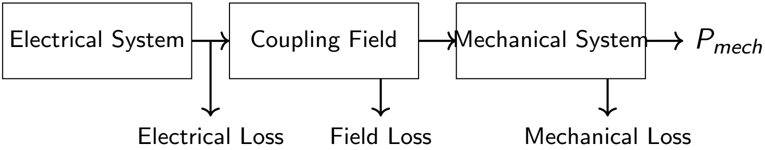
Electro Mechanical Energy Conversion

- Energy conversion takes place in machines with the help of magnetic fields.
- Machines such as motors and generators are used for continuous energy conversion.
- Some devices such relays and electromagnets do whenever necessary.
- However, they all operate on similar principles.

Energy Conversion Process

To calculate the force or torque developed in machines, the principle of conservation of energy is used.
For an electromechanical converter,

$$\text{Electrical energy input} = \text{Energy losses} + \text{Increase in stored energy in the field} \\ + \text{Mechanical energy output}$$



The energy balance equation can be written as

Electrical energy input from source	=	mechanical energy output	increase in stored field energy
−		+	+
resistance loss		friction and windage loss	core loss

In a lossless system,

Electrical energy input from source	=	Mechanical energy output	+	Increase in stored field energy

$$dW_e = dW_m + dW_f$$

where dW_e, dW_f and dW_m are incremental electrical energy input, stored field energy and mechanical energy output, respectively.

$$\frac{dW_f}{dt} = ei - P_{mech}$$

P_{mech} can be written as follows:

$$P_{mech} = f\frac{dx}{dt} \quad \text{for linear movement systems}$$

$$P_{mech} = T\frac{d\theta}{dt} \quad \text{for rotational systems}$$

$$\frac{dW_f}{dt} = ei - f\frac{dx}{dt}$$

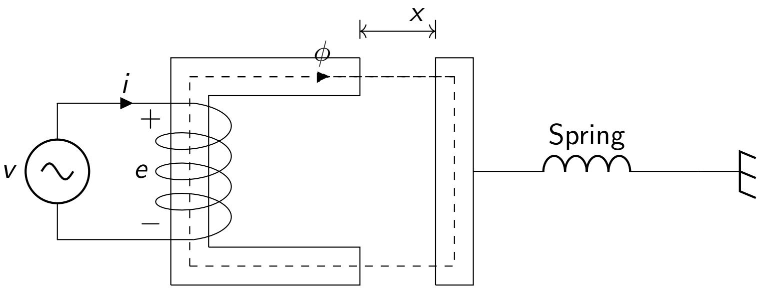
By Faraday's law, $e = \dfrac{d\lambda}{dt}$.

$$\frac{dW_f}{dt} = i\frac{d\lambda}{dt} - f\frac{dx}{dt}$$

In the differential form,

$$\boxed{dW_f = id\lambda - fdx}$$

Singly Excited System



It is assumed that the system is lossless.
By Faraday's law,

$$v = e = \frac{d\lambda}{dt}$$

The incremental energy balance equation is

$$dW_e = dW_f + dW_m$$

Let us assume that the movable part is held stationary.

$$dW_m = 0$$

Therefore,

$$dW_e = dW_f$$

Since it is a lossless system, all the incremental energy input is stored as incremental field energy.

$$e = \frac{d\lambda}{dt}$$

$$dW_e = eidt = id\lambda$$

Hence

$$dW_f = id\lambda$$

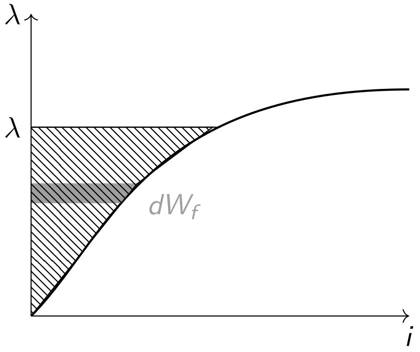


Figure: $\lambda - i$ characteristics for a particular x.

When the flux linkage is increased from 0 to λ, the filed energy is

$$W_f = \int_o^\lambda i d\lambda$$

Let

$$H_c = \text{magnetic intensity in the core}$$
$$H_g = \text{magnetic intensity in the air gap}$$
$$l_c = \text{length of the magnetic core material}$$
$$l_g = \text{length of the air gap}$$

We know

$$Ni = H_c l_c + H_g l_g$$

also

$$\lambda = N\phi = NBA$$

where A is the cross sectional area of the flux path and B is the flux density ($B_g = B_c = B$).

We can write

$$W_f = \int \frac{H_c l_c + H_g l_g}{N} NAdB$$

For the air-gap,

$$H_g = \frac{B}{\mu_0}$$

$$W_f = \int \left(H_c l_c + \frac{B l_g}{\mu_0} \right) A dB$$

$$\boxed{W_f = V_c \int H_c dB + \frac{B^2}{2\mu_0} V_g}$$

where V_c is the volume of core and V_g is the volume of air gap.

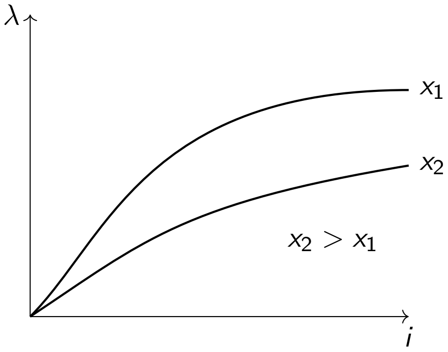


Figure: $\lambda - i$ characteristics for different x.

The field energy not only depends on λ but x.

$$W_f = W_f(\lambda, x)$$

$$dW_f = \left.\frac{\partial W_f}{\partial \lambda}\right|_x + \left.\frac{\partial W_f}{\partial x}\right|_\lambda$$

We know that

$$dW_f = id\lambda - fdx$$

By comparing them,

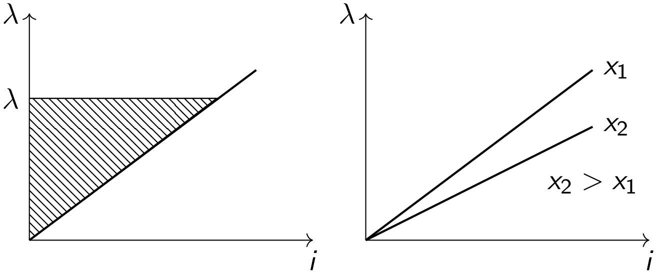
$$f = -\left.\frac{\partial W_f}{\partial x}\right|_{\lambda}$$

Similarly for rotational systems,

$$T = -\left.\frac{\partial W_f}{\partial \theta}\right|_{\lambda}$$

It shows how the force or Torque is obtained from the field energy.

Let us consider a linear magnetic system.



(a) for a particular x (b) for different x

Figure: $\lambda - i$ characteristics.

In a linear magnetic system, L is a function of geometry alone.

$$W_f = \int_0^{\lambda} i d\lambda$$

Since $\lambda = L(x)i$,

$$\boxed{W_f = \frac{1}{2}L(x)i^2 = \frac{\lambda^2}{2L(x)}}$$

The energy stored can also be found as follows:
For a linear magnetic system,

$$H_c = \frac{B}{\mu_c}$$

$$\boxed{W_f = \frac{B^2}{2\mu_c}V_c + \frac{B^2}{2\mu_0}V_g}$$

The force developed is

$$f = -\left.\frac{\partial W_f}{\partial x}\right|_{\lambda} = \frac{1}{2}\frac{\lambda^2}{L(x)^2}\frac{dL(x)}{dx}$$

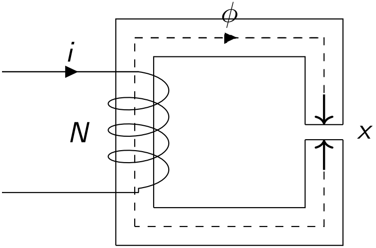
$$\boxed{f = \frac{1}{2}i^2\frac{dL(x)}{dx}\ \text{N}}$$

Similarly for rotational liner magnetic systems,

$$\boxed{T = \frac{1}{2}i^2\frac{dL(\theta)}{d\theta}\ \text{Nm}}$$

Example 1: The magnetic circuit has the following parameters:
$N = 500$, $i = 2$ A, $x = 1$ mm, $A_g = 4$ cm^2.
Neglect the reluctance of core, the leakage flux and fringing.

1. Determine the force of attraction between both sides of the air gap.
2. Determine the energy stored in the air gap.



1.

$$L = \frac{N^2}{\mathcal{R}}$$

Since the reluctance of the core is neglected,

$$L = \frac{N^2}{\mathcal{R}_g} = \frac{N^2 \mu_0 A_g}{x}$$

The force developed is

$$f = \frac{1}{2} i^2 \frac{dL(x)}{dx} = -\frac{i^2 N^2 \mu_0 A_g}{2x^2}$$

On substitution,

$$f = -251.3274 \text{ N}$$

The negative sign indicates that the force acts in such a direction to reduce the air gap length (the force of attraction).

2. The energy stored in the air gap,

$$W = \frac{B^2}{2\mu_0} V_g$$

Since $B = \dfrac{\mu_0 N i}{l_g}$,

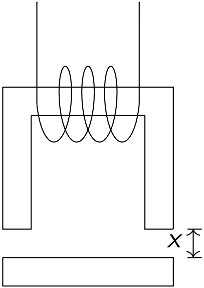
$$W = \mu_0 \frac{N^2 i^2}{2 l_g} A_g = \frac{1}{2} L i^2$$

and $l_g = x$. On substitution,

$$W = 0.2513 \text{ Joules}$$

Example 2 : The lifting magnetic system has $A = 36$ cm^2 and $x = 5$ mm. The coil has 300 turns and a resistance of 5 Ω. Neglect the reluctance of the magnetic core and fringing.

1. Find the lifting force when a dc source of 100 V is connected to the coil.
2. Find the lifting force when an ac source of 100 V, 50 Hz is connected to the coil.



1. When a dc source is connected,

$$I = \frac{100}{5} = 20 \text{ A}$$

Since the reluctance of the core is neglected,

$$L = \frac{N^2}{\mathcal{R}_g} = \frac{N^2 \mu_0 A_g}{2x}$$

The force developed is

$$f = -\frac{I^2 N^2 \mu_0 A_g}{4x^2}$$

On substitution, $f \approx -1629$ N .
The negative sign indicates that the force acts in such a direction to reduce the air gap length (the lifting force).
$|f| \approx 1629$ N.

2. When a 100 V ac source is connected,

$$I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

$$L = \frac{N^2 \mu_0 A_g}{2x} = 40.71 \text{ mH}$$

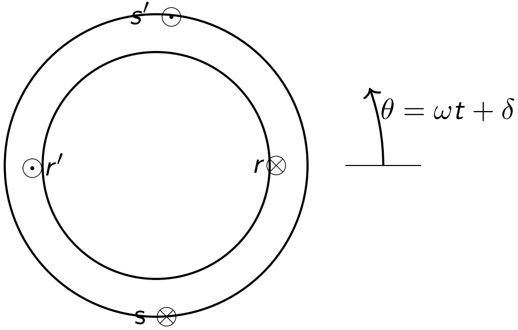
$$I = \frac{100}{\sqrt{5^2 + (2 \times \pi \times 50 \times 0.04071)^2}} = 7.28 \text{ A}$$

The average force developed is

$$f = -\frac{I^2 N^2 \mu_0 A_g}{4x^2}$$

On substitution, $f \approx -216$ N. The lifting force $|f| \approx 216$ N. The force obtained by the dc source is almost 8 times higher than the force obtained by the ac source.

Multiple Excitation System



Assume the system is lossless and the magnetic system is linear. By Faraday's law,

$$e_s = \frac{d\lambda_s}{dt} \quad e_r = \frac{d\lambda_r}{dt}$$

The flux linkages of a linear magnetic system are as follows:

$$\lambda_s = L_s i_s + M_{sr} i_r$$

$$\lambda_r = M_{rs} i_s + L_r i_r$$

where
L_s is the self inductance of the stator winding
L_r is the self inductance of the rotor winding
M_{sr} and M_{rs} are the mutual inductances between the stator and rotor windings.
For a linear magnetic system, $M_{sr} = M_{rs} = M$.
In a matrix form,

$$\begin{bmatrix} \lambda_s \\ \lambda_r \end{bmatrix} = \begin{bmatrix} L_s & M \\ M & L_r \end{bmatrix} \begin{bmatrix} i_s \\ i_r \end{bmatrix}$$

The incremental energy balance equation is

$$dW_e = dW_f + dW_m$$

Let us assume that the rotor is stationary.

$$dW_m = 0$$

Therefore,

$$dW_e = dW_f$$

Since,

$$dW_e = e_s i_s dt + e_r i_r dt$$

Therefore

$$dW_f = i_s d(L_s i_s + M i_r) + i_r d(M i_s + L_r i_r)$$

$$dW_f = L_s i_s di_s + L_r i_r di_r + Md(i_s, i_r)$$

The field energy is

$$W_f = \int_0^{i_s} L_s i_s di_s + \int_0^{i_r} L_r i_r di_r + \int_0^{(i_s, i_r)} Md(i_s, i_r)$$

$$W_f = \frac{1}{2} L_s i_s^2 + \frac{1}{2} L_r i_r^2 + M i_s i_r$$

The torque developed is

$$T = - \left. \frac{\partial W_f}{\partial \theta} \right|_{\lambda}$$

In a linear magnetic system, L and M are functions of θ.

$$\boxed{T = \frac{1}{2} i_s^2 \frac{dL_s}{d\theta} + \frac{1}{2} i_r^2 \frac{dL_r}{d\theta} + i_s i_r \frac{dM}{d\theta}}$$

- In salient pole machines, L_s and L_r vary with respect to rotor positions. This is because of the rotor structure.
- Whereas in cylindrical machines, L_s and L_r are constant irrespective of rotor positions.

$$T = i_s i_r \frac{dM}{d\theta}$$

Example 3: Two windings, one mounted on the stator and the other on a rotor, have a self and mutual inductances of

$$L_s = 4.5 \text{ H} \quad L_r = 2.5 \text{ H} \quad M = 2.8 \cos\theta \text{ H}$$

where θ is the angle between the axes of the windings. The resistances of the winding may be neglected. Rotor winding is short-circuited, and the current in stator winding i_s is $10 \sin \omega t$ A.

1. Find the expression of instantaneous torque on the rotor.
2. Compute the time averaged torque when $\theta = 45^\circ$.
3. If the rotor is allowed to move, will it rotate continuously? If not, when will it stop?

1. Since L_s and L_r are constant,

$$T = i_s i_r \frac{dM}{d\theta}$$

Since the rotor winding is short circuited,

$$v_r = \frac{d\lambda_r}{dt} = 0$$

$$\lambda_r = L_r i_r + M i_s = 0$$

$$i_r = -\frac{2.8 \cos\theta}{2.5} \times 10 \sin\omega t$$

The instantaneous torque is

$$T = 10 \sin\omega t \times \left(-\frac{2.8\cos\theta}{2.5} \times 10 \sin\omega t\right) \times (-2.8 \sin\theta)$$

$$T = 313.6 \sin^2 \omega t \sin\theta \cos\theta \text{ N-m}$$

2. When $\theta = 45^\circ$,

$$T_{ave} = \frac{313.6}{4} = 78.4 \text{ N-m}$$

3. - At $\theta = 0^\circ$ and $\theta = 90^\circ$, $T = 0$.
   - When θ is in between 0° and 90°, T is positive.
   - When θ is in between 90° and 180°, T is negative.
   - The net torque is zero. It will not rotate and will stop at 90°.