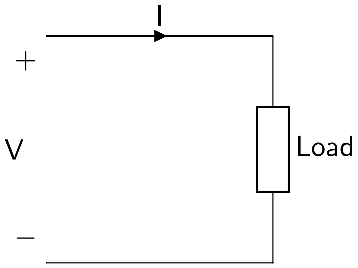
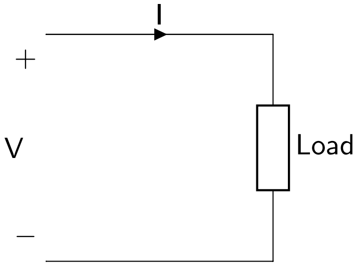
Power in DC Circuits



$$P = VI \text{ Watts}$$

Power in AC Circuits

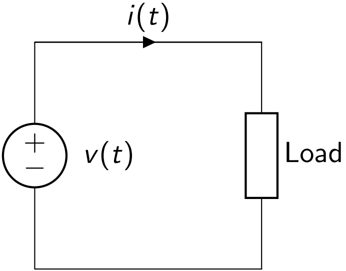


To find power,

- RMS values of V and I alone are not sufficient.
- The phase angle difference between them is also needed.

Power in AC Circuits

Let us consider the following circuit.



if $v = V_m \sin(\omega t)$, the steady state current in $R - L$ load is

$$i = I_m \sin(\omega t - \phi)$$

where $I_m = \dfrac{V_m}{\sqrt{R^2 + (\omega L)^2}}$ and $\phi = \tan^{-1} \dfrac{\omega L}{R}$.

Let $v(t) = \sqrt{2}V\sin(\omega t + \theta_V)$ and $i(t) = \sqrt{2}I\sin(\omega t + \theta_I)$ be the voltage across and current through a load.
The **instantaneous power** delivered to the load is

$$p(t) = v(t)i(t)$$

$$p(t) = V_m\sin(\omega t + \theta_V)I_m\sin(\omega t + \theta_I)$$

$$p(t) = \frac{V_m I_m}{2}(\cos(\theta_V - \theta_I) - \cos(2\omega t + \theta_V + \theta_I))$$

$$p(t) = \frac{V_m I_m}{2}\cos(\theta_V - \theta_I) - \frac{V_m I_m}{2}\cos(2\omega t + \theta_V + \theta_I)$$

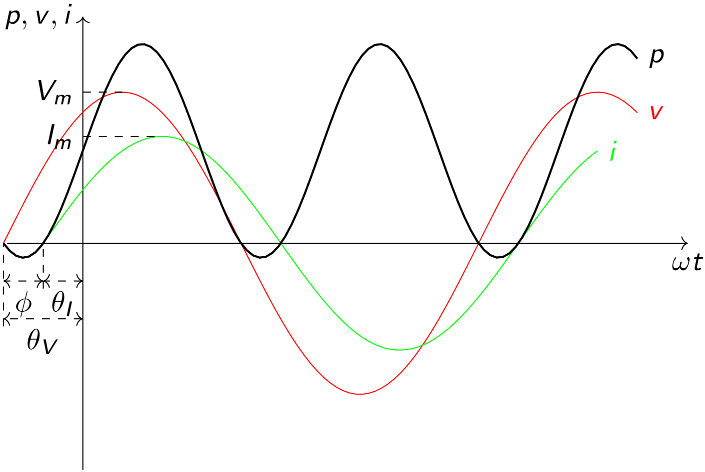


Figure: Voltage, current and power in RL circuit

Let $\theta_V - \theta_I$ be ϕ.

$$p(t) = VI\cos\phi - VI\cos(2\omega t + \theta_V + \theta_I)$$

$$p(t) = VI\cos\phi - VI\cos(2\omega t + \theta_V - \theta_I + 2\theta_I)$$

$$p(t) = VI\cos\phi - VI\cos(2\omega t + 2\theta_I - \phi)$$

$$p(t) = \underbrace{VI\cos\phi(1 - \cos(2\omega t + 2\theta_I))}_{p_I} - \underbrace{VI\sin\phi\sin(2\omega t + 2\theta_I)}_{p_{II}}$$

p_I has an average value of $VI\cos\phi$ which is called the average power.

p_{II} does not have an average. But it's maximum value is $VI\sin\phi$ which is called reactive power.

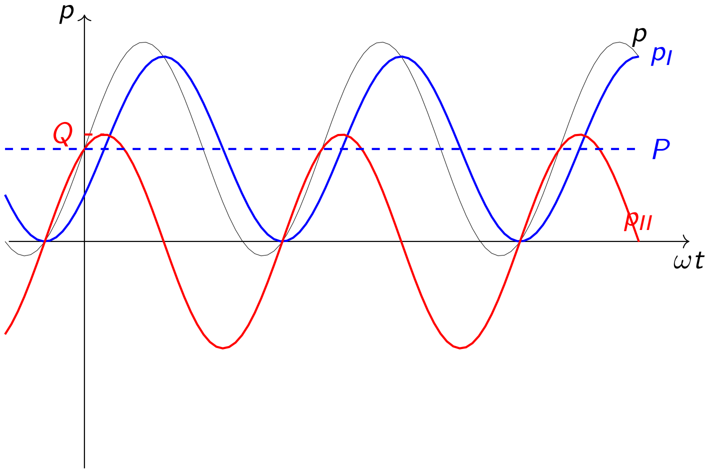


Figure: Power in RL circuit

Average Power / Real Power

The average power P is

$$\boxed{P = \frac{V_m I_m}{2}\cos(\theta_V - \theta_I) = VI\cos(\phi)}$$

where $\phi = \theta_V - \theta_I$. Its unit is watts (W).

1. Resistor: V and I are in phase.

$$P = VI\cos(0^\circ) = VI$$

2. Ideal Inductor : V leads I by 90°.

$$P = VI\cos(90^\circ) = 0$$

3. Ideal Capacitor : I leads V by 90°.

$$P = VI\cos(90^\circ) = 0$$

Reactive Power

This does not do any useful work. But it keeps oscillating between source and load with zero average.
The maximum value is

$$Q = VI \sin \phi$$

Its unit is VAr.

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$$Q = VI \sin(0^\circ) = 0$$

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3. Ideal Capacitor : I leads V by 90°.

$$Q = VI \sin(90^\circ) = VI$$

Apparent Power and Power Factor

If the voltage and the current were dc quantities, the power delivered to a load would be

$$|S| = VI$$

As dc values and RMS are equal,

$$|S| = V_{\text{RMS}} I_{\text{RMS}}$$

where $|S|$ is called as the **apparent power**. Its unit is **volt-ampere (VA)**.
The ratio of real power (P) to apparent power is called as the **power factor (pf)**.

$$\text{pf} = \frac{VI \cos\phi}{VI} = \cos\phi$$

Since $\cos\phi$ can never be greater than unity, $P \leq |S|$.

1. Resistor:

$$\phi = 0^\circ$$

$$\text{pf} = \cos(0^\circ) = 1$$

2. Ideal Inductor:

$$\phi = 90^\circ$$

$$\text{pf} = \cos(90^\circ) = 0$$

3. Ideal Capacitor:

$$\phi = -90^\circ$$

$$\text{pf} = \cos(-90^\circ) = 0$$

- RL Circuit: Let $\phi = 60^\circ$. pf $= \cos(60^\circ) = 0.5$.
- RC Circuit: Let $\phi = -60^\circ$. pf $= \cos(-60^\circ) = 0.5$

In order to differentiate these two cases, power factor in a RL circuit is mentioned **lagging** pf and in a RC circuit as **leading** pf.

Complex Power

Let us define voltage phasor and current phasor.

$$\mathsf{V} = V\angle\theta_V, \quad \mathsf{I} = I\angle\theta_I$$

The complex power S is

$$\mathsf{S} = \mathsf{V}\mathsf{I}^*$$

$$\mathsf{S} = V\angle\theta_V \; I\angle-\theta_I$$

$$= VI\angle(\theta_V - \theta_I)$$

$$\mathsf{S} = VI\cos\phi + \jmath VI\sin\phi$$

The real part of S is called the average power (P). The imaginary part of S is called the reactive power (Q).

$$\mathsf{S} = P + \jmath Q$$

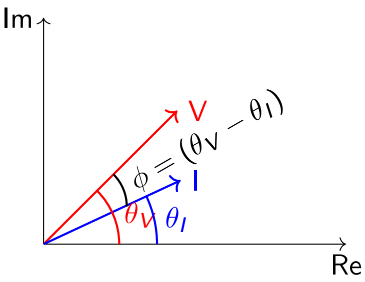


Figure: RL load

If V leads I ($\phi > 0$), power factor is lagging.

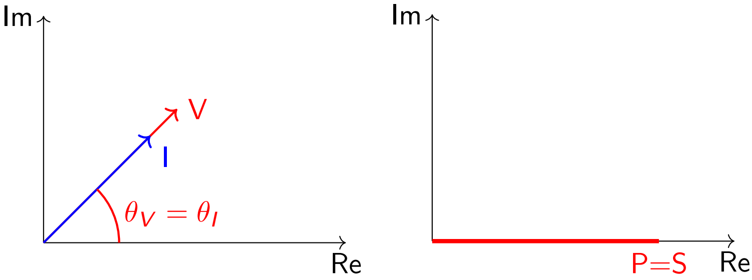


Figure: Resisitive Load

If V and I are in phase ($\phi = 0$), power factor is unity.

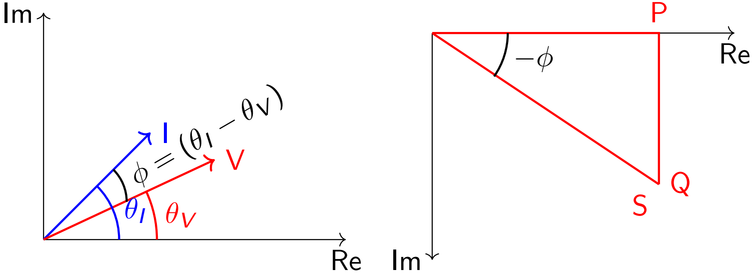
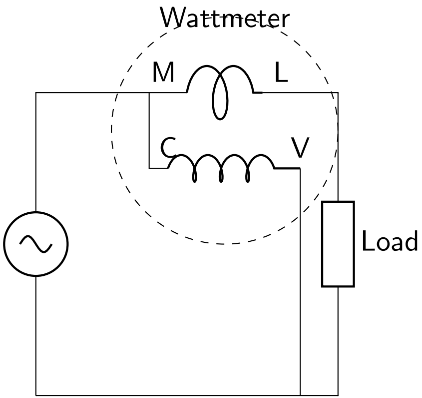


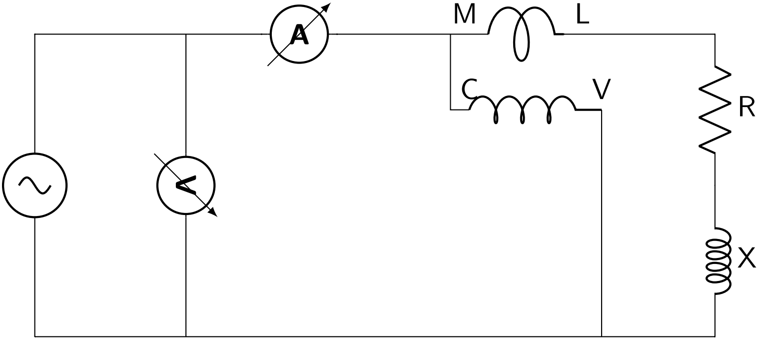
Figure: RC load

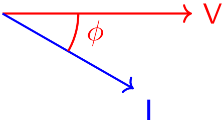
If I leads V ($\phi < 0$), power factor is leading.

Power Measurement - Single Phase



Measurement of R and X





$$Z = \frac{V}{I};\ R = \frac{P}{I^2}$$

$$X = \sqrt{Z^2 - R^2}$$

Power in Three Phase

Let v_a, v_b and v_c be the instantaneous voltages of a balanced three phase source.

$$\begin{aligned} v_a &= \sqrt{2}V\sin(\omega t + \theta_V) \\ v_b &= \sqrt{2}V\sin(\omega t + \theta_V - 120^\circ) \\ v_c &= \sqrt{2}V\sin(\omega t + \theta_V - 240^\circ) \end{aligned}$$

When it supplies a balanced load,

$$\begin{aligned} i_a &= \sqrt{2}I\sin(\omega t + \theta_I) \\ i_b &= \sqrt{2}I\sin(\omega t + \theta_I - 120^\circ) \\ i_c &= \sqrt{2}I\sin(\omega t + \theta_I - 240^\circ) \end{aligned}$$

The instantaneous power is

$$p = v_a i_a + v_b i_b + v_c i_c$$

$$\begin{aligned} p = & \sqrt{2} V_p \sin(\omega t + \theta_V) \times \sqrt{2} I_p \sin(\omega t + \theta_I) \\ & \sqrt{2} V_p \sin(\omega t + \theta_V - 120^\circ) \times \sqrt{2} I_p \sin(\omega t + \theta_I - 120^\circ) \\ & \sqrt{2} V_p \sin(\omega t + \theta_V - 240^\circ) \times \sqrt{2} I_p \sin(\omega t + \theta_I - 240^\circ) \end{aligned}$$

$$\begin{aligned} p = & V_p I_p \cos(\theta_V - \theta_I) + V_p I_p \cos(2\omega t + \theta_V + \theta_I) \\ & V_p I_p \cos(\theta_V - \theta_I) + V_p I_p \cos(2\omega t + \theta_V + \theta_I - 120^\circ) \\ & V_p I_p \cos(\theta_V - \theta_I) + V_p I_p \cos(2\omega t + \theta_V + \theta_I - 240^\circ) \end{aligned}$$

$$\boxed{p = 3 V_p I_p \cos\phi}$$

where $\phi = \theta_V - \theta_I$.

The instantaneous power in a 3 phase balanced system is constant.

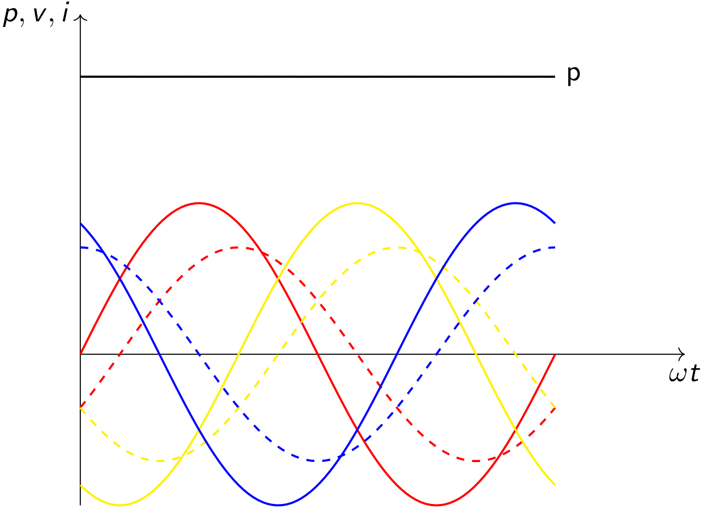


Figure: Voltage, current and power in a R-L load

The average/real power in a 3-phase system is

$$P = 3V_p I_p \cos\phi \quad \text{Watts}$$

In a Y connected load, $V_L = \sqrt{3}V_p$ and $I_L = I_p$,

$$P = \sqrt{3}V_L I_L \cos\phi$$

In a Δ connected load, $V_L = V_p$ and $I_L = \sqrt{3}I_p$,

$$P = \sqrt{3}V_L I_L \cos\phi$$

Therefore, the three phase real power is

$$P = 3V_p I_p \cos\phi = \sqrt{3}V_L I_L \cos\phi$$

Since the instantaneous power in a 3-phase balanced system is constant, it does not mean that there is no reactive power. Still the instantaneous power of individual phases is pulsating.

The 3-phase reactive power is

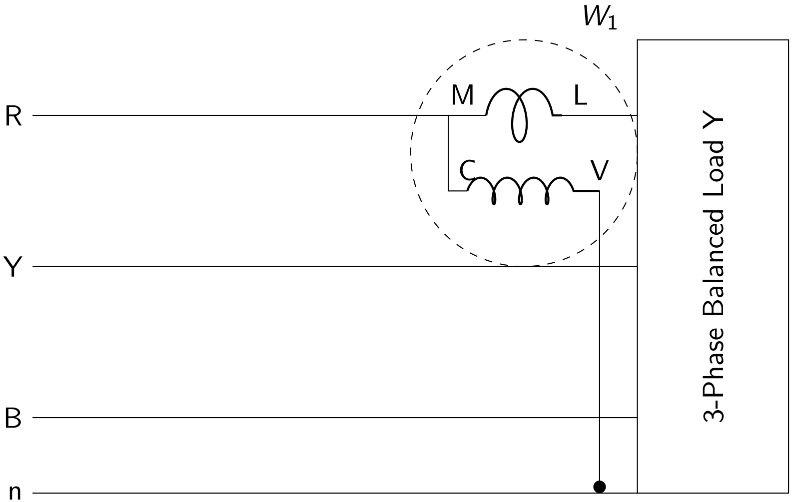
$$Q = 3V_p I_P \sin\phi = \sqrt{3} V_L I_L \sin\phi \quad \text{VAr}$$

The apparent power is

$$|S| = \sqrt{P^2 + Q^2} = 3V_p I_p = \sqrt{3} V_L I_L \quad \text{VA}$$

3-Phase Power Measurement

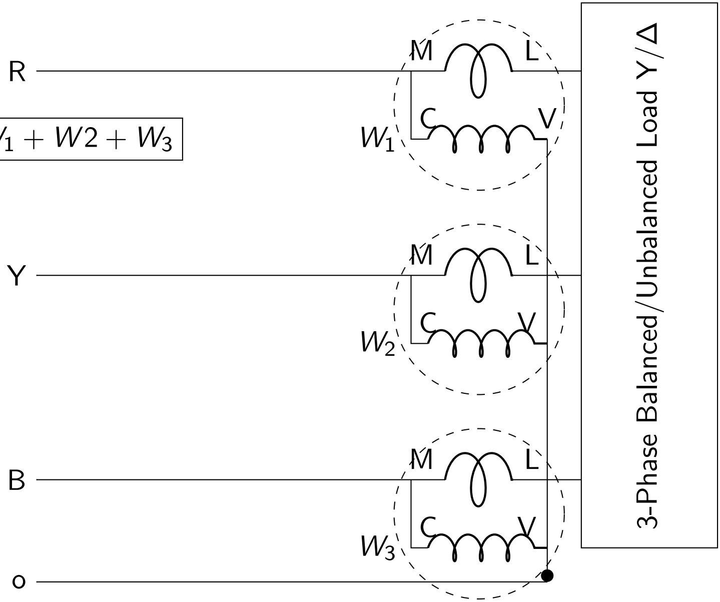
If the load is balanced,



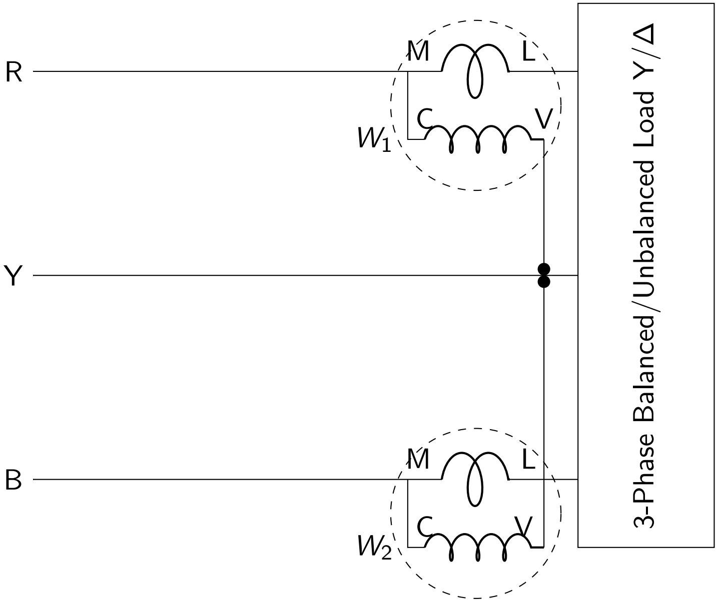
The total 3-phase power is $P = 3 \times W_1$

If the load is unbalanced,

$$P_T = W_1 + W2 + W_3$$



Two Wattmeter Method



The total average power absorbed the load is

$$P_T = W_1 + W_2$$

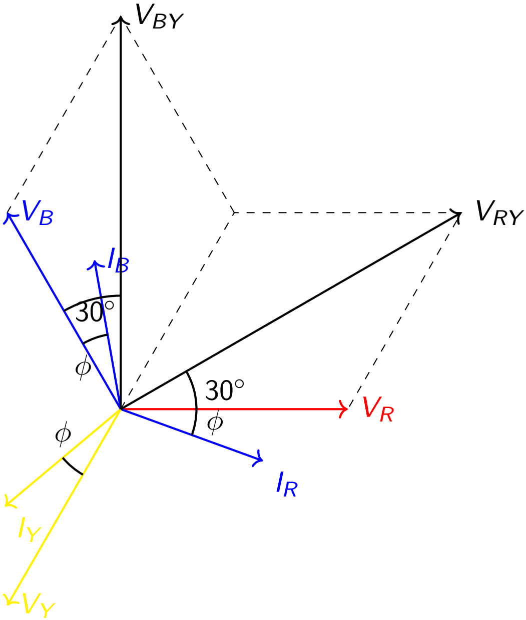
Proof:
Consider a balanced Y connected RL load.

$$V_p = V\angle 0^\circ;\ I_L = I_p = I\angle -\phi;\ V_L = \sqrt{3}V\angle 30^\circ$$

Therefore

$$W_1 = V_{RY} I_R \cos(\angle V_{RY} - \angle I_R)$$

$$W_2 = V_{BY} I_B \cos(\angle V_{BY} - \angle I_B)$$



V_{BY}
V_B
V_{RY}
I_B
30°
ϕ
30°
V_R
ϕ
ϕ
I_R
I_Y
V_Y

Since $V_{RY} = V_{BY} = V_L$,

$$W_1 = V_L I_L \cos(30^\circ + \phi)$$

$$W_2 = V_L I_L \cos(30^\circ - \phi)$$

$$W_1 + W_2 = V_L I_L \cos(30^\circ + \phi) + V_L I_L \cos(30^\circ - \phi)$$

$$W_1 + W_2 = \sqrt{3} V_L I_L \cos\phi$$

So, the total average power is

$$P = W_1 + W_2$$

Let us find $W_2 - W_1$.

$$W_2 - W_1 = V_L I_L \cos(30^\circ - \phi) - V_L I_L \cos(30^\circ + \phi)$$

$$W_2 - W_1 = V_L I_L \sin\phi$$

The total reactive power is

$$Q = \sqrt{3}(W_2 - W_1)$$

The power factor angle can be found as follows:

$$\phi = tan^{-1}\left(\frac{\sqrt{3}(W_2 - W_1)}{W_1 + W_2}\right)$$

1. If $W_1 = W_2$, the load is resistive.
2. If $W_2 > W_1$, the load is inductive.
3. If $W_2 < W_1$, the load is capacitive.