Transformers

- A Transformer is a static device. However, it plays a major role in the energy conversion process.
- It transfers energy from one side to the other side by changing the level of voltage and current.
- It primarily consists of two or more windings coupled by mutual magnetic flux.
- If one of the windings, the primary, is connected to an alternating voltage source, an alternating flux will be produced.
- The mutual flux will link the other winding, the secondary[1], and will induce a voltage in it.

[1] An alternate terminology, which refers to the windings as "High Voltage" and "Low Voltage" is often used as power can flow either way.

- The value of the induced voltage depends on the number of turns and the magnitude of the mutual flux.
- By properly proportioning the number of primary and secondary turns, the desired voltage ratio or ratio of transformation can be obtained.
- The essence is the existence of time varying mutual flux linking two windings.
- To make it link with two windings, air or ferromagnetic core can be used.
- Iron core transformers which provide high magnetic coupling are used in **high power applications**.
- Air core transformers which have poor magnetic coupling are used in **low power electronic circuits**.
- We discuss iron core transformers in this course.

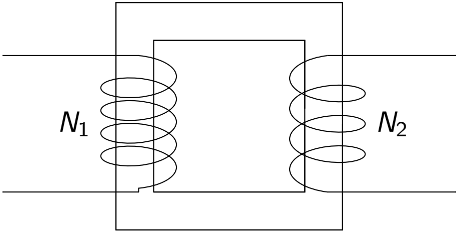


Figure: Two Winding Transformer - Iron Core

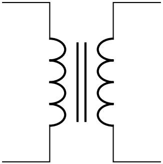
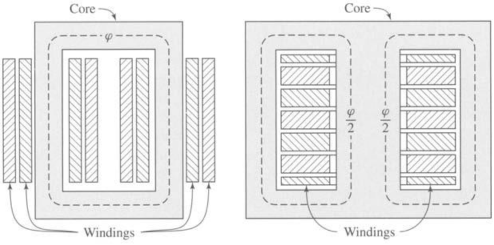


Figure: Schematic Representation

Types of Core Construction

a Core Type
- The windings are wound around two legs of a magnetic core.

b Shell Type
- The windings are wound around the center leg of a three legged magnetic core.

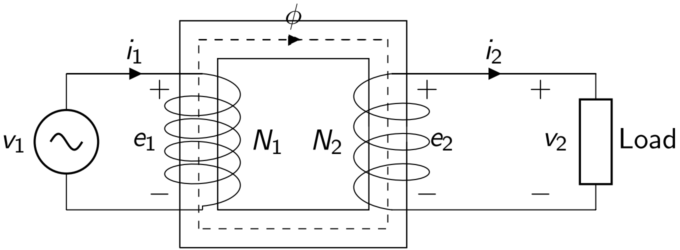


(a) (b)

- However, the windings are interleaved to avoid leakage flux in actual transformers.
- The low voltage winding is placed nearer the core and the high voltage winding on top.

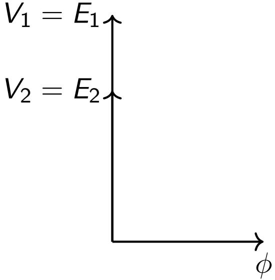
Ideal Transformer

- The winding resistances are negligible.
- All fluxes are confined to the core and link both windings. That is, there is no leakage flux.
- Core losses are are assumed to be negligible.
- The permeability of the core is infinite ($\mu = \infty$). Therefore, the exciting current required to produce flux in the core is negligible. Hence, net mmf required to produce flux in the core is zero.



$$v_1 = e_1 = N_1 \frac{d\phi}{dt}$$

$$v_2 = e_2 = N_2 \frac{d\phi}{dt}$$



$$\frac{v_1}{v_2} = \frac{N_1}{N_2}$$

Thus an ideal transformer transforms voltages in the ratio of the turns in its windings.

Let a load be connected to the secondary.

- A current i_2 will flow in the secondary winding and the secondary winding will provide an mmf of $N_2 i_2$ for the core.
- The secondary winding mmf will oppose the mutual flux as per Lenz's law.
- This will immediately make the primary winding current draw a current of i_1 so that $N_1 i_1$ opposes $N_2 i_2$.
- Otherwise, $N_2 i_2$ would make the core flux change drastically and the balance between v_1 and e_1 would be disturbed.
- The net mmf required to produce flux in the ideal transformer is zero.

$$N_1 i_1 - N_2 i_2 = \text{net mmf} = 0$$

$$N_1 i_1 = N_2 i_2$$

$$\frac{i_1}{i_2} = \frac{N_2}{N_1}$$

Thus an ideal transformer transforms currents in the inverse ratio of the turns in its windings.
From the two equations,

$$v_1 i_1 = v_2 i_2$$

If the supply voltage is sinusoidal,

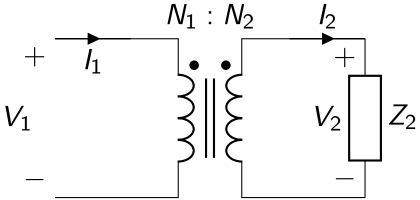
$$\frac{V_1}{V_2} = \frac{N_1}{N_2}$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_2}$$

$$V_1 I_1 = V_2 I_2$$

$$\text{Input VA} = \text{Output VA}$$

Impedance Transfer

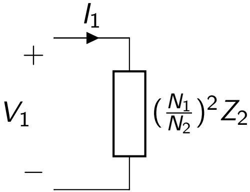


$$Z_2 = \frac{V_2}{I_2}$$

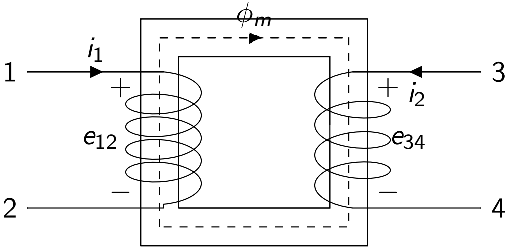
The input impedance

$$Z_1 = \frac{V_1}{I_1} = \frac{\frac{N_1}{N_2} V_2}{\frac{N_2}{N_1} I_2}$$

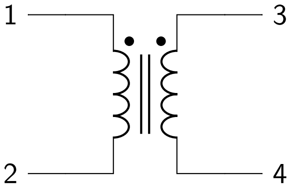
$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2 = Z_2'$$



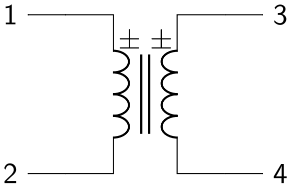
Polarity



- Terminals 1 and 3 are identical because current entering these terminals produce flux in the same direction.
- Terminals 2 and 4 are identical.
- e_{12} and e_{34} are in phase
- Identical terminals are marked by **dots** or $\pm$.

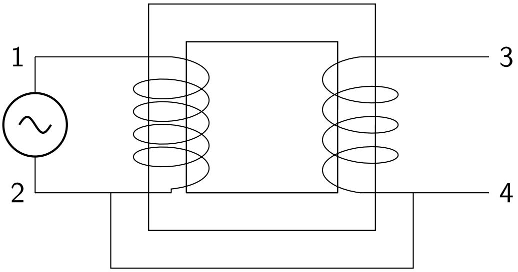


1
2
3
4



1
2
±
±
3
4

Polarity Test



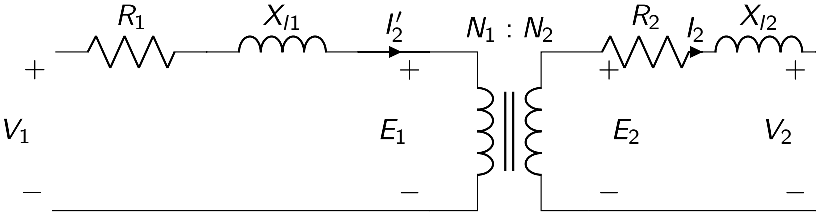
- Measure the voltages V_{12}, V_{34} and V_{13}.
- If $V_{13} = V_{12} - V_{34}$, terminals 1 and 3 are of the same polarity.
- If $V_{13} = V_{12} + V_{34}$, terminals 1 and 4 are of the same polarity.

Practical Transformer

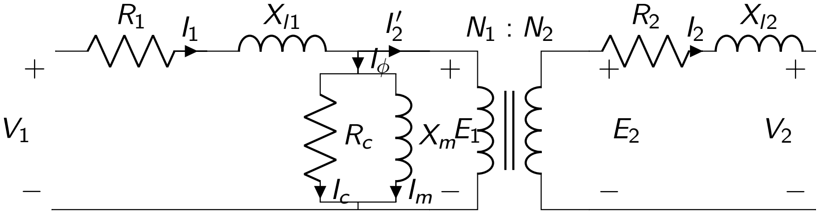
- The windings have resistances.
- All fluxes do not link both windings. That is, there is leakage flux.
- Core losses occur.
- The permeability of the core is finite. Therefore, exciting current is required to produce flux in the core.

Let us develop an equivalent circuit of a practical transformer by considering the above imperfections.

1. Considering R and X_l



2. Considering all imperfections



$R_1 =$ resistance of winding 1	$X_{l1} =$ leakage reactance of winding 1
$R_c =$ core loss resistance	$X_m =$ magnetizing reactance
$R_2 =$ resistance of winding 2	$X_{l2} =$ leakage reactance of winding 2

$$I_1 = I_\phi + I_2'$$

where

$$I_2' = \frac{N_2}{N_1} I_2$$

$$I_\phi = I_c + I_m$$

We can refer either secondary side parameters to the primary side or primary side parameters to the secondary side.

By referring the secondary side parameters to the primary side,

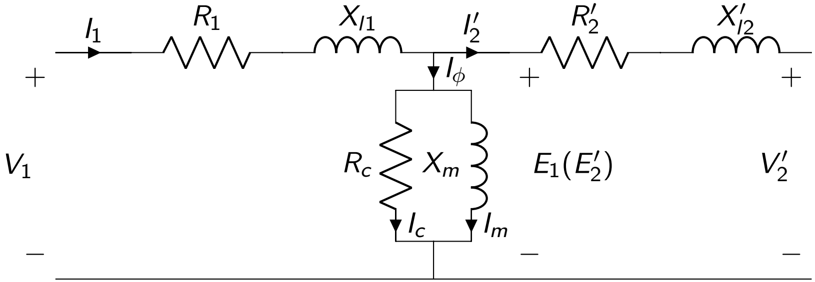


Figure: Exact Equivalent Circuit

$$R_2' = R_2 \left(\frac{N_1}{N_2}\right)^2 \quad X_{l2}' = X_{l2} \left(\frac{N_1}{N_2}\right)^2$$

$$V_2' = V_2 \frac{N_1}{N_2} \quad I_2' = I_2 \frac{N_2}{N_1}$$

Approximate Equivalent Circuit

The voltage drops $I_1 R_1$ and $I_1 X_{l1}$ are normally small.

$$E_1 \approx V_1$$

Therefore, the shunt branch can be moved to the supply side.

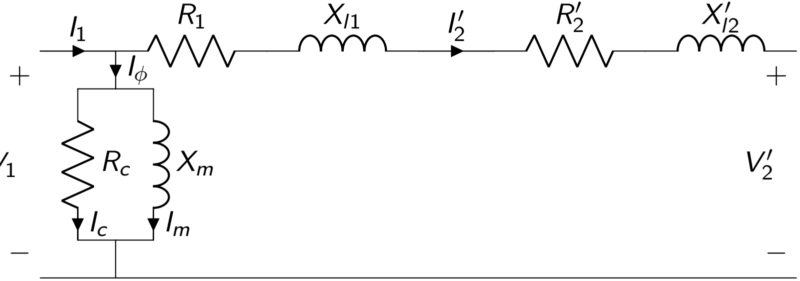


Figure: Approximate Equivalent Circuit

By combining winding resistances and leakage reactances,

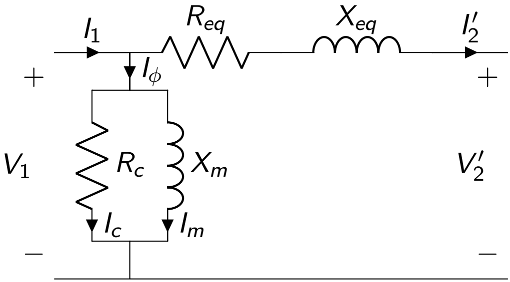
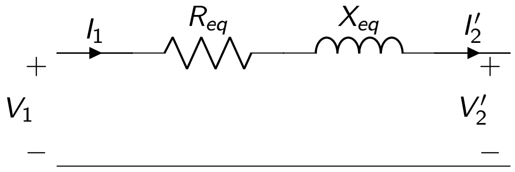


Figure: Approximate Equivalent Circuit

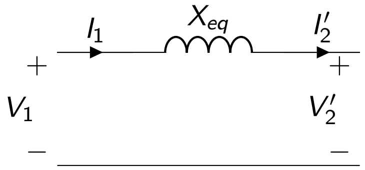
$$R_{eq} = R_1 + R_2' \quad X_{eq} = X_1 + X_2'$$

This is the circuit we use in EE280 and EE281.

Since I_ϕ is less than 5 % of the rated current, the shunt branch can be removed.



If the transformer is large (several hundred kVA), $R_{eq} << X_{eq}$,



This circuit is used in EE381 - Power Systems.

Transformer Rating

- The kilovolt-ampere (kVA) rating and voltage ratings of a transformer are marked on its nameplate.
- kVA rating indicates the rated apparent power it can deliver.
- The voltage ratio indicates the turns ratio.

For example, a single phase transformer has the following.

$$S = 50 \text{ kVA} \quad 2400/240 \text{ V}$$

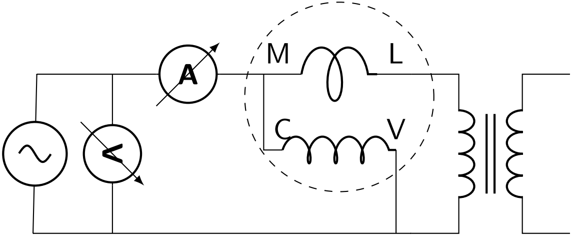
The current ratings of HV and LV windings are

$$I_{HV} = \frac{S}{V_{HV}} = \frac{50 \times 10^3}{2400} = 20.8 \text{ A}$$

$$I_{LV} = \frac{S}{V_{LV}} = \frac{50 \times 10^3}{240} = 208.3 \text{ A}$$

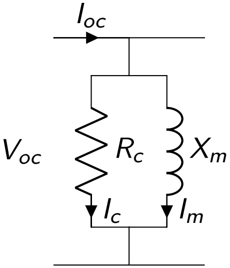
Determination of Parameters

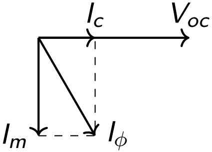
1. Open Circuit Test (No Load Test)
   - This test is performed on the low voltage side.
   - The rated voltage is applied to the low voltage side while the HV side is kept open.
   - This test is used to determine the shunt parameters.



Let

$$V_{oc} = \text{open circuit voltage in V}$$
$$I_{oc} = \text{open circuit current in A}$$
$$P_{oc} = \text{open circuit power in W}$$





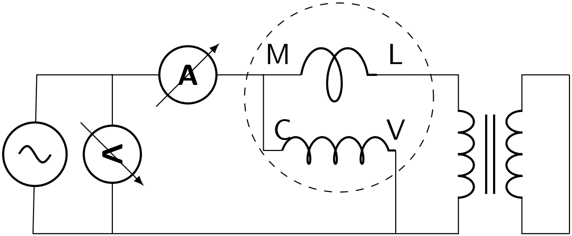
$$R_c = \frac{V_{oc}^2}{P_{oc}}$$

$$I_c = \frac{V_{oc}}{R_c}$$

$$I_m = \sqrt{I_{oc}^2 - I_c^2}$$

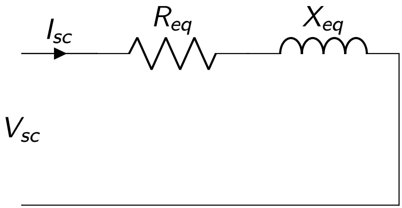
$$X_m = \frac{V_{oc}}{I_m}$$

2. Short Circuit Test
   - This test is performed on the high voltage side.
   - The voltage which is enough to supply the rated current in the HV side is applied while the LV side is short circuited.
   - This test is used to determine the series parameters.



Let

$$V_{sc} = \text{short circuit voltage in V}$$
$$I_{sc} = \text{short circuit current in A}$$
$$P_{sc} = \text{short circuit power in W}$$



$$R_{eq} = \frac{P_{sc}}{I_{sc}^2}$$

$$X_{eq} = \sqrt{\left(\frac{V_{sc}}{I_{sc}}\right)^2 - R_{eq}^2}$$

- Since the OC test is done on the LV side, the shunt parameters obtained reflect the LV side.
- Since the SC test is done on the HV side, the series parameters obtained reflect the HV side.
- They have to be referred to one side either HV or LV.

Voltage Regulation

The voltage regulation of a transformer is defined as the change in secondary terminal voltage from no load to full load and is usually expressed as a percentage of the full-load value.

$$\%\text{regn} = \frac{|V_2|_{NL} - |V_2|_{FL}}{|V_2|_{FL}} \times 100$$

- It is a figure of merit.
- It shows how far the secondary voltage varies when a full load is connected across it.
- It has to be as low as possible.

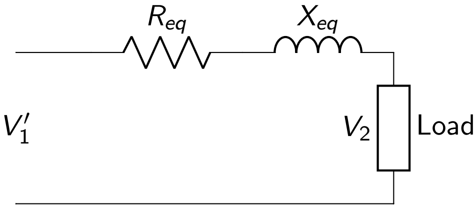


Figure: Referred to the secondary

$$V_{2,NL} = V_1' \quad V_{2,FL} = V_{2,rated}$$

$$\%\text{regn} = \frac{|V_1'| - |V_2|_{rated}}{|V_2|_{rated}} \times 100$$

From the equivalent circuit,

$$V_1' = V_2 + I_2 \times (R_{eq} + \jmath X_{eq})$$

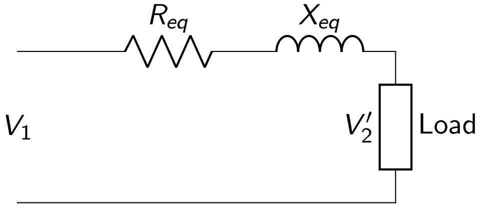


Figure: Referred to the primary

$$\%\text{regn} = \frac{|V_2'|_{NL} - |V_2'|_{FL}}{|V_2'|_{FL}} \times 100$$

Since

$$V_{2,NL}' = V_1 \quad V_{2,FL}' = V_{2,rated}'$$

$$\%\text{regn} = \frac{|V_1| - |V_2'|_{rated}}{|V_2'|_{rated}} \times 100$$

From the equivalent circuit,

$$V_1 = V_2' + I_2' \times (R_{eq} + \jmath X_{eq})$$

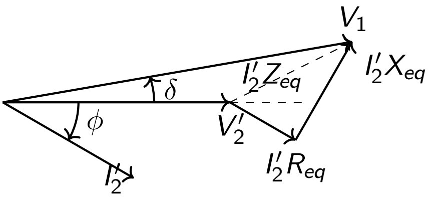


Figure: lagging power factor load

The % regn will be maximum when V_1 is in phase V_2'.

$$\theta_{eq} - \phi = 0$$

For lagging power factor load,

$$\phi = \theta_{eq}$$

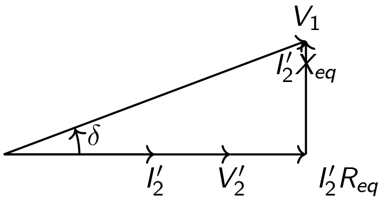


Figure: unity power factor load

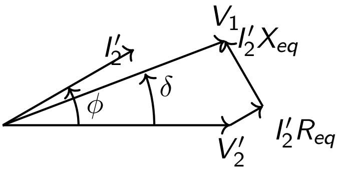


Figure: leading power factor load

If δ is small,

$$V_1 = V_2' + I_2' R_{eq} \cos\phi \pm I_2' X_{eq} \sin\phi$$

($+$ for lagging power factor; - for leading power factor)
Therefore

$$\%\text{regn} = \frac{|V_1| - |V_2'|_{rated}}{|V_2'|_{rated}} \times 100$$

$$\%\text{regn} = \frac{I_{2,rated}' R_{eq} \cos\phi \pm I_{2,rated}' X_{eq} \sin\phi}{|V_2'|_{rated}} \times 100$$

Efficiency

- Since the transformer is a static device, there are no rotational losses such as friction and windage losses.
- It's efficiency is high.
- In a well designed transformer, the efficiency can be as high as 99 %.

The efficiency is defined as follows:

$$\eta = \frac{\text{output power}(P_{out})}{\text{input power}(P_{in})}$$

$$\eta = \frac{P_{out}}{P_{out} + P_{losses}}$$

The losses in the transformer are core losses (P_c) and copper losses (P_{cu}).

$$\eta = \frac{P_{out}}{P_{out} + P_c + P_{cu}}$$

The P_{cu} can be determined as follows:

$$\begin{aligned} P_{cu} &= I_1^2 R_1 + I_2^2 R_2 \\ &= I_1^2 R_{eq} \\ &= I_2^2 R_{eq} \end{aligned}$$

- P_{cu} depends on the load current. Therefore, it is a variable loss.
- P_c depends on the flux density which in turns on depends on voltage. Since a transformer remains connected to an essentially constant voltage, the core loss is almost constant.

The efficiency can be determined from the equivalent circuit parameters.

$$P_{out} = V_2 I_2 \cos\phi$$

where $\cos\phi$ is the load power factor.

$$\eta = \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + P_c + I_2^2 R_{eq}}$$

For constant values of V_2 and ϕ, the maximum efficiency occurs at

$$\frac{d\eta}{dI_2} = 0$$

On differentiation and simplification, we get

$$I_2^2 R_{eq} = P_c$$

$$\text{copper loss} = \text{core loss}$$

The P_{cu} at full load is

$$P_{cu,FL} = I_{2,FL}^2 R_{eq}$$

Let $x = \dfrac{I_2}{I_{2,FL}}$. The copper loss at any load is

$$P_{cu} = x^2 I_{2,FL}^2 R_{eq} = x^2 P_{cu,FL}$$

From the maximum efficiency condition,

$$x^2 P_{cu,FL} = P_c$$

Therefore, the load corresponding to the max η is

$$x = \sqrt{\frac{P_c}{P_{cu,FL}}}$$

For constant values of V_2 and I_2, the maximum efficiency occurs at

$$\frac{d\eta}{d\phi} = 0$$

On differentiation and simplification, we get

$$\phi = 0$$

$$\cos\phi = 1$$

Therefore, the max η occurs for constant load at unity power factor.

All day efficiency η_{AD}

- Power transformers which are used in power plants operate near their full capacity. Therefore, they are designed to produce η_{max} near the rated output.
- Distribution transformers which are used to supply power to our houses and the locality operate well below the rated out for most of time.
- Therefore, it is desirable to design a distribution transformer for η_{max} at the average output power.
- η_{AD} is used to represent the efficiency performance of a distribution transformer.

The η_{AD} is defined as follows:

$$\eta_{AD} = \frac{\text{energy output over 24 hours}}{\text{energy input over 24 hours}}$$

$$\eta_{AD} = \frac{\text{energy output over 24 hours}}{\text{energy output over 24 hours} + \text{losses over 24 hours}}$$

Example 1 (Fitzgerald: 2.6) : A 50-kVA, single phase, 2400:240-V transformer gives the following measurements.
OC Test : 240 V, 5.41 A, 186 W
SC Test : 48 V, 20.8 A, 617 W
Determine the efficiency and the voltage regulation at full load, 0.80 power factor lagging.
From OC Test,

$$R_c = \frac{V_{oc}^2}{P_{oc}} = \frac{240^2}{186} = 309.677\ \Omega$$

$$Z_{oc} = \frac{V_{oc}}{I_{oc}} = \frac{240}{5.41} = 44.36\ \Omega$$

$$X_m = \frac{1}{\sqrt{(\frac{1}{Z_{oc}})^2 - (\frac{1}{R_c})^2}} = 44.8246\ \Omega$$

From SC Test,

$$R_{eq} = \frac{P_{sc}}{I_{sc}^2} = \frac{617}{20.8^2} = 1.42\ \Omega$$

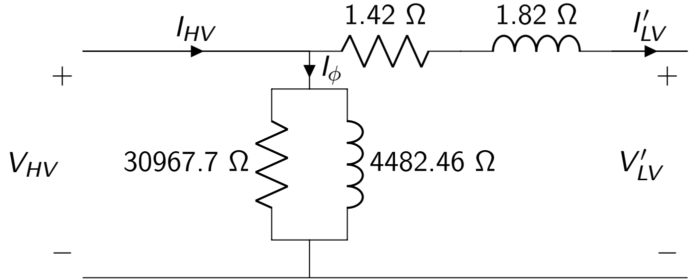
$$X_{eq} = \sqrt{Z_{eq}^2 - R_{eq}^2} = \sqrt{(48/20.8)^2 - 1.42^2} = 1.82\ \Omega$$

Since R_{eq} and X_{eq} are on the HV side, let us refer R_c and X_m to the HV side.

$$R_c' = R_c \times \left(\frac{N_{HV}}{N_{LV}}\right)^2 = 309.677 \times \left(\frac{2400}{240}\right)^2 = 30967.7\ \Omega$$

$$X_m' = 44.8246 \times \left(\frac{2400}{240}\right)^2 = 4482.46\ \Omega$$

The approximate equivalent circuit referred to the HV side is



1. The efficiency at full load 0.8 power factor lagging:

$$\eta = \frac{P_{out}}{P_{out} + P_c + P_{cu}}$$

$$\eta = \frac{50 \times 10^3 \times 0.8}{50 \times 10^3 \times 0.8 + 186 + 617} \times = 98.0\%$$

2. Regulation

$$V_{HV} = V'_{LV} + I'_{LV,rated}(R_{eq} + \jmath X_{eq})$$

$$I_{LV,rated} = \frac{S}{V_{LV}} = \frac{50 \times 10^3}{240} = 208.33\ A$$

$$I'_{LV,rated} = I_{LV,rated} \times \frac{N_{LV}}{N_{HV}} = 20.8\ A$$

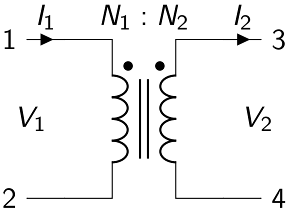
$$V_{HV} = 2400\underline{/0^\circ} + 20.8\underline{/-36.87^\circ} \times (1.42 + \jmath 1.82)$$

$$V_{HV} = 2446\underline{/0.3^\circ}\ V$$

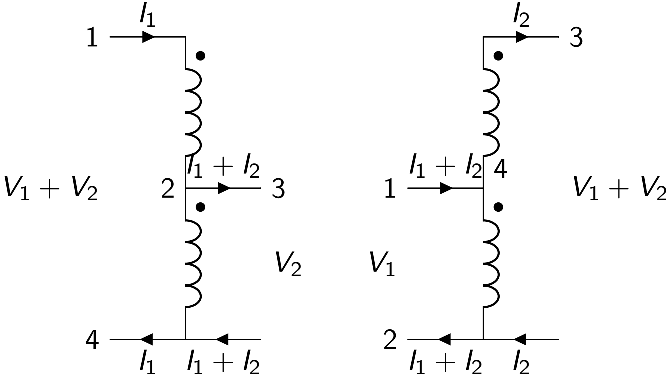
$$\%\ \text{regn} = \frac{2446 - 2400}{2400} \times 100 = 1.92\ \%$$

Auto Transformers

- Unlike two winding transformers, auto transformers have one winding which is common to both primary and secondary.
- Auto transformers have lower leakage reactances, lower losses and smaller exciting current.
- Auto transformers cost less than two winding transformers when the voltage ratio does not vary widely from 1.
- However, electrical isolation is lost in auto transformers since the two windings are electrically connected.



It can be connected as an auto transformer as shown below.



(a) $(V_1 + V_2)/V_2$ (b) $V_1/(V_1 + V_2)$

Let S_{TW} be the rating of the two winding transformer.

$$S_{TW} = V_1 I_1 = V_2 I_2$$

To find the rating of the auto transformer,

(a)

$$S_{auto} = (V_1 + V_2) I_1 = V_1 I_1 (1 + V_2/V_1)$$

$$S_{auto} = S_{TW}(1 + N_2/N_1)$$

(b)

$$S_{auto} = V_1 (I_1 + I_2) = V_1 I_1 (1 + I_2/I_1)$$

$$S_{auto} = S_{TW}(1 + N_1/N_2)$$

Example 2 (Fitzgerald 2.7) : The 50 kVA, 2400/240 V transformer given in Example 1 is connected as an auto transformer of 2400/2640 V.

1. Compute the kVA rating of an auto transformer.
2. Compute the full-load efficiency as an auto transformer operating with a rated load of 0.80 power factor lagging.

1. kVA rating

$$S_{auto} = 50 \times (1 + 10) = 550 \text{ kVA}$$

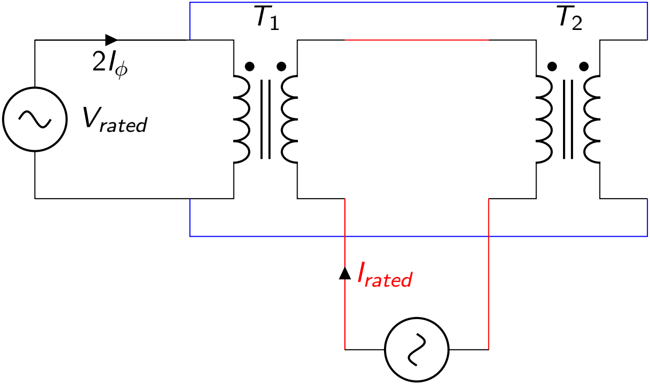
2. The full load efficiency at 0.8 power factor lagging

$$\eta = \frac{550 \times 10^3 \times 0.8}{550 \times 10^3 \times 0.8 + P_c + P_{cu}}$$

The losses are the same as the two winding transformer.

$$\eta = \frac{550 \times 10^3 \times 0.8}{550 \times 10^3 \times 0.8 + 186 + 617} \times 100 = 99.82\%$$

Sumpner's Test



- This test is to apply both rated voltage and rated current without adding load.
- This test is also called as heat run test.
- Two identical transformers are needed.

per unit

The per unit is defined as

$$\text{per unit} = \frac{\text{actual value in any unit}}{\text{base value in the same unit}}$$

There are normally four quantities.

$$S, V, I, Z$$

How to find base quantities?

- Choose any two. Normally S_{base} and V_{base} are chosen.
- Find the remaining two using their relations.

For a single phase system,

$$S_b = S_{1\phi} \text{ MVA}; \quad V_b = V_{1\phi} \text{ kV}$$

$$I_b = \frac{S_b(\text{MVA})}{V_b(\text{kV})} \text{ kA}$$

$$Z_b = \frac{V_b(\text{kV})}{I_b(\text{kA})} \, \Omega$$

Substituting I_b in Z_b,

$$Z_b = \frac{V_b^2(\text{ in kV})}{S_b(\text{ in MVA})} \, \Omega$$

$$Z_{\text{p.u.}} = \frac{Z_{\text{actual}}(\Omega)}{Z_b(\Omega)}$$

$$\boxed{\therefore Z_{\text{p.u.}} = \frac{Z_{\text{actual}}(\Omega) \times S_b(\text{ } 1\phi \text{ MVA})}{V_b^2(\text{ L- N in kV})}}$$

Advantages:

- The per unit values of impedance, voltage and current of a transformer are the same regardless of whether they are referred to the HV side or LV side. This is possible by choosing base voltages on either side of the transformer using the voltage ratio of the transformer.
- The parameters and variables fall in a narrow numerical range. This simplifies computations.

Example 3: A 50-kVA, 2400:240-V transformer has an equivalent impedance of $1.42 + \jmath 1.82\Omega$ referred to the high-voltage side. Find the per unit impedance on both sides.

1. per unit impedance on the HV side

$$S_b = 50 \text{ kVA}, V_{b,HV} = 2400 \text{ V}$$

$$Z_{eq,HV} = (1.42 + \jmath 1.82) \times \frac{0.05}{2.4^2} = 0.0123 + \jmath 0.0158 \text{ p.u.}$$

2. per unit impedance on the LV side

$$S_b = 50 \text{ kVA}$$

$$V_{b,LV} = V_{b,HV} \times \frac{240}{2400} = 2400 \times \frac{240}{2400} = 240 \text{ V}$$

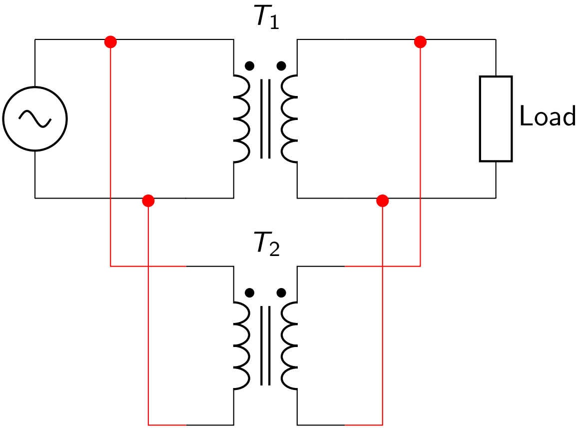
Let us first refer Z_{eq} to the LV side.

$$Z'_{eq} = (1.42 + \jmath 1.82) \times \left(\frac{240}{2400}\right)^2 = 0.0142 + \jmath 0.0182\ \Omega$$

$$Z_{eq,LV} = 0.0142 + \jmath 0.0182 \times \frac{0.05}{0.24^2} = 0.0123 + \jmath 0.0158 \text{ p.u.}$$

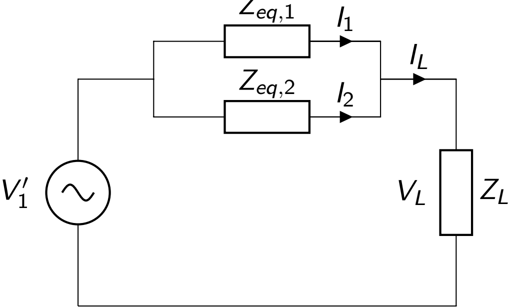
Parallel Operation

- To supply additional load economically.
- To improve reliability



The following conditions have to be met before connecting transformers in parallel.

- The voltage ratio must be the same.
- The per unit impedance of each machine on its own base must be the same.
- The polarity must be the same, so that there is no circulating current between the transformers.
- The phase sequence must be the same and no phase difference must exist between the voltages of the two transformers.



To find the power shared by each transformer,

$$I_1 = I_L \frac{Z_{eq,2}}{Z_{eq,1} + Z_{eq,2}}$$

$$I_2 = I_L \frac{Z_{eq,1}}{Z_{eq,1} + Z_{eq,2}}$$

If the load voltage is V_L, the complex power delivered by each transformer is

$$S_1 = V_L I_1^* \quad S_2 = V_L I_2^*$$

In order to share load in proportion to their ratings,

$$|S_1| = |V_L|I_1| = |V_L||I_L|\frac{|Z_{eq,2}|}{|Z_{eq,1} + Z_{eq,2}|}$$

$$|S_2| = |V_L|I_2| = |V_L||I_L|\frac{|Z_{eq,1}|}{|Z_{eq,1} + Z_{eq,2}|}$$

By dividing,

$$\frac{S_1}{S_2} = \frac{Z_{eq,2}}{Z_{eq,1}}$$

Let us multiply and divide by $S_{1,b}$ and $S_{2,b}$

$$\frac{S_1}{S_2} = \frac{Z_{eq,2}}{Z_{eq,1}} \times \frac{S_{1,b} \times S_{2,b}}{S_{1,b} \times S_{2,b}}$$

Since $S_b = \dfrac{V_b^2}{Z_b}$,

$$\frac{S_1}{S_2} = \frac{Z_{eq,2}}{Z_{eq,1}} \times \frac{S_{1,b} \times V_{2,b}^2/Z_{2,b}}{S_{2,b} \times V_{1,b}^2/Z_{1,b}}$$

Since $V_{1,b} = V_{2,b}$,

$$\frac{S_1}{S_2} = \frac{Z_{eq,2}(\text{p.u.})}{Z_{eq,1}(\text{p.u.})} \times \frac{S_{1,b}}{S_{2,b}}$$

If the per unit impedance of each machine on its own base is the same,

$$\frac{S_1}{S_2} = \frac{S_{1,b}}{S_{2,b}}$$

Transformers will share the load in proportion to their ratings.

Three Phase Transformers

- Since a three phase ac is generated and transmitted, three phase transformers are required to step up or step down voltages.
- A three phase transformer can be built
    1. by suitably connecting a bank of three single phase transformers.
    2. by constructing a three phase transformer on a common magnetic structure.
- The primary and secondary windings may be connected in either Y or Δ.

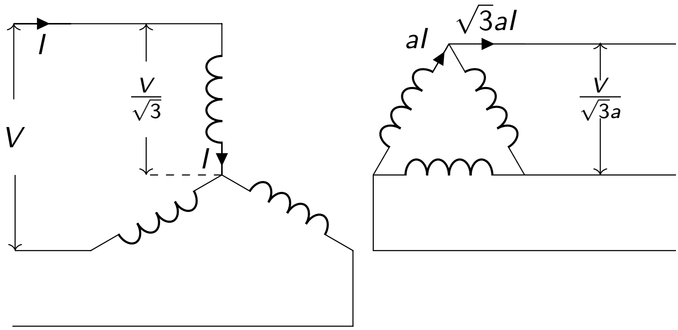
There are four possible connections.

1. $Y - \Delta$: This connection is commonly used to step down a high voltage to a lower voltage. The neutral point on the HV side is grounded.
2. $\Delta - Y$: This connection is commonly used to step up voltage.
3. $\Delta - \Delta$: This connection has advantage that one transformer can be removed for repair and the remaining two can continue to deliver power at a reduced rating.
4. $Y - Y$: This connection is rarely used because of problems with the exciting current and induced voltages.

- The total kVA of the three phase transformer is shared equally by each phase for all connections.
- The voltage and current ratings depend on the connections.

$Y - \Delta$

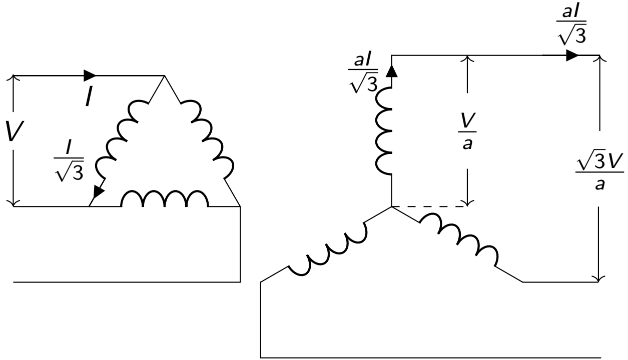
Let $\frac{N_1}{N_2} = a$.



$$S_Y = \sqrt{3}VI$$

$$S_\Delta = \sqrt{3}\frac{V}{\sqrt{3}a} \times \sqrt{3}aI = \sqrt{3}VI$$

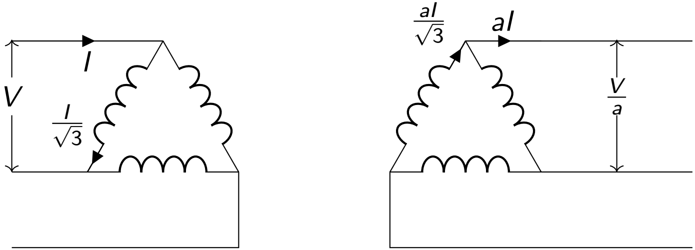
$\Delta - Y$



$$S_\Delta = \sqrt{3}VI$$

$$S_Y = \sqrt{3}\frac{\sqrt{3}V}{a} \times \frac{aI}{\sqrt{3}} = \sqrt{3}VI$$

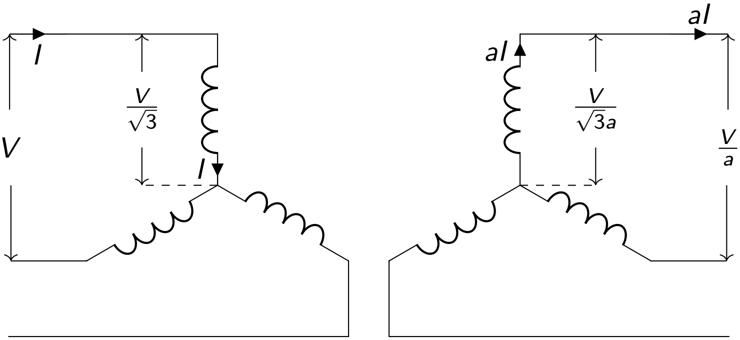
$\Delta - \Delta$



$$S_\Delta = \sqrt{3}VI$$

$$S_\Delta = \sqrt{3}\frac{V}{a} \times aI = \sqrt{3}VI$$

$Y - Y$



$$S_Y = \sqrt{3}VI$$

$$S_Y = \sqrt{3}\frac{V}{a} \times aI = \sqrt{3}VI$$

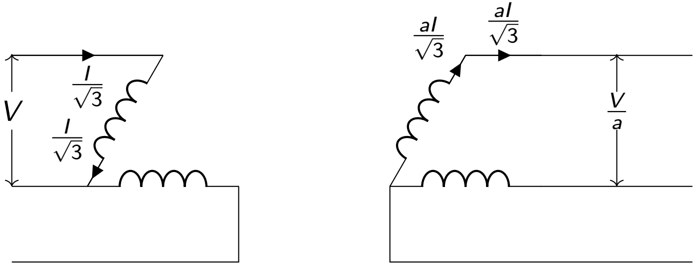
Example 4: A three phase transformer bank is made of 3 single phase transformers of 50 kVA 2400/240 V. What would be the power and voltage rating of the three phase transformer if it is connected as follows:

1. $\Delta - \Delta$
2. $Y - \Delta$

1. $\Delta - \Delta$
   The power rating is
   $$S_{3\phi} = 3 \times S_{1\phi} = 150 \text{ kVA}$$
   In Δ, $V_L = V_{ph}$. The voltage rating is 2400/240 V.
2. $Y - \Delta$
   The power rating is
   $$S_{3\phi} = 3 \times S_{1\phi} = 150 \text{ kVA}$$
   In Y, $V_L = \sqrt{3}V_{ph}$. The voltage rating is $2400 \times \sqrt{3}/240$ V.

Open Delta (V)

Let $\frac{I}{\sqrt{3}}$ be the rated current each transformer can carry.



$$S_V = \sqrt{3}V\frac{I}{\sqrt{3}} = VI$$

$$S_V = \sqrt{3}\frac{V}{a} \times \frac{aI}{\sqrt{3}} = VI$$

$$\frac{S_V}{S_\Delta} = \frac{VI}{\sqrt{3}VI} = \frac{1}{\sqrt{3}} = 58\%$$