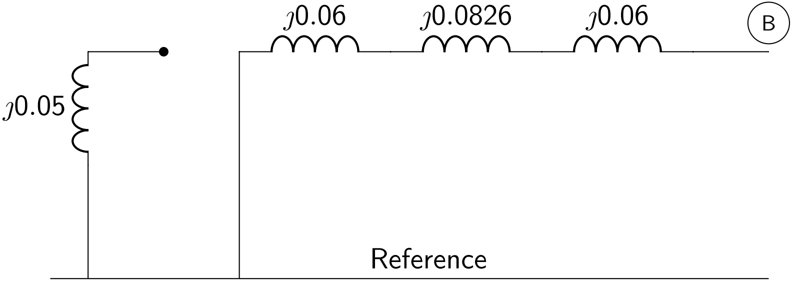
The ratings of the equipment are as follows:
Generator G:15 MVA, 6,6 kV, $X_1 = 15\%$, $X_2 = 10\%$ and $X_0 = 5\%$
Transformers : 15 MVA, 6.6 kV Δ / 33 kV Y,
$X_1 = X_2 = X_0 = 6\%$
Line : $X_1 = X_2 = 2\ \Omega$ and $X_0 = 6\ \Omega$
Find the fault current in Amp for a line to ground fault on one of the bus bars at B.



(a) Positive sequence circuit

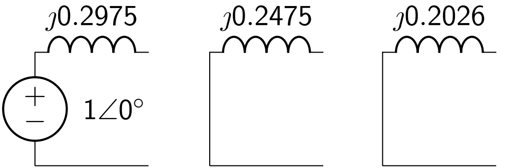


(b) Negative sequence circuit



(c) Zero sequence circuit

Figure: Sequence Circuits



(a) Positive sequence circuit

(b) Negative sequence circuit

(c) Zero sequence circuit

Figure: Thevenin Equivalent Circuits with respect to B

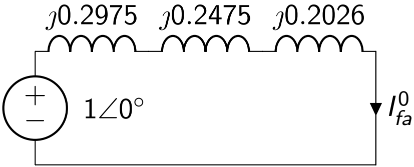


Figure: SLG Fault at B

$$I_{fa}^{0} = \frac{1\angle 0^{\circ}}{\jmath 0.2975 + \jmath 0.2475 + \jmath 0.2026} = 1.3376\angle -90^{\circ}$$

$$I_{fa} = 3 \times I_{fa}^{0} = 4.0128 \text{ p.u}$$

$$I_{fa} = 4.0128 \times \frac{15 \times 10^{3}}{\sqrt{3} \times 33} = 1.05 \text{ kA}$$

Line to Line Fault

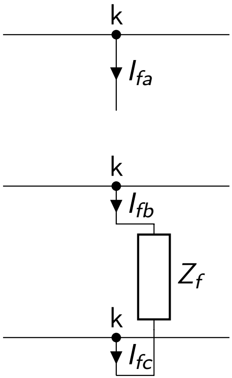


Figure: Line to Line fault

The relations at the fault point are

$$I_{fa} = 0 \quad I_{fb} = -I_{fc} \quad V_{kb} - V_{kc} = I_{fb} Z_f$$

Since $I_{fb} = -I_{fc}$ and $I_{fa} = 0$,

$$\begin{bmatrix} I_{fa}^0 \\ I_{fa}^1 \\ I_{fa}^2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} 0 \\ I_{fb} \\ -I_{fb} \end{bmatrix}$$

We get,

$$I_{fa}^0 = 0$$
$$I_{fa}^1 = -I_{fa}^2 = \frac{1}{3}(\alpha - \alpha^2) I_{fb}$$

The voltages throughout the zero sequence network must be zero since there are no zero sequence sources.
Hence line to line fault calculations do not involve the zero sequence network.

To satisfy $I_{fa}^1 = -I_{fa}^2$, the thevenin equivalents of positive and negative sequence networks are connected in parallel.

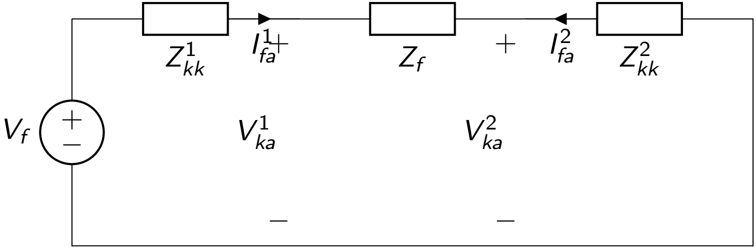


Figure: Connection of the Thevenin equivalents for a LL fault

To show that

$$V_{kb} - V_{kc} = I_{fb} Z_f$$

$$V_{kb} - V_{kc} = (V_{kb}^1 + V_{kb}^2) - (V_{kc}^1 + V_{kc}^2) \quad = (V_{kb}^1 - V_{kc}^1) + (V_{kb}^2 - V_{kc}^2)$$
$$= (\alpha^2 - \alpha)V_{ka}^1 + (\alpha - \alpha^2)V_{ka}^2 \qquad = (\alpha^2 - \alpha)(V_{ka}^1 - V_{ka}^2)$$

$$I_{fb}Z_f = (I_{fb}^1 + I_{fb}^2)Z_f = (\alpha^2 I_{fa}^1 + \alpha I_{fa}^2)Z_f$$

Substituting $I_{fa}^1 = -I_{fa}^2$ and equating both equations, we get

$$(\alpha^2 - \alpha)(V_{ka}^1 - V_{ka}^2) = (\alpha^2 - \alpha)I_{fa}^1 Z_f$$

$$V_{ka}^1 - V_{ka}^2 = I_{fa}^1 Z_f$$

From the connection of the thevenin equivalents,

$$I_{fa}^1 = -I_{fa}^2 = \frac{V_f}{Z_{kk}^1 + Z_{kk}^2 + Z_f}$$

For a bolted line to line fault $(Z_f = 0)$,

$$I_{fa}^1 = -I_{fa}^2 = \frac{V_f}{Z_{kk}^1 + Z_{kk}^2}$$

To find the fault current ($I_f = I_{fb} = -I_{fc}$),

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_{fa}^0 \\ I_{fa}^1 \\ I_{fa}^2 \end{bmatrix}$$

Since $I_{fa} = 0$ and $I_{fb} = -I_{fc} = I_f$,

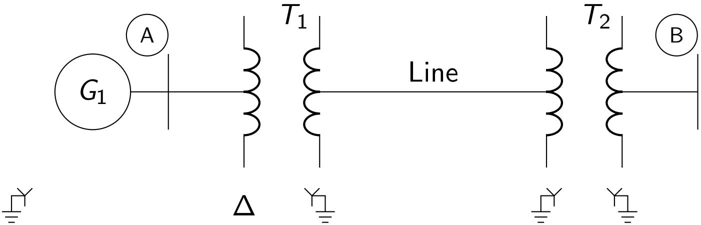
$$I_f = I_{fb} = I_{fa}^0 + \alpha^2 I_{fa}^1 + \alpha I_{fa}^2$$

We know that $I_{fa}^0 = 0$ and $I_{fa}^1 = -I_{fa}^2$.

$$\boxed{I_f = I_{fa}^1(\alpha^2 - \alpha)}$$

Example

A 3-phase power system is represented by one-line diagram as shown in the figure below:



The ratings of the equipment are as follows:
Generator G:15 MVA, 6,6 kV, $X_1 = 15\%$, $X_2 = 10\%$ and $X_0 = 5\%$
Transformers : 15 MVA, 6.6 kV Δ / 33 kV Y,
$X_1 = X_2 = X_0 = 6\%$
Line : $X_1 = X_2 = 2\ \Omega$ and $X_0 = 6\ \Omega$
Find the fault current in Amp for a line to line fault on the bus bars at B.

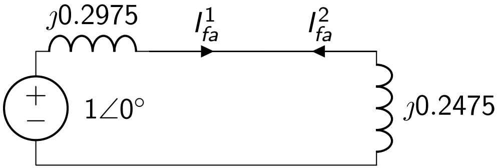


Figure: LL Fault at B

$$I_{fa}^1 = \frac{1\angle 0^\circ}{\jmath 0.2975 + \jmath 0.2475} = 1.8349\angle{-90^\circ}$$

$$I_f = I_{fa}^1(\alpha^2 - \alpha) = -3.1781 \text{ p.u}$$

$$I_f = 834 \text{ A}$$

-ve sign indicates the direction of the fault current is opposite.

Double Line to Ground Fault

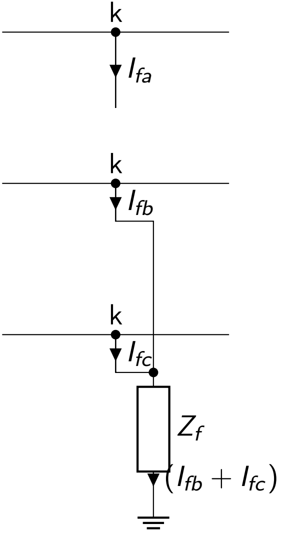


Figure: Double line to ground fault

The relations at the fault bus are

$$I_{fa} = 0 \quad V_{kb} = V_{kc} = (I_{fb} + I_{fc})Z_f$$

The zero sequence current is

$$I_{fa}^0 = \frac{1}{3}(I_{fa} + I_{fb} + I_{fc})$$

Since $I_{fa} = 0$,

$$I_{fa}^0 = \frac{1}{3}(I_{fb} + I_{fc})$$

The voltage at the fault point is

$$V_{kb} = V_{kc} = 3I_{fa}^0 Z_f$$

Since $I_{fa} = 0$,

$$I_{fa}^0 + I_{fa}^1 + I_{fa}^2 = 0$$

Substituting for V_{kb} for V_{kc},

$$\begin{bmatrix} V_{ka}^0 \\ V_{ka}^1 \\ V_{ka}^2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} V_{ka} \\ V_{kb} \\ V_{kb} \end{bmatrix}$$

$$V_{ka}^1 = V_{ka}^2$$

$$3V_{ka}^0 = V_{ka} + 2V_{kb} = (V_{ka}^0 + V_{ka}^1 + V_{ka}^2) + 2(3Z_f I_{fa}^0)$$

Substituting $V_{ka}^1 = V_{ka}^2$,

$$2V_{ka}^0 = 2V_{ka}^1 + 2(3Z_f I_{fa}^0)$$

$$V_{ka}^1 = V_{ka}^2 = V_{ka}^0 - 3Z_f I_{fa}^0$$

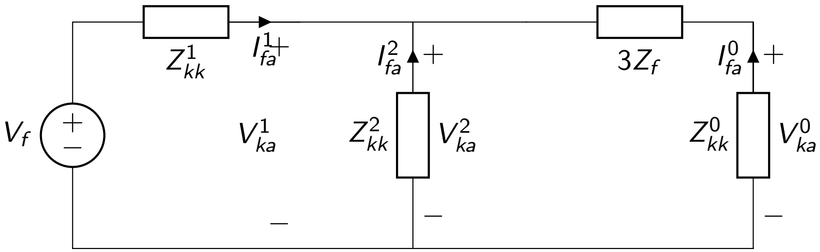


Figure: Connection of the Thevenin equivalents for a LLG fault

$$I_{fa}^1 = \frac{V_f}{Z_{kk}^1 + \left[\frac{Z_{kk}^2(Z_{kk}^0 + 3Z_f)}{Z_{kk}^2 + Z_{kk}^0 + 3Z_f}\right]}$$

$$I_{fa}^2 = -I_{fa}^1 \left[\frac{Z_{kk}^0 + 3Z_f}{Z_{kk}^2 + Z_{kk}^0 + 3Z_f}\right] \qquad I_{fa}^0 = -I_{fa}^1 \left[\frac{Z_{kk}^2}{Z_{kk}^2 + Z_{kk}^0 + 3Z_f}\right]$$

For a bolted fault, $Z_f = 0$.

To find the fault current ($I_f = I_{fb} + I_{fc}$),

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} I_{fa}^0 \\ I_{fa}^1 \\ I_{fa}^2 \end{bmatrix}$$

$$I_f = I_{fa}^0 + \alpha^2 I_{fa}^2 + \alpha I_{fa}^2 + I_{fa}^0 + \alpha I_{fa}^1 + \alpha^2 I_{fa}^2$$

Since $1 + \alpha + \alpha^2 = 0$,

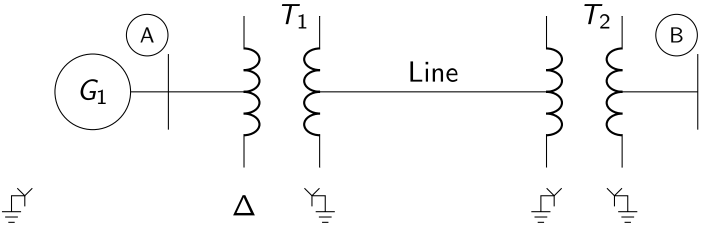
$$I_f = 2I_{fa}^0 - I_{fa}^1 - I_{fa}^2$$

Since $I_{fa}^0 + I_{fa}^1 + I_{fa}^2 = 0$,

$$\boxed{I_f = 3I_{fa}^0}$$

Example

A 3-phase power system is represented by one-line diagram as shown in the figure below:



The ratings of the equipment are as follows:
Generator G:15 MVA, 6,6 kV, $X_1 = 15\%$, $X_2 = 10\%$ and $X_0 = 5\%$
Transformers : 15 MVA, 6.6 kV Δ / 33 kV Y,
$X_1 = X_2 = X_0 = 6\%$
Line : $X_1 = X_2 = 2\ \Omega$ and $X_0 = 6\ \Omega$
Find the fault current in Amp for a double line to ground fault on the bus bars at B.

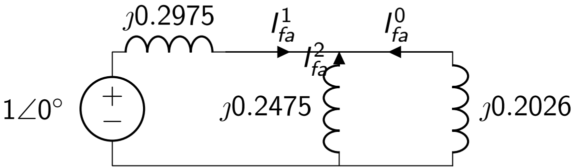


Figure: LLG Fault at B

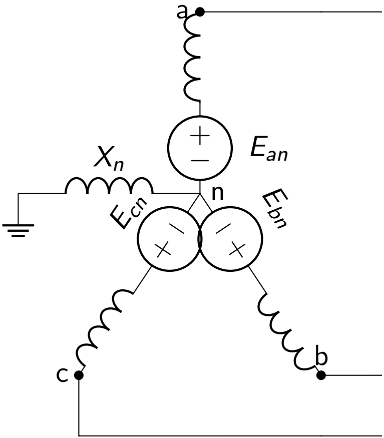
$$I_{fa}^1 = \frac{1\angle 0^\circ}{\jmath 0.2975 + \frac{\jmath 0.2475 \times \jmath 0.2026}{\jmath 0.2475 \times \jmath 0.2026}} = 2.4456\angle{-90^\circ}$$

$$I_{fa}^0 = -I_{fa}^1 \times \frac{\jmath 0.2475}{\jmath 0.2475 + \jmath 0.2026} = 1.3448\angle 90^\circ$$

$$I_f = 3I_{fa}^0 = 4.0343 \text{ p.u}$$

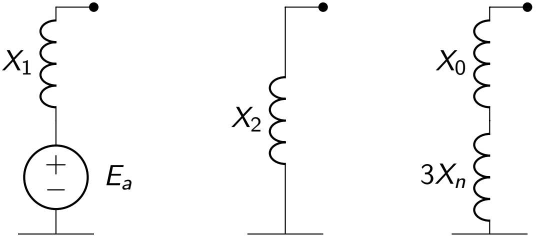
$$I_f = 1.06 \text{ kA}$$

Observation on Neutral Connection



Let X_1, X_2 and X_0 be the positive, negative and zero sequence reactances respectively.

$$X_1 = X_2 > X_0$$



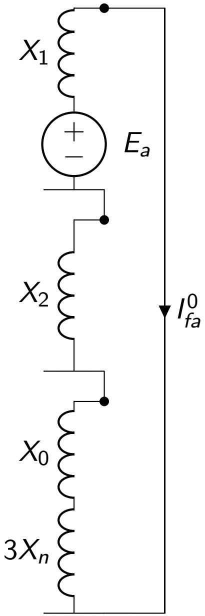
(a) Positive sequence circuit

(b) Negative sequence circuit

(c) Zero sequence circuit

Figure: Sequence Circuits

For a SLG fault on Phase a,



$$I_{fa}^0 = I_{fa}^1 = I_{fa}^2 = \frac{I_{fa}}{3}$$

$$I_{fa} = \frac{3E_a}{X_1 + X_2 + X_0 + 3X_n}$$

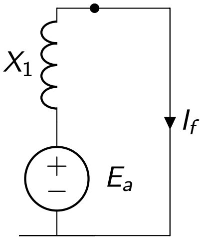
Since $X_1 = X_2$,

$$I_{fa} = \frac{3E_a}{2X_1 + X_0 + 3X_n}$$

If the neutral is solidly grounded,

$$I_{fa} = \frac{3E_a}{2X_1 + X_0}$$

For a three phase short circuit fault,



$$I_f = \frac{E_a}{X_1} = \frac{3E_a}{3X_1}$$

It is clear that

$$I_{f_{\text{SLG}}} > I_{f_{\text{3L}}}$$

If the generator neutral is solidly grounded, SLG fault is more severe than three phase short circuit.