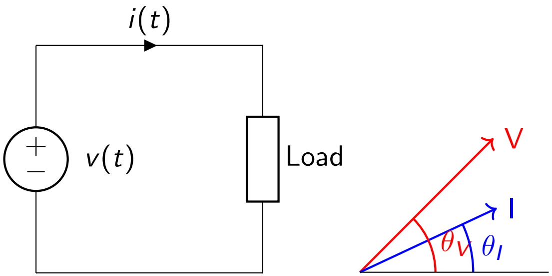
Power in Single Phase AC Circuits

Let us consider the following circuit.



Let

$$v(t) = \sqrt{2}V\sin(\omega t + \theta_V)$$

$$i(t) = \sqrt{2}I\sin(\omega t + \theta_I)$$

The **instantaneous power** delivered to the load is

$$p(t) = v(t)i(t)$$

$$p(t) = V_m \sin(\omega t + \theta_V) I_m \sin(\omega t + \theta_I)$$

$$p(t) = \frac{V_m I_m}{2}(\cos(\theta_V - \theta_I) - \cos(2\omega t + \theta_V + \theta_I))$$

$$p(t) = \frac{V_m I_m}{2}\cos(\theta_V - \theta_I) - \frac{V_m I_m}{2}\cos(2\omega t + \theta_V + \theta_I)$$

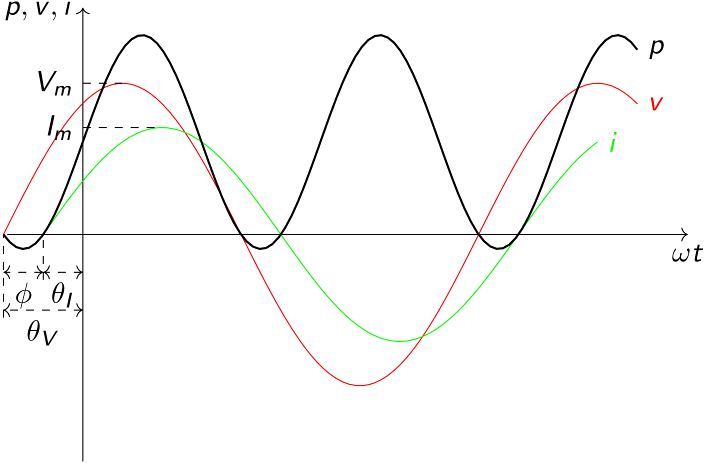


Figure: Voltage, current and power in RL circuit

Let $\theta_V - \theta_I$ be ϕ.

$$p(t) = VI\cos\phi - VI\cos(2\omega t + \theta_V + \theta_I)$$

$$p(t) = VI\cos\phi - VI\cos(2\omega t + \theta_V - \theta_I + 2\theta_I)$$

$$p(t) = VI\cos\phi - VI\cos(2\omega t + 2\theta_I - \phi)$$

$$p(t) = \underbrace{VI\cos\phi(1 - \cos(2\omega t + 2\theta_I))}_{p_I} - \underbrace{VI\sin\phi\sin(2\omega t + 2\theta_I)}_{p_{II}}$$

p_I has an average value of $VI\cos\phi$ which is called the average power.

p_{II} does not have an average. But it's maximum value is $VI\sin\phi$ which is called reactive power.

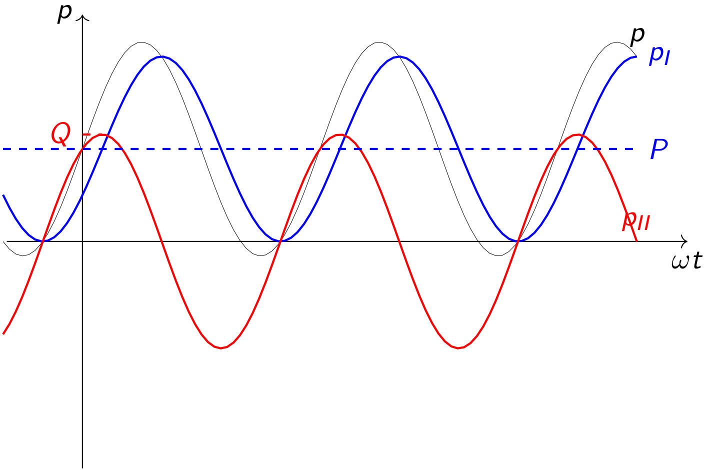


Figure: Power in RL circuit

Power

The average power P is

$$\boxed{P = \frac{V_m I_m}{2}\cos(\theta_V - \theta_I) = VI\cos(\phi)}$$

where $\phi = \theta_V - \theta_I$. Its unit is watts (W).
The reactive power Q is

$$Q = VI\sin\phi$$

The apparent power S is

$$|S| = VI$$

Its unit is **volt-ampere (VA)**.
The ratio of real power (P) to apparent power is called as the **power factor (pf)**.

$$\text{pf} = \frac{VI\cos\phi}{VI} = \cos\phi$$

Since $\cos\phi$ can never be greater than unity, $P \leq |S|$.

Complex Power

Let us define voltage phasor and current phasor.

$$\mathsf{V} = V\angle\theta_V, \quad \mathsf{I} = I\angle\theta_I$$

The complex power S is

$$\mathsf{S} = \mathsf{V}\mathsf{I}^*$$

$$\mathsf{S} = V\angle\theta_V \; I\angle-\theta_I$$

$$= VI\angle(\theta_V - \theta_I)$$

$$\mathsf{S} = VI\cos\phi + \jmath VI\sin\phi$$

The real part of S is called the average power (P). The imaginary part of S is called the reactive power (Q).

$$\mathsf{S} = P + \jmath Q$$

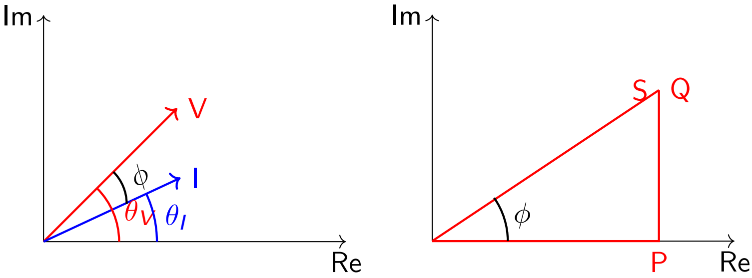


Figure: RL load

If V leads I ($\phi > 0$), power factor is lagging.

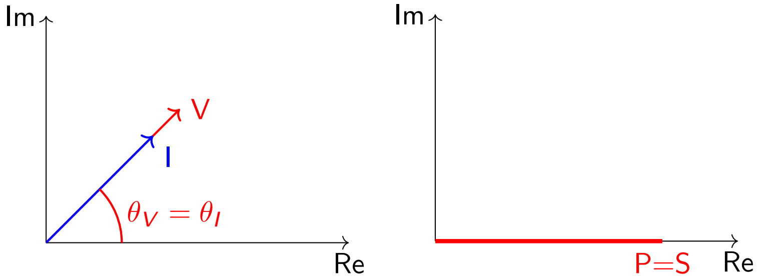


Figure: Resisitive Load

If V and I are in phase ($\phi = 0$), power factor is unity.

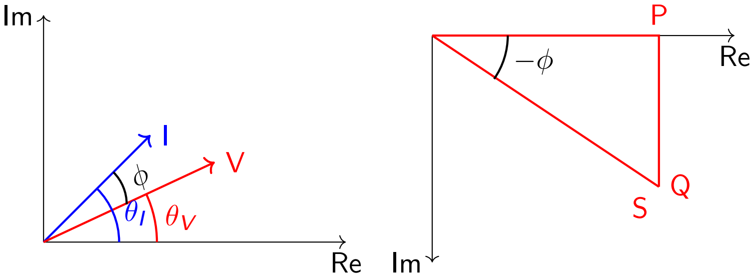
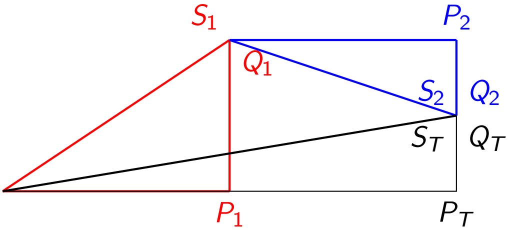


Figure: RC load

If I leads V ($\phi < 0$), power factor is leading.

For two loads (inductive and capacitive) in parallel,



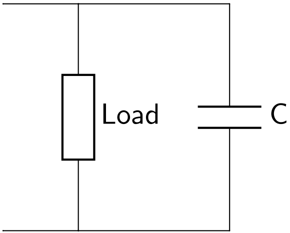
$$P_T = P_1 + P_2; \quad Q_T = Q_1 + Q_2$$

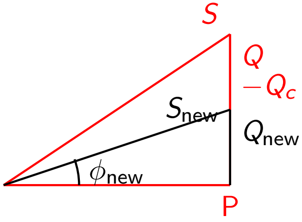
But

$$|S_T| \neq |S_1| + |S_2|$$

Power Factor Control

- If *pf* decreases, the current will increase to supply the same real power.
- This will increase the line loss. (It is an additional cost to a utility.)
- Capacitors which supply reactive power are connected in parallel to improve the power factor.





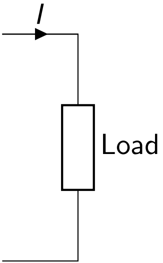
Example 1 : A single-phase inductive load draws to 1 kW at 0.6 power-factor lagging from a 230 V AC supply.

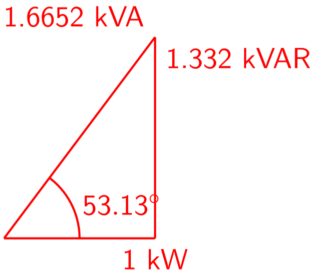
1. Find the current it draws.
2. Find the value of a capacitor to be connected in parallel with the load to raise the power factor to 0.9 lagging. Determine the current under this condition.

1.

$$I = \frac{1000}{230 \times 0.6} = 7.24 \text{ A}$$

$$Q = 230 \times 7.24 \times 0.8 = 1.332 \text{ kVAr}$$





2.

$$pf_{\text{new}} = 0.9; \quad \phi_{\text{new}} = 25.84^\circ$$

$$Q_{\text{new}} = Q_L - Q_c$$

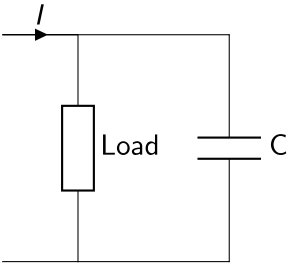
$$Q_{\text{new}} = P \times \tan 25.84^\circ = 484.32 \text{ VAr}$$

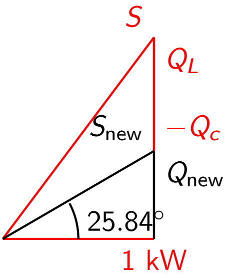
$$Q_c = 847.68 \text{ VAr}$$

$$Q_c = V^2 \omega C$$

$$C = 51 \ \mu F$$

$$I = \frac{1000}{230 \times 0.9} = 4.83 \ A$$





Power in Balanced Three Phase Circuits

Let v_a, v_b and v_c be the instantaneous voltages of a balanced three phase source.

$$
\begin{aligned}
v_a &= \sqrt{2}V\sin(\omega t + \theta_V) \\
v_b &= \sqrt{2}V\sin(\omega t + \theta_V - 120^\circ) \\
v_c &= \sqrt{2}V\sin(\omega t + \theta_V - 240^\circ)
\end{aligned}
$$

When it supplies a balanced load,

$$
\begin{aligned}
i_a &= \sqrt{2}I\sin(\omega t + \theta_I) \\
i_b &= \sqrt{2}I\sin(\omega t + \theta_I - 120^\circ) \\
i_c &= \sqrt{2}I\sin(\omega t + \theta_I - 240^\circ)
\end{aligned}
$$

The instantaneous power is

$$p = v_a i_a + v_b i_b + v_c i_c$$

$$\begin{aligned} p =& \sqrt{2} V_p \sin(\omega t + \theta_V) \times \sqrt{2} I_p \sin(\omega t + \theta_I) \\ & \sqrt{2} V_p \sin(\omega t + \theta_V - 120^\circ) \times \sqrt{2} I_p \sin(\omega t + \theta_I - 120^\circ) \\ & \sqrt{2} V_p \sin(\omega t + \theta_V - 240^\circ) \times \sqrt{2} I_p \sin(\omega t + \theta_I - 240^\circ) \end{aligned}$$

$$\begin{aligned} p =& V_p I_p \cos(\theta_V - \theta_I) + V_p I_p \cos(2\omega t + \theta_V + \theta_I) \\ & V_p I_p \cos(\theta_V - \theta_I) + V_p I_p \cos(2\omega t + \theta_V + \theta_I - 120^\circ) \\ & V_p I_p \cos(\theta_V - \theta_I) + V_p I_p \cos(2\omega t + \theta_V + \theta_I - 240^\circ) \end{aligned}$$

$$\boxed{p = 3 V_p I_p \cos\phi}$$

where $\phi = \theta_V - \theta_I$.

The instantaneous power in a 3 phase balanced system is constant.

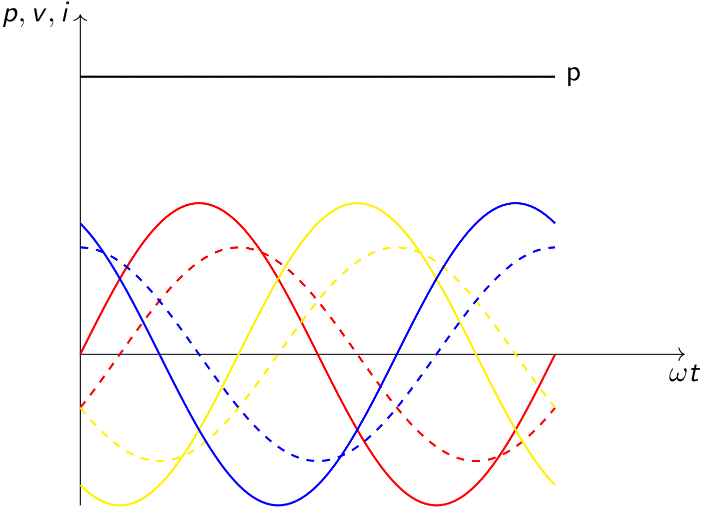


Figure: Voltage, current and power in a R-L load

The average/real power in a 3-phase system is

$$P = 3V_p I_p \cos\phi \quad \text{Watts}$$

In a Y connected load, $V_L = \sqrt{3}V_p$ and $I_L = I_p$,

$$P = \sqrt{3}V_L I_L \cos\phi$$

In a Δ connected load, $V_L = V_p$ and $I_L \sqrt{3} I_p$,

$$P = \sqrt{3}V_L I_L \cos\phi$$

Therefore, the three phase real power is

$$P = 3V_p I_p \cos\phi = \sqrt{3}V_L I_L \cos\phi$$

Since the instantaneous power in a 3-phase balanced system is constant, it does not mean that there is no reactive power. Still the instantaneous power of individual phases is pulsating.

The 3-phase reactive power is

$$Q = 3V_p I_P \sin\phi = \sqrt{3} V_L I_L \sin\phi \quad \text{VAr}$$

The apparent power is

$$|S| = \sqrt{P^2 + Q^2} = 3V_p I_p = \sqrt{3} V_L I_L \quad \text{VA}$$

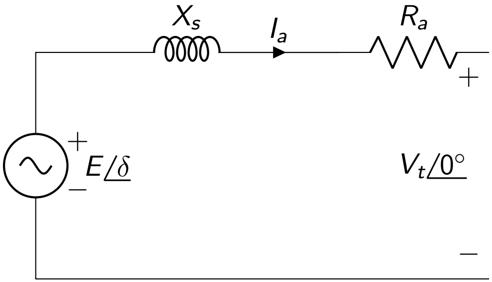
Per Phase Analysis

If a three phase system is balanced and there is no mutual inductance between phases, it is enough to analyze it on per phase basis.

1. Convert all Δ connected sources and loads into equivalent Y connections.
2. Solve for phase a variables using the phase a circuit with neutrals connected.
3. Other phase variables can be found from the phase a variables using the symmetry.
4. If necessary, find line-line variables from the original circuit.

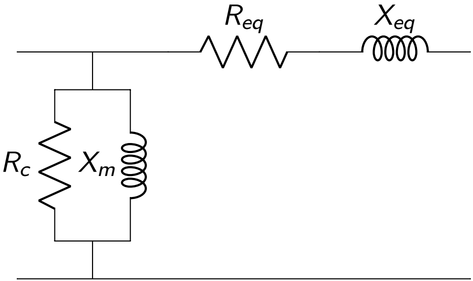
Synchronous Machine Model

The per phase equivalent circuit of a synchronous machine is

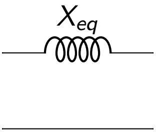


Transformer Model

The per phase equivalent circuit of a transformer is



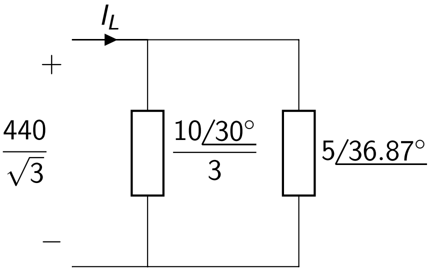
- Since the impedance of the shunt path is larger, R_c and X_m are neglected.
- Since R_{eq} is much smaller than X_{eq}, R_{eq} can be eliminated.



Example 2: Consider a system where a three phase 440 V, 50 Hz source is supplying power to two loads. Load 1 is a Δ connected load with a phase impedance of $10\angle 30^\circ$ Ω and load 2 is a Y connected load with a phase impedance of $5\angle 36.87^\circ$ Ω.

1. Find the line current and the overall power factor of the system.
2. Determine the capacitance per phase in μF of a three phase bank of delta connected capacitors to be added in parallel to the load to improve the overall power factor unity. Find the line current under this condition.

1. The per phase equivalent circuit after using Δ to Y transformation,



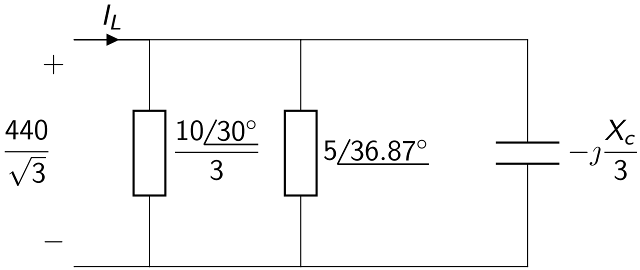
$$I_L = \frac{440/\sqrt{3}}{10/3\angle 30^\circ} + \frac{440/\sqrt{3}}{5\angle 36.87^\circ}$$

$$I_L = 106.64 - \jmath 68.6\text{A}$$

$$I_L = 126.8\angle -32.75^\circ \text{ A}$$

$$pf = \cos(-32.75^\circ) = 0.84 \text{ lag}$$

2. The per phase equivalent circuit with a capacitor bank



To make overall power factor unity, I_L must be in phase with the voltage.

$$\therefore I_c = \jmath 68.6\text{A}$$

$$X_c = \frac{3 \times 440}{\sqrt{3} \times 68.6} = 11.11\ \Omega$$

$$C = 286.52\ \mu F$$

Example 3: Consider a system where a three phase 400 V, 50 Hz source is supplying power to two loads. Load 1 draws 5 kW at 0.8 pf lagging and load 2 draws 5 kW at unity power factor. The voltage across the loads is 400 V.

1. Find the line current and the overall power factor.
2. Find the value of kVAR required from a bank of capacitors connected across the loads to improve the overall power factor to unity. Determine the line current under this condition.

1.

$$I_{L1} = \frac{5000}{\sqrt{3} \times 400 \times 0.8} = 9 \text{ A}$$

$$I_{L2} = \frac{5000}{\sqrt{3} \times 400 \times 1} = 7.2; \text{A}$$

$$I_L = 9\underline{/-36.87^\circ} + 7.2\underline{/0^\circ} = 15.38\underline{/-20.56^\circ} \text{ A}$$

$$pf = 0.9363 \text{ lag}$$

$$Q_T = Q_1 + Q_2 = 3.75 + 0 = 3.75 \text{ kVAR}$$

2. To make overall power factor unity,

$$Q_C + Q_T = 0$$

$$Q_C = -3.75 \text{ kVAR}$$

(-ve indicates that the capacitor supplies reactive power.)

$$\therefore Q_C = 3.75 \text{ kVAR}$$

When pf is unity, $S = P$.

$$I_L = \frac{P_T}{\sqrt{3}V_L} = \frac{10 \times 10^3}{\sqrt{3} \times 400} = 14.4 \text{ A}$$

$$I_L = 14.4\underline{/0^\circ} \text{ A}$$