Magnetic Circuits

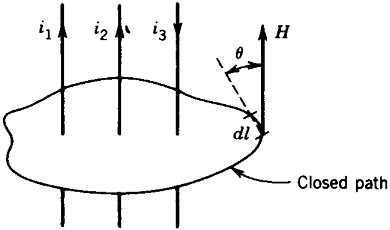
- Magnetic fields act as a medium in the energy conversion and transfer process.
- In most electrical machines, except permanent magnet machines, the magnetic field is produced by passing an electrical current through coils wound on ferromagnetic materials.
- Energy transfer takes place in transformers with the help of magnetic fields.
- Energy conversion takes place in machines with the help of magnetic fields.
- Magnetic circuits produce high flux density which results in large torque or large machine output.

Therefore, It is imperative that magnetic circuits be analyzed.

Ampère's circuital law:

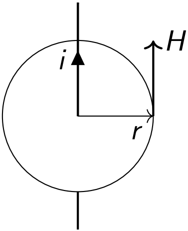
The line integral of the tangential component of the magnetic filed intensity H around a closed contour is equal to the total current linked by the contour.

$$\oint \mathrm{H} \cdot \mathrm{dl} = i_{net}$$



$$\oint H dl \cos\theta = \sum i = i_1 - i_2 + i_3$$

Let us consider a conductor carrying current i.



$$\oint \mathsf{H} \cdot \mathsf{dl} = i$$

Because of symmetry,H and dl are in the same direction. $\theta = 0^\circ$.

$$H\ 2\pi r = i$$

$$H = \frac{i}{2\pi r} \quad \mathrm{AT/m}$$

H produces the magnetic flux density B.

$$B = \mu H \quad \text{Wb}/m^2 \text{ or Tesla}$$

$$B = \mu_0 \mu_r H \quad T$$

where

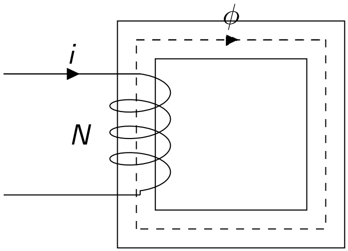
$\mu =$ the permeability of the medium.

$\mu_0 =$ the permeability of free space and is $4\pi \times 10^{-7}$ Henry/meter

$\mu_r =$ the relative permeability of the medium.

- For free space or electrical conductors or insulators, $\mu_r = 1$.
- For ferromagnetic materials, μ_r varies from several hundred to several thousand.
- A large value of μ_r means that a small current can produce a large flux density in the machine.

Magnetic Circuit



- Flux is mostly confined in the core material
- The flux outside the core called *leakage flux* is so small and it can be neglected.

From Ampère's circuital law.

$$\oint \mathrm{H} \cdot \mathrm{dl} = Ni$$

$$Hl = Ni$$

where
l is the mean length of the core in meter.
Ni is the magneto motive force (F) in ampere-turn.

$$Hl = Ni = F$$

$$H = \frac{Ni}{l} \text{ At/m}$$

We know that

$$B = \mu_0 \mu_r H$$

$$B = \mu_0 \mu_r \frac{Ni}{l} \quad \text{T}$$

If all the fluxes are confined in the core (no leakage flux),

$$\phi = \int B dA$$

$$\phi = BA \quad \text{Wb}$$

where
B is the average flux density in the core in Tesla.
A is the area of cross section of the core in m^2.
Substituting B,

$$\phi = \mu_0 \mu_r \frac{Ni}{l} A = \frac{Ni}{l/\mu_0 \mu_r A}$$

$$\phi = \frac{F}{\mathcal{R}}$$

where
F is magneto motive force (mmf) in At.
$\mathcal{R} = \frac{l}{\mu_0 \mu_r A}$ is the reluctance of the magnetic path in At/Wb.

$$\phi = F\mathcal{P}$$

where $\mathcal{P} = \dfrac{1}{\mathcal{R}}$ is the permeance of the magnetic path.

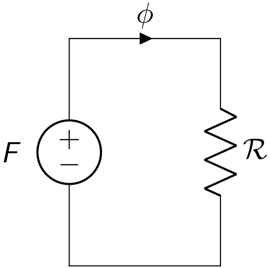
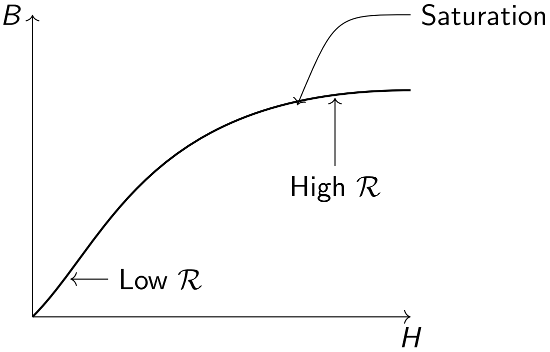


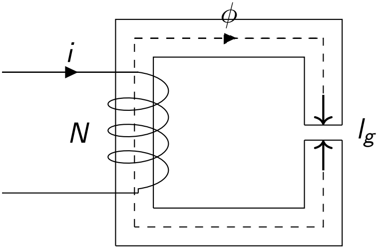
Figure: Magnetic circuit

$B - H$ curve



- The reluctance ($\mathcal{R}$) of the magnetic path is dependent of flux density (B).
- It is low when B is low.It is high when B is high.
- This is how magnetic circuits differ from electric circuits (R is independent of current).

Magnetic Circuit with Air Gap



Flux lines pass through two media.

$$\mathcal{R}_c = \frac{l_c}{\mu_0 \mu_r A_c}$$

$$\mathcal{R}_g = \frac{l_g}{\mu_0 A_g}$$

$$\phi = \frac{Ni}{\mathcal{R}_c + \mathcal{R}_g}$$

$$Ni = H_c l_c + H_g l_g$$

where
l_c is the mean length of the core
l_g is the mean length of the air gap
The flux densities are

$$B_c = \frac{\phi_c}{A_c}$$

$$B_g = \frac{\phi_g}{A_g}$$

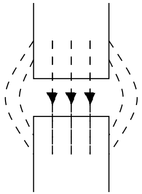


Figure: Fringing Effect

- In the air gap, magnetic flux lines bulge outward somewhat. This is called the *Fringing* of flux.
- The fringing effect increases the cross sectional area of the air gap.
- If the air gap length is small, this effect can be neglected.

Assume no fringing. $A_g = A_c$

$$B_g = B_c = \frac{\phi}{A_c}$$

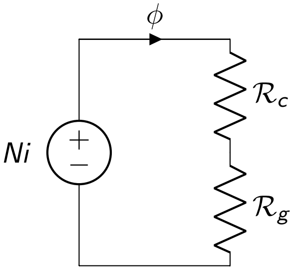


Figure: Magnetic circuit

Inductance

It is defined as the flux linkage of the coil per current.

$$\lambda = N\phi$$

where λ is the flux linkage.

$$L = \frac{\lambda}{i}$$

$$L = \frac{N\phi}{i} = \frac{NBA}{i} = \frac{N\mu HA}{i}$$

We know that

$$Ni = Hl$$

Therefore, inductance is

$$L = \frac{N^2\mu A}{l} = \frac{N^2}{l/\mu A}$$

$$L = \frac{N^2}{\mathcal{R}} \text{ H}$$

Example: A magnetic circuit has the following dimensions:

$$l_c = 30 \text{ cm};\ A_c = 9 \text{ cm}^2;\ N = 500 \text{ turns}$$

Assume $\mu_r = 20000$. Find

1. the current required to produce $B = 1$ T in the circuit and the inductance.
2. the current required to produce $B = 1$ T in the circuit and the inductance if an air gap of 1 mm is introduced.

1. Without Air gap:

$$\phi = \frac{Ni}{\mathcal{R}_c}$$

Since $B = 1$ T,

$$\phi = BA_c = 1 \times 9 \times 10^{-4} = 0.9 \text{ mWb}$$

$$\mathcal{R}_c = \frac{l_c}{\mu_0 \mu_r A_c} = \frac{30 \times 10^{-2}}{4\pi \times 10^{-7} \times 20000 \times 9 \times 10^{-4}} = 1.3263 \times 10^4 \text{ At/V}$$

Therefore,

$$i = \frac{\phi \mathcal{R}_c}{N} = \frac{9 \times 10^{-4} \times 1.3263 \times 10^4}{500} = 0.024 \text{ A}$$

Inductance

$$L = \frac{N^2}{\mathcal{R}} = \frac{500^2}{1.3263 \times 10^4} = 18.85 \text{ H}$$

2. With Air gap of 1mm: (No fringing)

$$\phi = \frac{Ni}{\mathcal{R}_c + \mathcal{R}_g}$$

$$\mathcal{R}_c = \frac{l_c}{\mu_0 \mu_r A_c} = \frac{29.9 \times 10^{-2}}{4\pi \times 10^{-7} \times 20000 \times 9 \times 10^{-4}} = 1.3219 \times 10^4 \text{ At/W}$$

$$\mathcal{R}_g = \frac{l_g}{\mu_0 A_g} = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 9 \times 10^{-4}} = 8.842 \times 10^5 \text{ At/Wb}$$

Notice $\mathcal{R}_g > \mathcal{R}_c$.

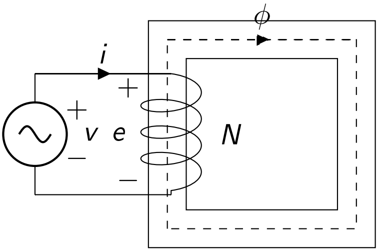
Therefore,

$$i = \frac{\phi(\mathcal{R}_c + \mathcal{R}_g)}{N} = 1.6154 \text{ A}$$

Because of air gap, the magnetic circuit needs more current to produce the same flux density.

Inductance

$$L = \frac{N^2}{\mathcal{R}_c + \mathcal{R}_g} = 0.2786 \text{ H}$$



If i is time varying, ϕ will also vary with time. By Faraday's law,

$$e = \frac{d\lambda}{dt}$$

The direction of the induced voltage e is defined to take care of Lenz's law.
Since there is no coil resistance R, $v = e$.

$$e = \frac{d\lambda}{dt}$$

$$e = \frac{d(Li)}{dt}$$

In a magnetic linear circuit ($L \propto i$), L is a function of geometry alone. $L = \frac{N^2}{\mathcal{R}} = N^2\mathcal{P}$.

1. If the geometry of a magnetic circuit is fixed, L will not vary with time.

$$e = L\frac{di}{dt}$$

2. If the geometry of a magnetic circuit changes with time, L will vary with time.

$$e = L\frac{di}{dt} + i\frac{dL}{dt}$$

To find the energy stored in the coil,

$$p = ei = i\frac{d\lambda}{dt}$$

$$W = \int_0^{t_1} p dt = \int_0^{\lambda_1} i d\lambda$$

In a linear magnetic system,

$$\lambda = Li$$

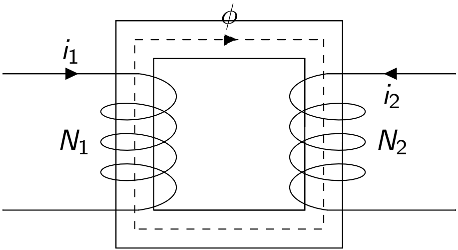
$$W = \int_0^{\lambda_1} \frac{\lambda}{L} d\lambda$$

$$W = \frac{1}{2L}\lambda_1^2 = \frac{1}{2}Li_1^2$$

In general,

$$W = \frac{1}{2L}\lambda^2 = \frac{1}{2}Li^2 \text{ J}$$

Mutual Inductance



The directions of currents are such that they produce flux in the same direction. Therefore, the net mmf

$$F = N_1 i_1 + N_2 i_2$$

The resultant flux

$$\phi = \frac{F}{\mathcal{R}} = \frac{N_1 i_1 + N_2 i_2}{\mathcal{R}}$$

The flux linkages of coil 1

$$\lambda_1 = N_1\phi = \frac{N_1^2 i_1}{\mathcal{R}} + \frac{N_1 N_2 i_2}{\mathcal{R}}$$

This can be written

$$\lambda_1 = L_1 i_1 + M_{12} i_2$$

where L_1 is the self inductance of coil 1.

$$L_1 = \frac{N_1^2}{\mathcal{R}}$$

M_{12} is the mutual inductance between coils 1 and 2.

$$M_{12} = \frac{N_1 N_2}{\mathcal{R}}$$

Similarly, the flux linkages of coil 2 is

$$\lambda_2 = N_2\phi = \frac{N_1 N_2 i_1}{\mathcal{R}} + \frac{N_2^2 i_2}{\mathcal{R}}$$

This can be written

$$\lambda_2 = M_{21} i_1 + L_2 i_2$$

where L_2 is the self inductance of coil 2.

$$L_2 = \frac{N_2^2}{\mathcal{R}}$$

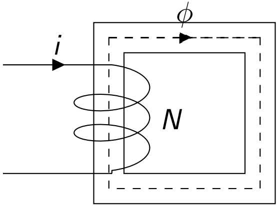
M_{21} is the mutual inductance between coils 1 and 2.

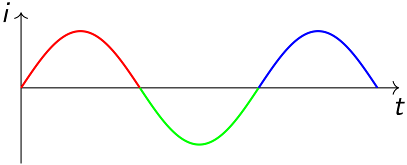
$$M_{21} = \frac{N_1 N_2}{\mathcal{R}}$$

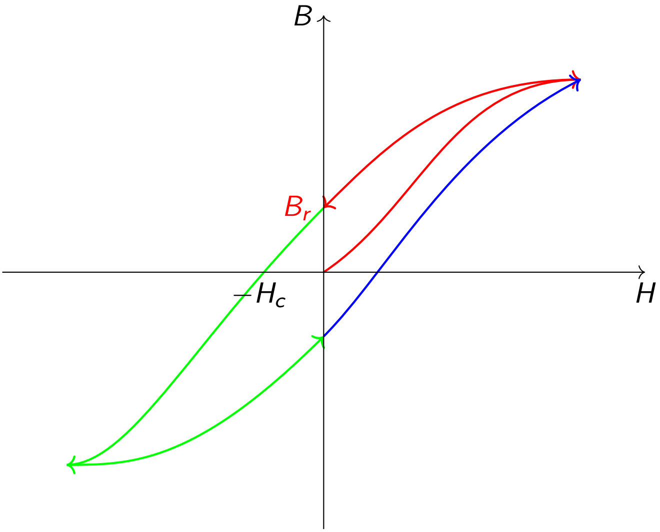
In a linear magnetic circuit,

$$M_{12} = M_{21}$$

Hysteresis







B_r is the residual flux density and $-H_c$ is the coercive force.

Hysteresis Loss

When i varies,

- some energy flows from source to the coil during some interval.
- some energy returns to the source during some other interval.
- However, there is net energy flow from the source to the coil.
- The energy loss called as *hysteresis loss* goes to heat the core.

Assume the coil has no resistance.

$$e = N\frac{d\phi}{dt}$$

The energy transfer from t_1 to t_2 is,

$$W = \int_{t_1}^{t_2} pdt = \int_{t_1}^{t_2} eidt$$

$$W = \int N\frac{d\phi}{dt}idt = \int_{\phi_1}^{\phi_2} Nid\phi$$

Since $\phi = BA$ and $i = \dfrac{Hl}{N}$,

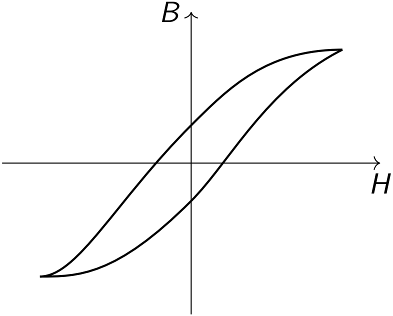
$$W = \int_{B_1}^{B_2} N \cdot \frac{Hl}{N} A dB$$

$$W = (lA) \int_{B_1}^{B_2} H dB$$

$$W = V_{core} \int_{B_1}^{B_2} H dB$$

where $V_{core} = lA$ represents the volume of the core. The energy transfer over one cycle is

$$W_{cycle} = V_{core} \oint H dB$$



$$W_{cycle} = V_{core} \times \text{ Area of the B- H loop}$$

The power loss in the core due to hysteresis is

$$P_h = W_{cycle} \times f$$

$$P_h = V_{core} \times f \times \oint HdB$$

- Since the $B - H$ curve is nonlinear and multi valued, no simple mathematical expression can describe the loop.
- Hence, it is difficult to evaluate the area of the loop.
- However, it has been approximately found by conducting a large number of experiments.

$$\text{Area of the B-H loop} = KB_{max}^{n}$$

$$P_h = K_h B_{max}^{n} f \quad \text{Watts}$$

where n varies in the range 1.5 to 2.5.
If B_{max} is constant,

$$\boxed{P_h \propto f}$$

Eddy Current Loss

- When the flux density varies, there will be induced emf in the core.
- This will circulate a current in the core. This current is called eddy current.
- Hence a power loss i^2R as heat will occur in the core.

The eddy current loss in a magnetic core is given as follows:

$$P_e = K_e B_{max}^2 f^2$$

The eddy current loss can be minimized in the following ways.

1. A high resistivity core material may be used.
2. A laminated core may be used. The thin laminations are insulated from each other. The lamination thickness varies from 0.5 to 5 mm in electrical machines.

If B_{max} is constant,

$$\boxed{P_e \propto f^2}$$

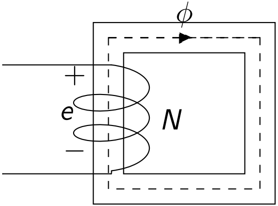
Core Loss

The core loss is the combination of the hysteresis loss and the eddy current loss.

$$P_c = P_h + P_e$$

- Since electrical machines are operated at constant voltage and frequency, the core loss is constant.
- Using a wattmeter, the core loss can be easily measured.
- However, it is difficult to know how much is due to hysteresis and how much is due to eddy currents.

Sinusoidal Excitation



Assume that the flux $\phi(t)$ varies sinusoidally with time.

$$\phi(t) = \phi_{max} \sin \omega t$$

where
ϕ_{max} is the maximum value of the core flux
$\omega = 2\pi f$ is the angular frequency
f is the frequency

From Faraday's law,

$$e(t) = N\frac{d\phi}{dt}$$

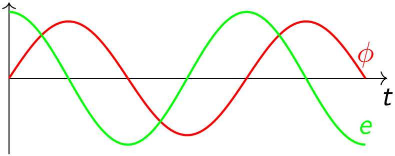
$$e(t) = N\phi_{max}\omega\cos\omega t$$

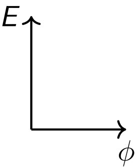
$$e(t) = E_{max}\cos\omega t$$

The rms value of the induced voltage is

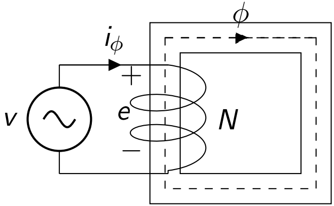
$$E_{rms} = \frac{E_{max}}{\sqrt{2}} = \frac{N\phi_{max}\omega}{\sqrt{2}}$$

$$\boxed{E_{rms} = 4.44f\phi_{max}}$$



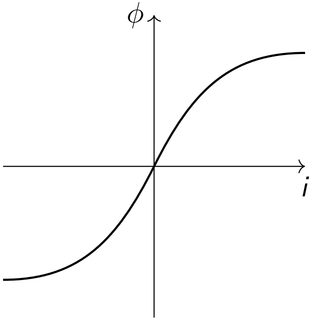


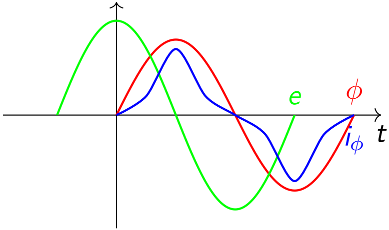
Exciting Current



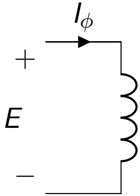
- If the coil is connected a sinusoidal voltage source, a current flows in the coil to establish a sinusoidal flux in the core.
- This current is called the exciting current.
- If the $B - H$ characteristics of the core is nonlinear, the exciting current will be non-sinusoidal.

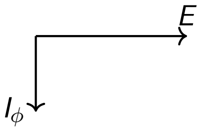
1. No Hysteresis



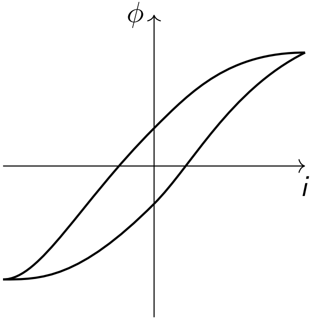


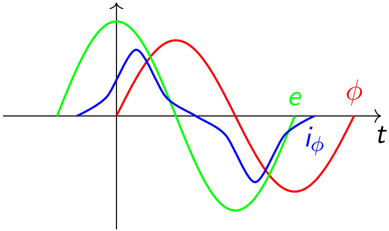
- i_ϕ is non-sinusoidal.
- i_ϕ and ϕ are in phase.
- i_ϕ lags e by 90°.





2. Hysteresis





- i_ϕ is non-sinusoidal.
- i_ϕ and ϕ are not in phase.
- i_ϕ lags e by less than 90°.