CS5201: Advanced Artificial Intelligence

Probabilistic Reasoning

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Need for probabilistic model

- **Till now, we have considered that intelligent agent has**
  - **Known environment**
  - **Full observability**
  - **Deterministic world**

- **Reasons for using probability**
  - **Example:** $Toothache \implies Cavity$
    - **Toothache can be caused by many other ways also, eg.** $Toothache \implies GumDisease \lor Cavity \lor WisdomTeeth \lor \ldots$
  - **Specifications become too large**
  - **Theoretical ignorance**
  - **Practical ignorance**

Probabilistic reasoning

- Useful for prediction
  - Analyzing causal effect, predicting outcome
    - Given that I have cavity, what is the chance that I will have toothache?
- Useful for diagnosis
  - Analyzing causal effect, finding out reasons for a given effect
    - Given that I have toothache, what is the chance that it is caused by a cavity?
- Require a methodology to analyze both the scenarios

Axioms of probability

- **All probabilities lie between 0 and 1, ie.,** $0 \leq P(A) \leq 1$
- $P(True) = 1$ **and** $P(False) = 0$
- $P(A \vee B) = P(A) + P(B) - P(A \wedge B)$

- $P(A \wedge B) = P(A|B) \times P(B) = P(B|A) \times P(A)$
  - $P(A) = \dfrac{P(A|B) \times P(B)}{P(B|A)}$
  - **Bayes' rule**

Joint probability

Color \ Type	EV	SUV	Sedan	Truck
Red	0.05	0.20	0.00	0.10
Green	0.10	0.00	0.10	0.00
Black	0.00	0.10	0.05	0.10
White	0.10	0.00	0.10	0.00

- **What is the probability of a vehicle to be EV and Red?**

Joint probability

Color \ Type	EV	SUV	Sedan	Truck
Red	0.05	0.20	0.00	0.10
Green	0.10	0.00	0.10	0.00
Black	0.00	0.10	0.05	0.10
White	0.10	0.00	0.10	0.00

- **What is the probability of a vehicle to be EV and Red?**
- **What is the probability of a Red vehicle?**

Joint probability

Color \ Type	EV	SUV	Sedan	Truck
Red	0.05	0.20	0.00	0.10
Green	0.10	0.00	0.10	0.00
Black	0.00	0.10	0.05	0.10
White	0.10	0.00	0.10	0.00

- **What is the probability of a vehicle to be EV and Red?**
- **What is the probability of a Red vehicle?**
  - **Marginal probability:** $\sum_{T} P(C = Red \wedge T = *)$

Joint probability

Type / Color	EV	SUV	Sedan	Truck
Red	0.05	0.20	0.00	0.10
Green	0.10	0.00	0.10	0.00
Black	0.00	0.10	0.05	0.10
White	0.10	0.00	0.10	0.00

- **What is the probability of a vehicle to be EV and Red?**
- **What is the probability of a Red vehicle?**
  - **Marginal probability:** $\sum_{T} P(C = Red \wedge T = *)$
- **What is the probability of a vehicle to be EV given that it is Red?**

Joint probability

Color \ Type	EV	SUV	Sedan	Truck
Red	0.05	0.20	0.00	0.10
Green	0.10	0.00	0.10	0.00
Black	0.00	0.10	0.05	0.10
White	0.10	0.00	0.10	0.00

- **What is the probability of a vehicle to be EV and Red?**
- **What is the probability of a Red vehicle?**
  - **Marginal probability:** $\sum_{T} P(C = Red \wedge T = *)$
- **What is the probability of a vehicle to be EV given that it is Red?**
  - **Conditional probability:** $\dfrac{P(C = Red \wedge T = EV)}{P(C = Red)}$

Independence

- **Two variables are independent if $P(X, Y) = P(X) \times P(Y)$ holds**
  - **It means that their joint distribution factors into a product two distributions**
  - **This can be expressed as $P(X|Y) = P(X)$**
- **Independence is a simplifying modeling assumption**
  - **Empirical joint distributions can be at best close to independent**

Example: Independence

T	W	P(T,W)
Hot	Sun	0.4
Hot	Rain	0.1
Cold	Sun	0.2
Cold	Rain	0.3

Example: Independence

T	W	P(T,W)
Hot	Sun	0.4
Hot	Rain	0.1
Cold	Sun	0.2
Cold	Rain	0.3

- **Are T and W independent?**

Example: Independence

T	W	P(T,W)
Hot	Sun	0.4
Hot	Rain	0.1
Cold	Sun	0.2
Cold	Rain	0.3

- **Are *T* and *W* independent?**
- **Find marginal probabilities**

Example: Independence

T	W	P(T,W)
Hot	Sun	0.4
Hot	Rain	0.1
Cold	Sun	0.2
Cold	Rain	0.3

- **Are T and W independent?**
- **Find marginal probabilities**
  - $P(T = Hot) = ?$, $P(T = Cold) = ?$
  - $P(W = Sun) = ?$, $P(W = Rain) = ?$

Example: Independence

T	W	P(T,W)
Hot	Sun	0.4
Hot	Rain	0.1
Cold	Sun	0.2
Cold	Rain	0.3

- **Are T and W independent?**
- **Find marginal probabilities**
  - $P(T = Hot) =?$, $P(T = Cold) =?$
  - $P(W = Sun) =?$, $P(W = Rain) =?$
- **Now check for independence**

Example: Independence

- **Tossing of N fair coins**

$P(X_1)$	
H	0.5
T	0.5

$P(X_2)$	
H	0.5
T	0.5

$\ldots\ldots$

$P(X_n)$	
H	0.5
T	0.5

$P(X_1, X_2, \ldots, X_n)$

X_1	X_2	X_3	$\ldots$	$\ldots$	X_n	P
H					H	
$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	$\ldots$	
T					H	

2^n

Chain rule

- **Product rule**

$$P(X, Y) = P(X) \times P(Y|X)$$

- **Chain rule**

$$\begin{aligned} P(X_1, X_2, \ldots, X_n) &= P(X_1) \times P(X_2|X_1) \times P(X_3|X_1, X_2) \times \ldots \times P(X_N|X_1, \ldots, X_{n-1}) \\ &= \prod_i P(X_i|X_1, \ldots, X_{i-1})) \end{aligned}$$

Conditional independence

- $P(Toothache, Cavity, Catch)$
- **If I have a cavity, the probability that the probe catches in it doesn't depend on whether I have a toothache:**

$$P(+catch| + toothache, +cavity) = P(+catch| + cavity)$$

- **The same independence holds if I don't have a cavity:**

$$P(+catch| + toothache, -cavity) = P(+catch| - cavity)$$

- **Catch is conditionally independent of Toothache given Cavity:**

$$P(Catch|Toothache, Cavity) = P(Catch|Cavity)$$

Conditional independence

- Absolute independence is very rare
- Conditional independence is the most basic and robust form of knowledge about un-certain environments
- Mathematically it is defined as - X is conditionally independent of Y given Z

$$P(X, Y|Z) = P(X|Z) \times P(Y|Z)$$

- It can also be shown that

$$P(X|Z, Y) = P(X|Z)$$

Belief network

- Qualitative information
  - A belief network is a graph consist of the following:
  - Nodes – Set of random variables
  - Edges – Dependency of nodes. $X \rightarrow Y$ means X has direct influence on Y
- Quantitative information
  - Each node has conditional probability table that quantifies the effects the parents have on the node

- The network is a directed acyclic graph (DAG), ie., without any cycle

Example

- **Consider the following knowledge base**
  - *Rain* **- Raining**
  - *Traffic* **- There may be traffic if it rains**
  - *Umbrella* **- People may carry umbrella if it rains**

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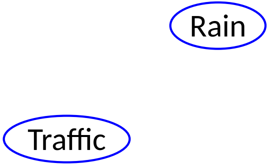

Example

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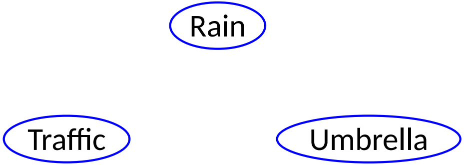

Example

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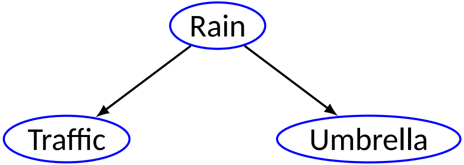
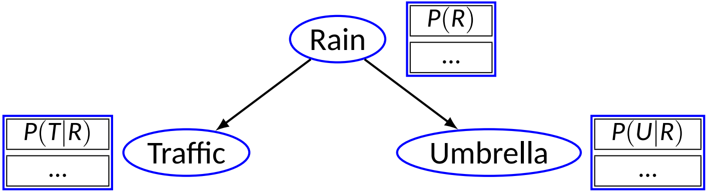
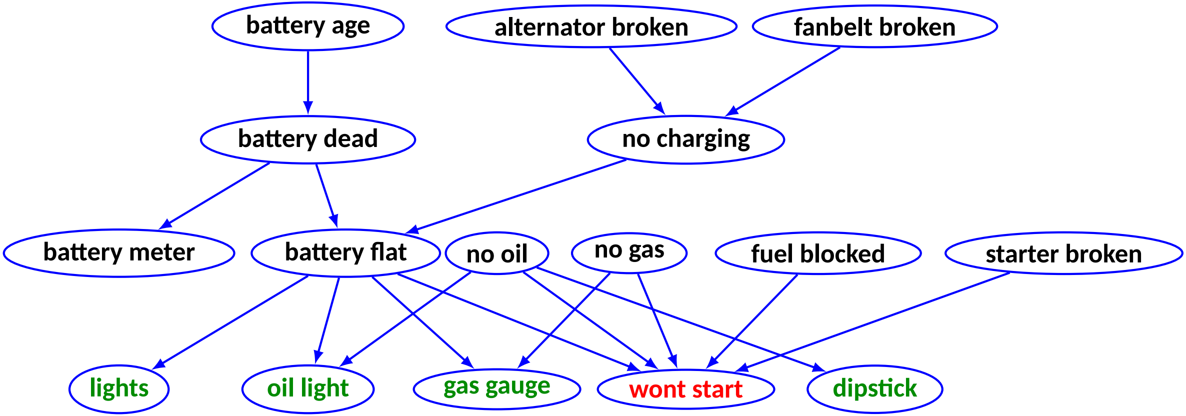


Example

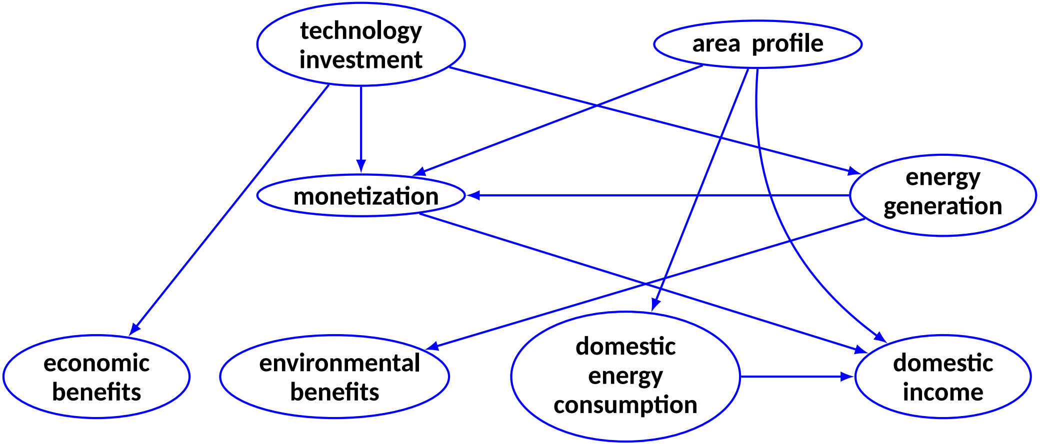
- **Consider the following knowledge base**
  - *Rain* - **Raining**
  - *Traffic* - **There may be traffic if it rains**
  - *Umbrella* - **People may carry umbrella if it rains**



Belief network example: car



Belief network example: renewable energy



Classical example

- Burglar alarm at a home
  - It works almost perfectly
  - Sometime alarm goes off if there is an earthquake
- John calls police when he hears the alarm but sometimes he misinterpret telephone ringing as alarm and calls police too.
- Mary also calls police. But she loves loud music, therefore, she misses the alarm sometime.

Belief network example

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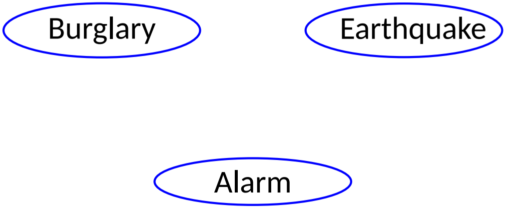
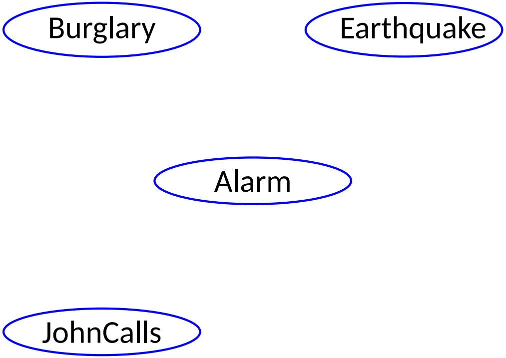
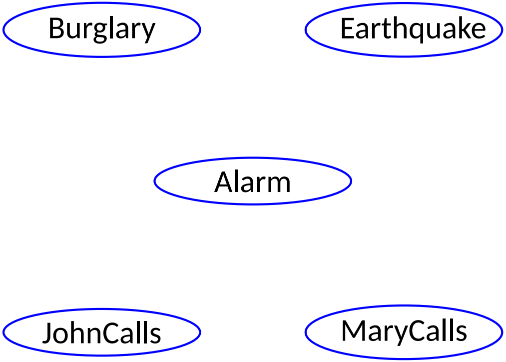


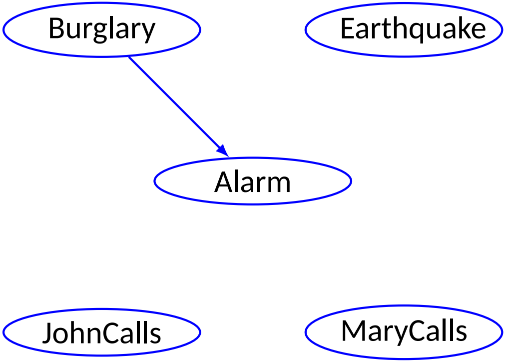
Belief network example



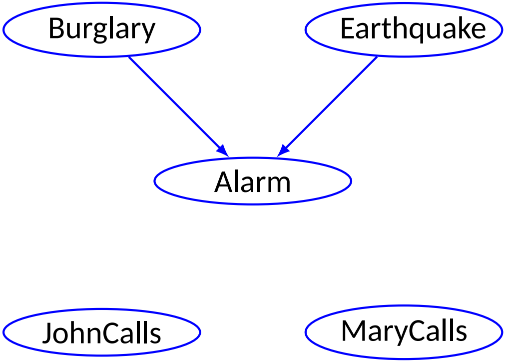
Belief network example



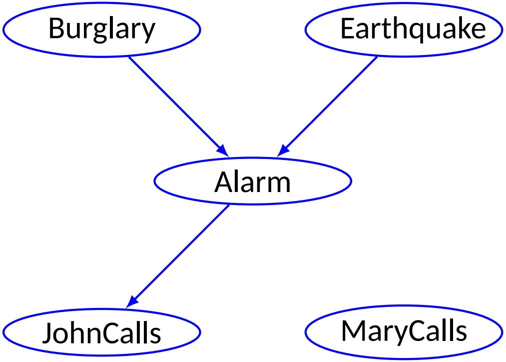
Belief network example



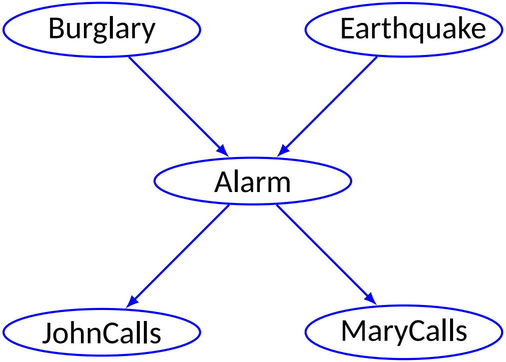
Belief network example



Belief network example



Belief network example



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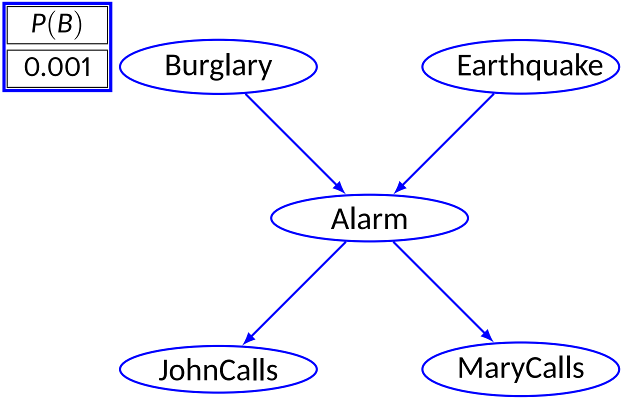

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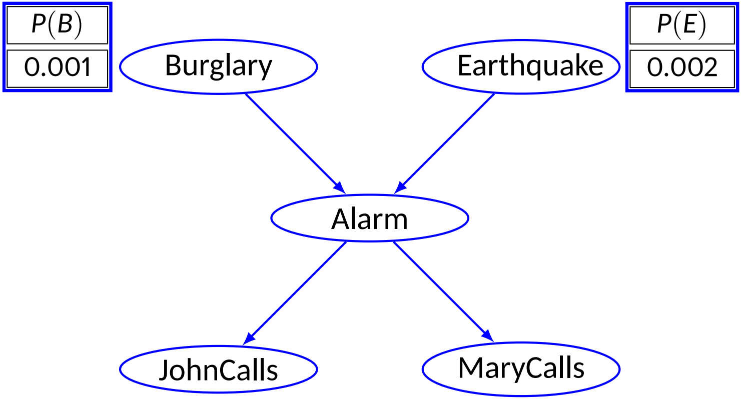
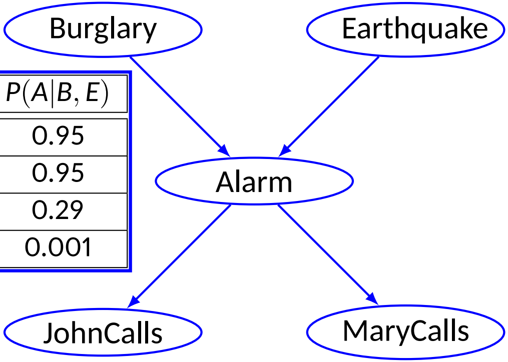


Belief network example

$P(B)$
0.001

$P(E)$
0.002

B	E	$P(A \mid B, E)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001



Belief network example

$P(B)$
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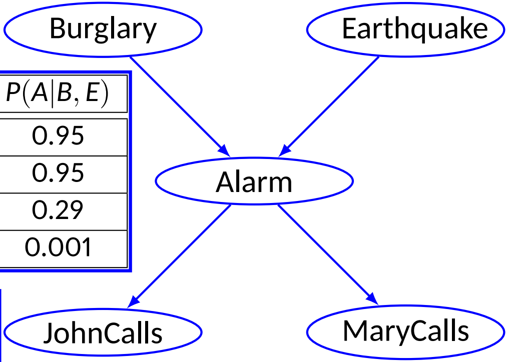
$P(E)$
0.002



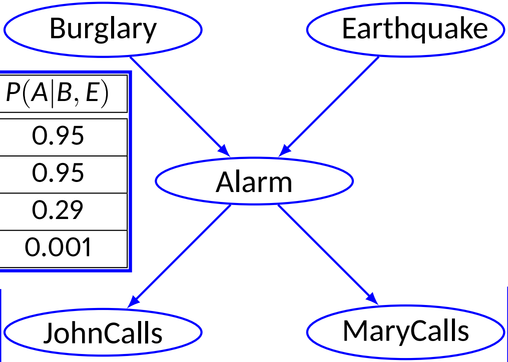
B	E	$P(A\|B, E)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J\|A)$
T	0.90
F	0.05

Belief network example

$P(B)$
0.001

$P(E)$
0.002



B	E	$P(A\|B,E)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J\|A)$
T	0.90
F	0.05

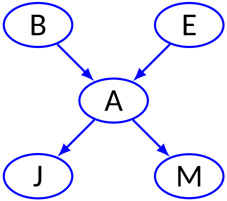
A	$P(M\|A)$
T	0.70
F	0.01

Probabilities in Belief Nets

- **Bayes' net implicitly encode joint distributions**
  - **This can be determined from local conditional distributions**
  - **Need to multiply relevant conditional probabilities**
  - $P(X_1, X_2, \ldots, X_n) = \prod_{i}^{n} P(X_i | parents(X_i))$

Joint probability distribution

- **Probability of the event that the alarm has sounded but neither a burglary nor an earthquake has occurred, and both Mary and John call:**



$P(B)$
0.001

$P(E)$
0.002

B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

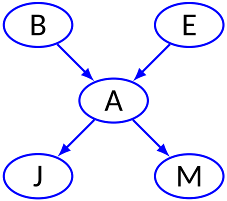
A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Joint probability distribution

- **Probability of the event that the alarm has sounded but neither a burglary nor an earthquake has occurred, and both Mary and John call:**

  $P(J \wedge M \wedge A \wedge \neg B \wedge \neg E)$



$P(B)$
0.001

$P(E)$
0.002

B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

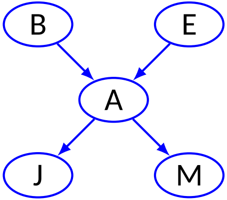
A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Joint probability distribution

- **Probability of the event that the alarm has sounded but neither a burglary nor an earthquake has occurred, and both Mary and John call:**

$$
\begin{aligned}
&P(J \land M \land A \land \neg B \land \neg E) \\
&\quad = P(J|A) \times P(M|A) \times P(A|\neg B \land \neg E) \times P(\neg B) \times P(\neg E) \\
&\quad = 0.9 \times 0.7 \times 0.001 \times 0.999 \times 0.998 \\
&\quad = 0.00062
\end{aligned}
$$



$P(B)$
0.001

$P(E)$
0.002

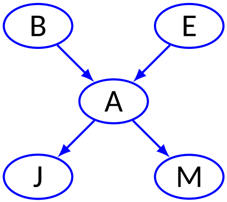
B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Joint probability distribution: $P(A)$

$P(A)$



$P(B)$
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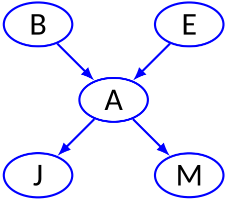
B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Joint probability distribution: $P(A)$

$$
\begin{aligned}
&P(A) \\
&\quad = P(A\bar{B}\bar{E}) + P(A\bar{B}E) + P(AB\bar{E}) + P(ABE)
\end{aligned}
$$



$P(B)$
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B	E	$P(A)$
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T	F	0.95
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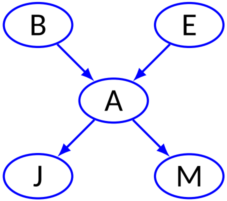
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Joint probability distribution: $P(A)$

$$
\begin{aligned}
P(A) & \\
&= P(A\bar{B}\bar{E}) + P(A\bar{B}E) + P(AB\bar{E}) + P(ABE) \\
&= P(A|\bar{B}\bar{E})\times P(\bar{B}\bar{E}) + P(A|\bar{B}E)\times P(\bar{B}E) + P(A|B\bar{E})\times P(B\bar{E}) + P(A|BE)\times P(BE)
\end{aligned}
$$



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B	E	$P(A)$
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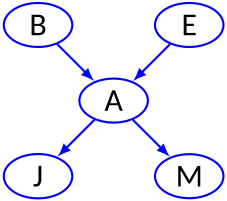
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Joint probability distribution: $P(A)$

$$
\begin{aligned}
&P(A)\\
&\quad = P(A\bar{B}\bar{E}) + P(A\bar{B}E) + P(AB\bar{E}) + P(ABE)\\
&\quad = P(A|\bar{B}\bar{E})\times P(\bar{B}\bar{E}) + P(A|\bar{B}E)\times P(\bar{B}E) + P(A|B\bar{E})\times P(B\bar{E}) + P(A|BE)\times P(BE)\\
&\quad = 0.001\times 0.999\times 0.998 + 0.29\times 0.999\times 0.002 + 0.95\times 0.001\times 0.998\\
&\qquad\quad +0.95\times 0.001\times 0.002\\
&\quad = 0.0025
\end{aligned}
$$



$P(B)$
0.001

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B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

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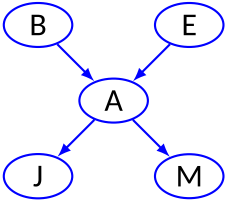
F	0.05

A	$P(M)$
T	0.70
F	0.01

Joint probability distribution: $P(J)$

$P(J)$



$P(B)$
0.001

$P(E)$
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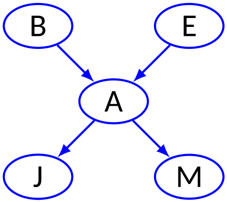
B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Joint probability distribution: $P(J)$

$$P(J)$$
$$= P(JA) + P(J\bar{A})$$



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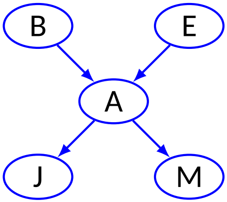
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Joint probability distribution: $P(J)$

$$
\begin{aligned}
& P(J) \\
& \quad = P(JA) + P(J\bar{A}) \\
& \quad = P(J|A) \times P(A) + P(J|\bar{A}) \times P(\bar{A}) \\
& \quad = 0.9 \times 0.0025 + 0.05 \times (1 - 0.0025) \\
& \quad = 0.052125
\end{aligned}
$$



$P(B)$
0.001

$P(E)$
0.002

B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

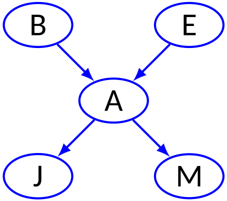
A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Joint probability distribution: $P(J)$

$$
\begin{aligned}
P(J) & \\
&= P(JA) + P(J\bar{A}) \\
&= P(J|A)\times P(A) + P(J|\bar{A})\times P(\bar{A}) \\
&= 0.9\times 0.0025 + 0.05\times(1 - 0.0025) \\
&= 0.052125
\end{aligned}
$$

$P(AB)$



$P(B)$
0.001

$P(E)$
0.002

B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

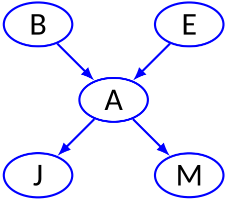
A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Joint probability distribution: $P(J)$

$$
\begin{aligned}
&P(J)\\
&\quad = P(JA) + P(J\bar{A})\\
&\quad = P(J|A)\times P(A) + P(J|\bar{A})\times P(\bar{A})\\
&\quad = 0.9\times 0.0025 + 0.05\times(1 - 0.0025)\\
&\quad = 0.052125
\end{aligned}
$$

$$
\begin{aligned}
&P(AB)\\
&\quad = P(ABE) + P(AB\bar{E})
\end{aligned}
$$



$P(B)$
0.001

$P(E)$
0.002

B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

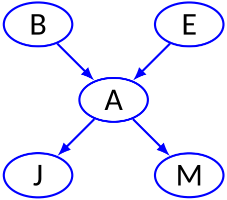
A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Joint probability distribution: $P(J)$

$$P(J)$$
$$= P(JA) + P(J\bar{A})$$
$$= P(J|A)\times P(A) + P(J|\bar{A})\times P(\bar{A})$$
$$= 0.9\times 0.0025 + 0.05\times(1 - 0.0025)$$
$$= 0.052125$$

$$P(AB)$$
$$= P(ABE) + P(AB\bar{E})$$
$$= 0.95\times 0.001\times 0.002 + 0.95\times 0.001\times 0.998$$
$$= 0.00095$$



$P(B)$
0.001

$P(E)$
0.002

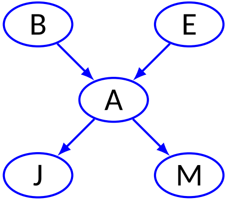
B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Joint probability distribution: $P(JB)$

$P(JB)$



$P(B)$
0.001

$P(E)$
0.002

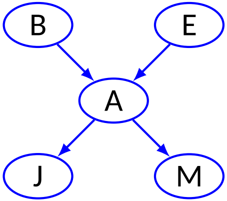
B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Joint probability distribution: $P(JB)$

$$P(JB)$$
$$= P(JBA) + P(JB\bar{A})$$



$P(B)$
0.001

$P(E)$
0.002

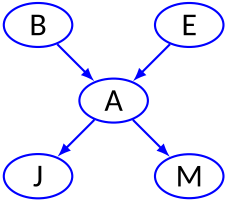
B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Joint probability distribution: $P(JB)$

$$P(JB)$$
$$= P(JBA) + P(JB\bar{A})$$
$$= P(J|AB) \times P(AB) + P(J|\bar{A}B) \times P(\bar{A}B)$$



$P(B)$
0.001

$P(E)$
0.002

B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

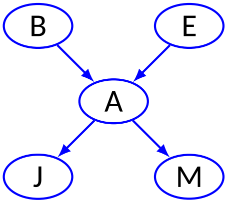
A	$P(M)$
T	0.70
F	0.01

Joint probability distribution: $P(JB)$

$P(JB)$

$$
\begin{aligned}
&= P(JBA) + P(JB\bar{A}) \\
&= P(J|AB)\times P(AB) + P(J|\bar{A}B)\times P(\bar{A}B) \\
&= P(J|A)\times P(AB) + P(J|\bar{A})\times P(\bar{A}B) \\
&= 0.9\times 0.00095 + 0.05\times 0.00005 \\
&= 0.00086
\end{aligned}
$$

$P(J|B)$



$P(B)$
0.001

$P(E)$
0.002

B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

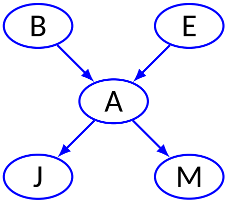
Joint probability distribution: $P(JB)$

$P(JB)$

$$= P(JBA) + P(JB\bar{A})$$
$$= P(J|AB)\times P(AB) + P(J|\bar{A}B)\times P(\bar{A}B)$$
$$= P(J|A)\times P(AB) + P(J|\bar{A})\times P(\bar{A}B)$$
$$= 0.9\times 0.00095 + 0.05\times 0.00005$$
$$= 0.00086$$

$P(J|B)$

$$= \frac{P(JB)}{P(B)} = \frac{0.00086}{0.001} = 0.86$$



$P(B)$
0.001

$P(E)$
0.002

B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

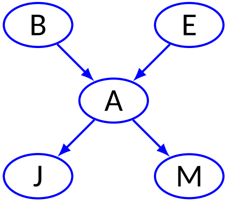
Inferences using belief networks

- **Diagnostic inferences (effects to causes)**
- **Given that JohnCalls, infer that** $P(Burglary|JohnCalls) = 0.016$

- **Causal inferences (causes to effects)**
- **Given Burglary, infer that**

  $P(JohnCalls|Burglary) = 0.86$

  $P(MaryCalls|Burglary) = 0.67$



$P(B)$
0.001

$P(E)$
0.002

B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

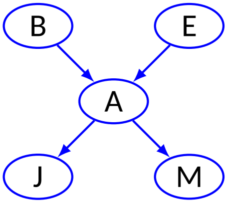
A	$P(M)$
T	0.70
F	0.01

Inferences using belief networks

- **Inter-causal inferences (between causes to a common effect)**
- **Given Alarm, we have** $P(Burglary|Alarm) = 0.376$
- **Also, if it is given that Earthquake is true, then**
  $P(Burglary|Alarm \wedge Earthquake) = 0.003$

- **Mixed inferences**

  $P(Alarm|JohnCalls \wedge \neg Earthquake) = 0.003$



$P(B)$
0.001

$P(E)$
0.002

B	E	$P(A)$
T	T	0.95
T	F	0.95
F	T	0.29
F	F	0.001

A	$P(J)$
T	0.90
F	0.05

A	$P(M)$
T	0.70
F	0.01

Example: traffic

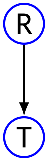
- Consider the following variables
  - R - It is Raining
  - T - There is traffic

Example: traffic

- Consider the following variables
  - R - It is Raining
  - T - There is traffic

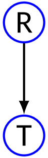
Example: traffic

- Consider the following variables
  - R - It is Raining
  - T - There is traffic



Example: traffic

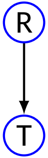
- **Consider the following variables**
  - R **- It is Raining**
  - T **- There is traffic**



$P(R)$
0.25

Example: traffic

- Consider the following variables
  - R - It is Raining
  - T - There is traffic

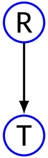


$P(R)$
0.25

	$P(T \mid R)$
R	0.75
$\neg R$	0.50

Example: traffic

- Consider the following variables
  - R - It is Raining
  - T - There is traffic



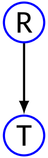
$P(R)$
0.25

	$P(T\|R)$
R	0.75
$\neg R$	0.50

		$P(T,R)$
R	T	3/16
R	$\neg T$	1/16
$\neg R$	T	6/16
$\neg R$	$\neg T$	6/16

Example: traffic

- Consider the following variables
  - R - It is Raining
  - T - There is traffic



$P(R)$
0.25

	$P(T\|R)$
R	0.75
$\neg R$	0.50

		$P(T,R)$
R	T	3/16
R	$\neg T$	1/16
$\neg R$	T	6/16
$\neg R$	$\neg T$	6/16

$P(T)$
9/16

Example: reverse traffic

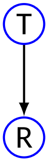
- **Consider the following variables**
  - R **- It is Raining**
  - T **- There is traffic**

Example: reverse traffic

- **Consider the following variables**
  - R **- It is Raining**
  - T **- There is traffic**

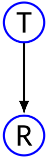
Example: reverse traffic

- Consider the following variables
  - R - It is Raining
  - T - There is traffic



Example: reverse traffic

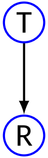
- **Consider the following variables**
  - R - **It is Raining**
  - T - **There is traffic**



$P(T)$
$9/16$

Example: reverse traffic

- **Consider the following variables**
  - R **- It is Raining**
  - T **- There is traffic**

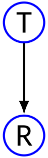


$P(T)$
9/16

	$P(R\|T)$
T	1/3
$\neg T$	1/7

Example: reverse traffic

- **Consider the following variables**
  - R - **It is Raining**
  - T - **There is traffic**



$P(T)$
$9/16$

	$P(R\|T)$
T	$1/3$
$\neg T$	$1/7$

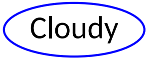
		$P(T,R)$
R	T	$3/16$
R	$\neg T$	$1/16$
$\neg R$	T	$6/16$
$\neg R$	$\neg T$	$6/16$

Example: Sprinkler

- There is some chance that tomorrow will be cloudy. If it is cloudy, then chances of rain is high. Also, there is a sprinkler to water the grass. When there is a prediction of cloud, it is less likely to run sprinkler. We want to model whether grass is wet or not.

Example: Sprinkler

- **There is some chance that tomorrow will be cloudy. If it is cloudy, then chances of rain is high. Also, there is a sprinkler to water the grass. When there is a prediction of cloud, it is less likely to run sprinkler. We want to model whether grass is wet or not.**



Example: Sprinkler

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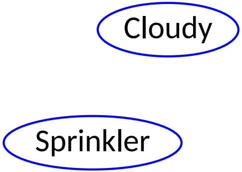

Example: Sprinkler

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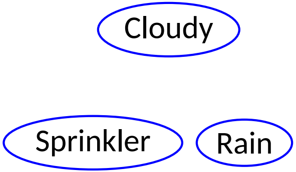

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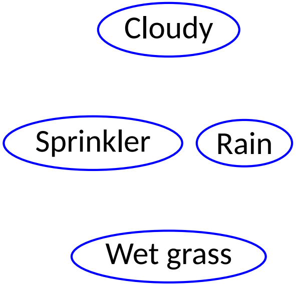

Example: Sprinkler

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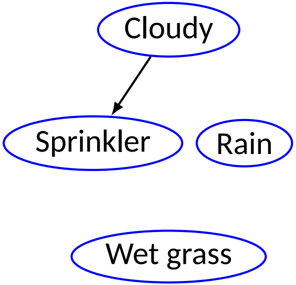

Example: Sprinkler

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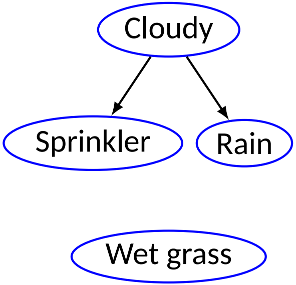

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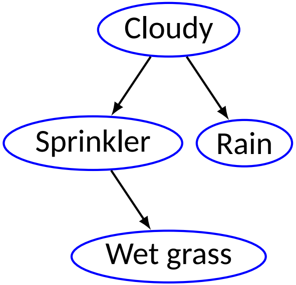

Example: Sprinkler

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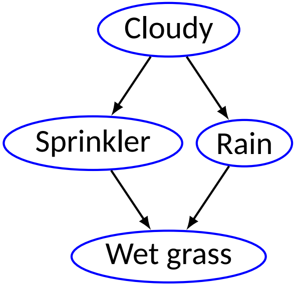

Example: Sprinkler

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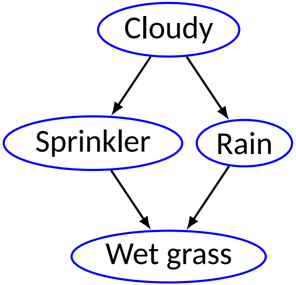
$P(C)$
0.50



Example: Sprinkler

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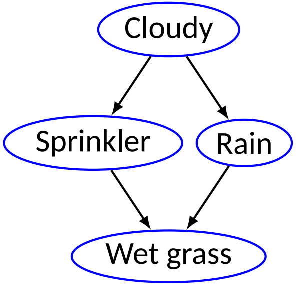
$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50



Example: Sprinkler

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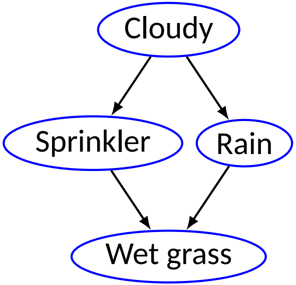
$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50

	$P(R)$
C	0.80
$\neg C$	0.20



Example: Sprinkler

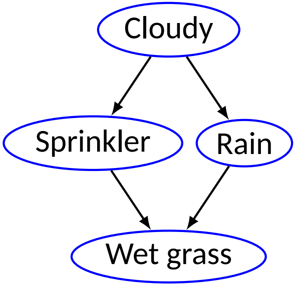
- **There is some chance that tomorrow will be cloudy. If it is cloudy, then chances of rain is high. Also, there is a sprinkler to water the grass. When there is a prediction of cloud, it is less likely to run sprinkler. We want to model whether grass is wet or not.**

$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50

	$P(R)$
C	0.80
$\neg C$	0.20

		$P(W)$
S	R	0.99
s	$\neg R$	0.90
$\neg S$	R	0.90
$\neg S$	$\neg R$	0.00



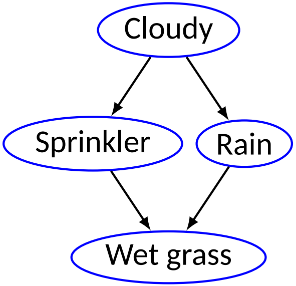
Example: Sprinkler

$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50

	$P(R)$
C	0.80
$\neg C$	0.20

		$P(W)$
S	R	0.99
S	$\neg R$	0.90
$\neg S$	R	0.90
$\neg S$	$\neg R$	0.00



- **Find** $P(W, S, R, C)$

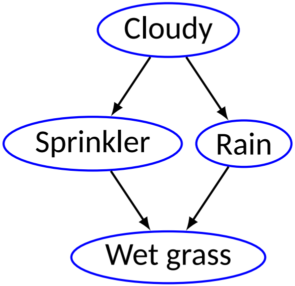
Example: Sprinkler

$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50

	$P(R)$
C	0.80
$\neg C$	0.20

		$P(W)$
S	R	0.99
S	$\neg R$	0.90
$\neg S$	R	0.90
$\neg S$	$\neg R$	0.00



- **Find** $P(W, S, R, C)$

$$= P(C) \times P(S|C) \times P(R|C) \times P(W|S, R)$$

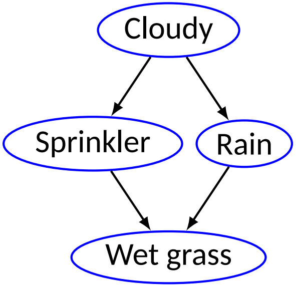
Example: Sprinkler

$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50

	$P(R)$
C	0.80
$\neg C$	0.20

		$P(W)$
S	R	0.99
S	$\neg R$	0.90
$\neg S$	R	0.90
$\neg S$	$\neg R$	0.00



- **Find** $P(W, S, R, C)$

$$= P(C) \times P(S|C) \times P(R|C) \times P(W|S, R)$$

- **Find** $P(W = T|C = T)$

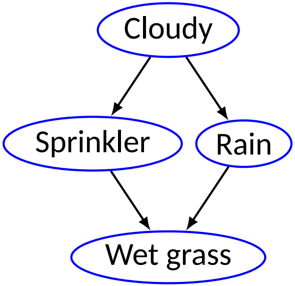
Example: Sprinkler

$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50

	$P(R)$
C	0.80
$\neg C$	0.20

		$P(W)$
S	R	0.99
S	$\neg R$	0.90
$\neg S$	R	0.90
$\neg S$	$\neg R$	0.00



- **Find** $P(W, S, R, C)$
$$= P(C) \times P(S|C) \times P(R|C) \times P(W|S, R)$$
- **Find** $P(W = T|C = T)$
$$= \frac{P(W = T, S, R, C = T)}{P(C = T)}$$

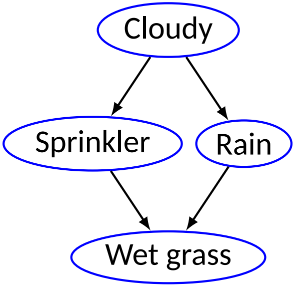
Example: Sprinkler

$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50

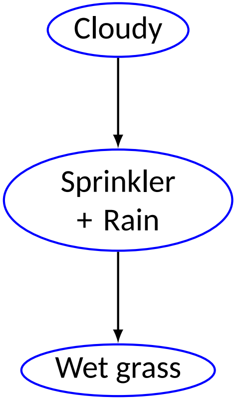
	$P(R)$
C	0.80
$\neg C$	0.20

		$P(W)$
S	R	0.99
S	$\neg R$	0.90
$\neg S$	R	0.90
$\neg S$	$\neg R$	0.00



- **Find** $P(W, S, R, C)$
$$= P(C) \times P(S|C) \times P(R|C) \times P(W|S, R)$$
- **Find** $P(W = T|C = T)$
$$= \frac{P(W = T, S, R, C = T)}{P(C = T)} = 0.7452$$

Example: Sprinkler - merging nodes



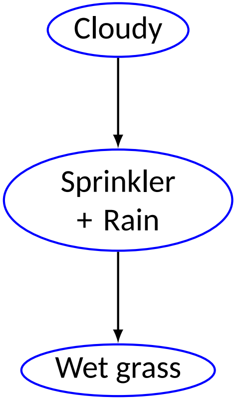
Example: Sprinkler - merging nodes

$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50

	$P(R)$
C	0.80
$\neg C$	0.20

		$P(W)$
S	R	0.99
s	$\neg R$	0.90
$\neg S$	R	0.90
$\neg S$	$\neg R$	0.00

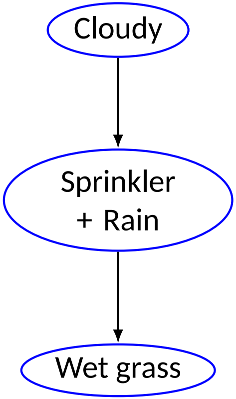


Example: Sprinkler - merging nodes

$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50

	$P(R)$
C	0.80
$\neg C$	0.20



$P(S \vee R)$

		$P(W)$
S	R	0.99
s	$\neg R$	0.90
$\neg S$	R	0.90
$\neg S$	$\neg R$	0.00

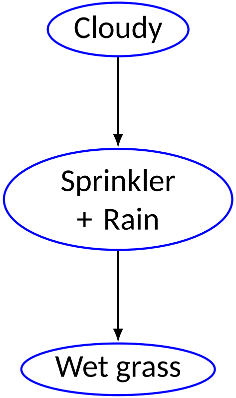
Example: Sprinkler - merging nodes

$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50

	$P(R)$
C	0.80
$\neg C$	0.20

		$P(W)$
S	R	0.99
s	$\neg R$	0.90
$\neg S$	R	0.90
$\neg S$	$\neg R$	0.00



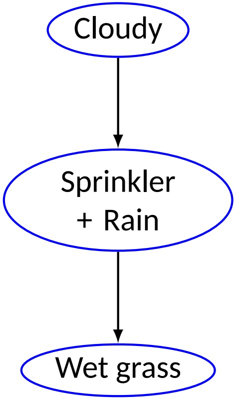
$P(S \vee R)$				
S+R / C	TT	TF	FT	FF
C				

Example: Sprinkler - merging nodes

$P(C)$
0.50

	$P(S)$
C	0.10
$\neg C$	0.50

	$P(R)$
C	0.80
$\neg C$	0.20



$P(S \vee R)$				
S+R / C	TT	TF	FT	FF
C	0.08	0.02	0.72	0.18
$\neg C$	0.40	0.10	0.40	0.10

		$P(W)$
S	R	0.99
s	$\neg R$	0.90
$\neg S$	R	0.90
$\neg S$	$\neg R$	0.00

Incremental network construction

- Select the set of random variables (X_i) to describe the domains
- Select an appropriate ordering of the variables
- Steps:
  - Select a variable (X) and add a node for it
  - Set $Parents(X)$ to some minimal set of existing nodes such that the conditional independence property is satisfied
  - Define the conditional probability table for X

Burglar-Alarm

- Variable ordering
  - MaryCalls, JohnCalls, Alarm, Burglary, Earthquake

Burglar-Alarm

- **Variable ordering**
  - **MaryCalls, JohnCalls, Alarm, Burglary, Earthquake**

Burglar-Alarm

- **Variable ordering**
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Burglar-Alarm

- **Variable ordering**
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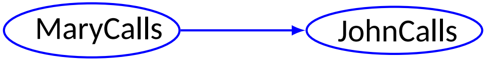
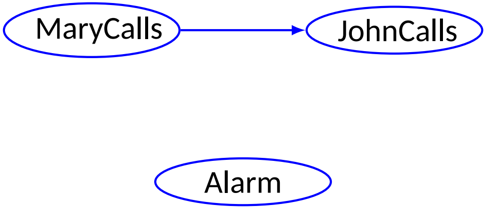


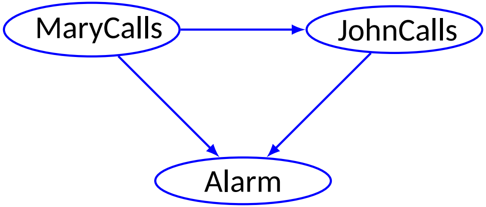
Burglar-Alarm

- **Variable ordering**
  - **MaryCalls, JohnCalls, Alarm, Burglary, Earthquake**



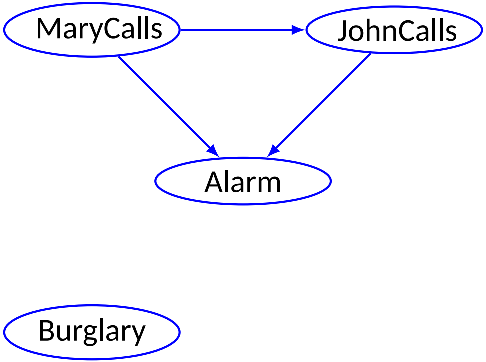
Burglar-Alarm

- **Variable ordering**
  - **MaryCalls, JohnCalls, Alarm, Burglary, Earthquake**



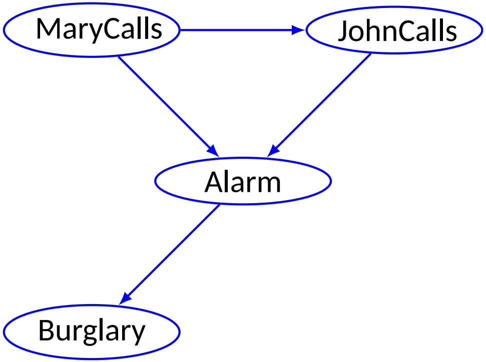
Burglar-Alarm

- **Variable ordering**
  - **MaryCalls, JohnCalls, Alarm, Burglary, Earthquake**



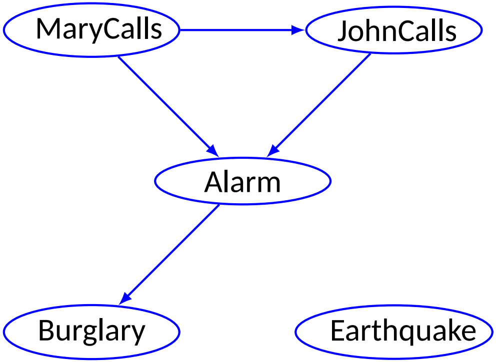
Burglar-Alarm

- **Variable ordering**
  - **MaryCalls, JohnCalls, Alarm, Burglary, Earthquake**



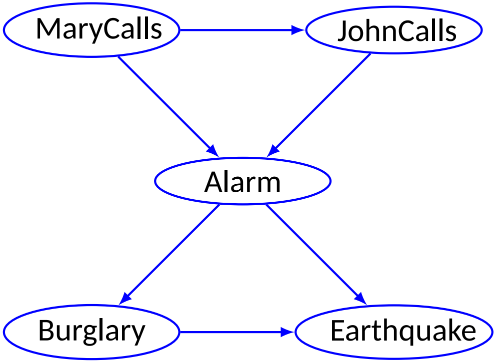
Burglar-Alarm

- **Variable ordering**
  - **MaryCalls, JohnCalls, Alarm, Burglary, Earthquake**



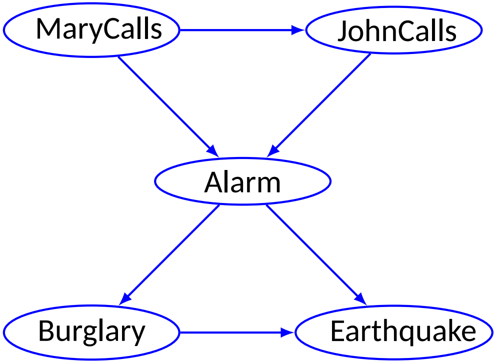
Burglar-Alarm

- **Variable ordering**
  - **MaryCalls, JohnCalls, Alarm, Burglary, Earthquake**



Burglar-Alarm

- **Variable ordering**
  - **MaryCalls, JohnCalls, Alarm, Burglary, Earthquake**
  - **MaryCalls, JohnCalls, Earthquake, Burglary, Alarm**



Burglar-Alarm

- **Variable ordering**
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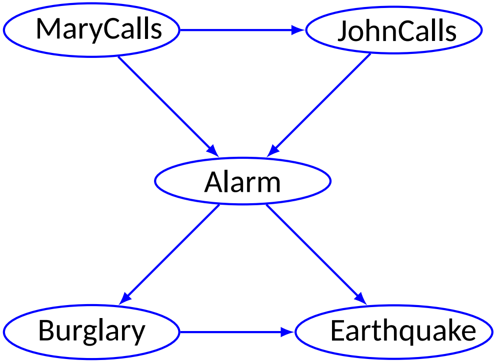

Burglar-Alarm

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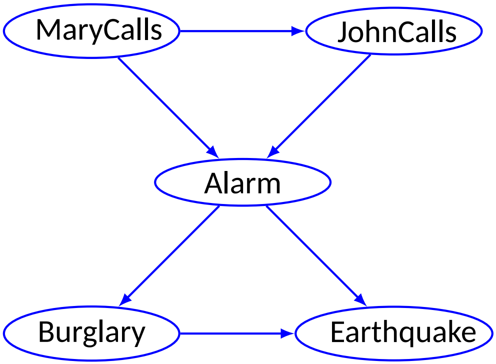
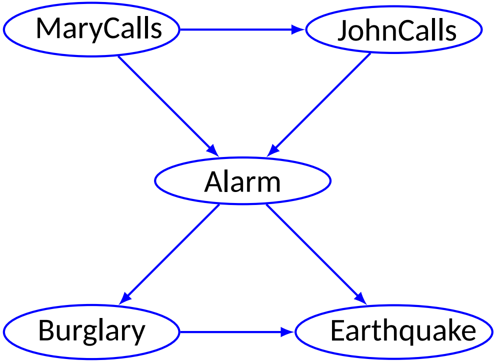


Burglar-Alarm

- Variable ordering
  - MaryCalls, JohnCalls, Alarm, Burglary, Earthquake
  - MaryCalls, JohnCalls, Earthquake, Burglary, Alarm



Burglar-Alarm

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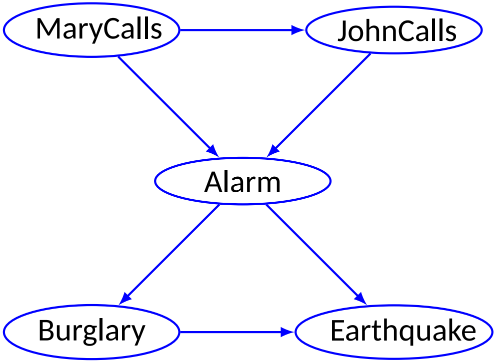

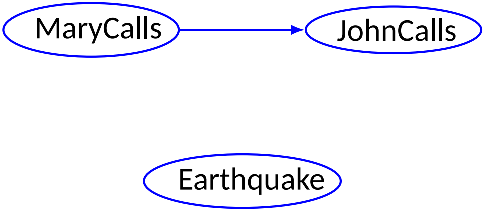

Burglar-Alarm

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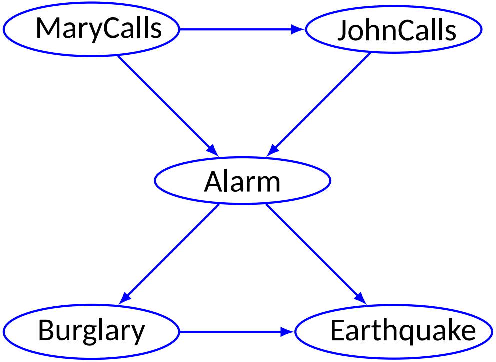

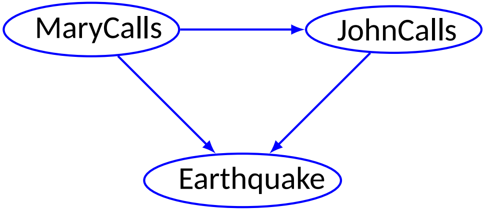

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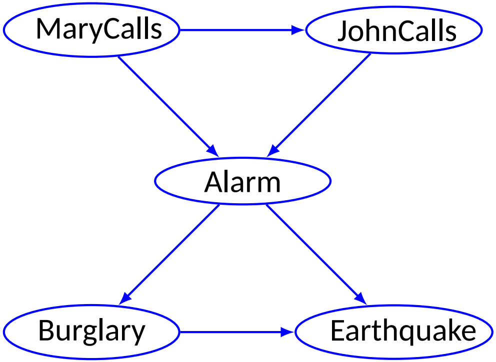

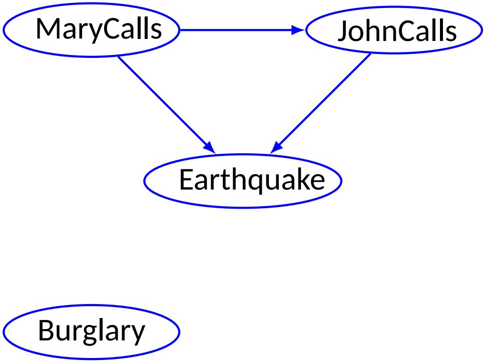

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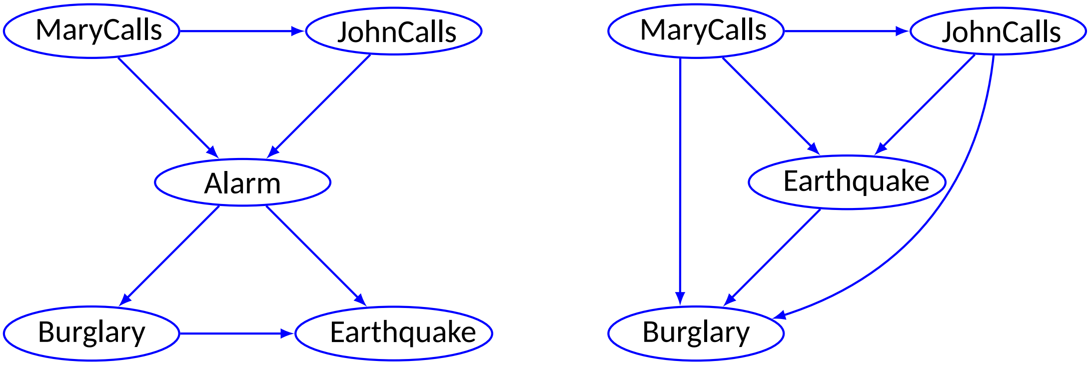

Burglar-Alarm

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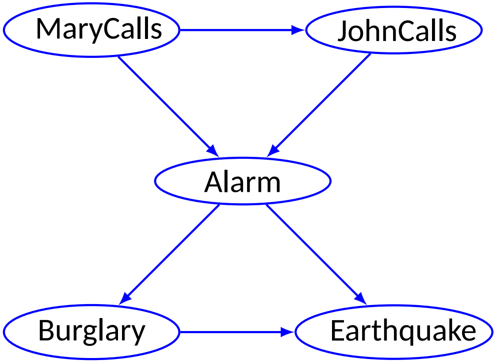

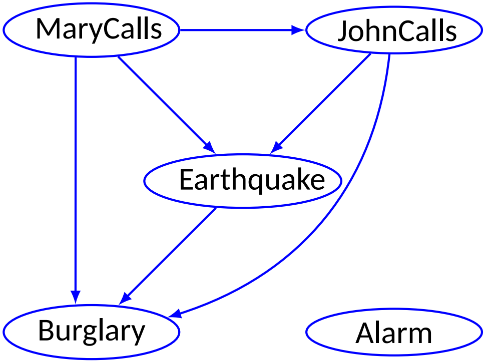

Burglar-Alarm

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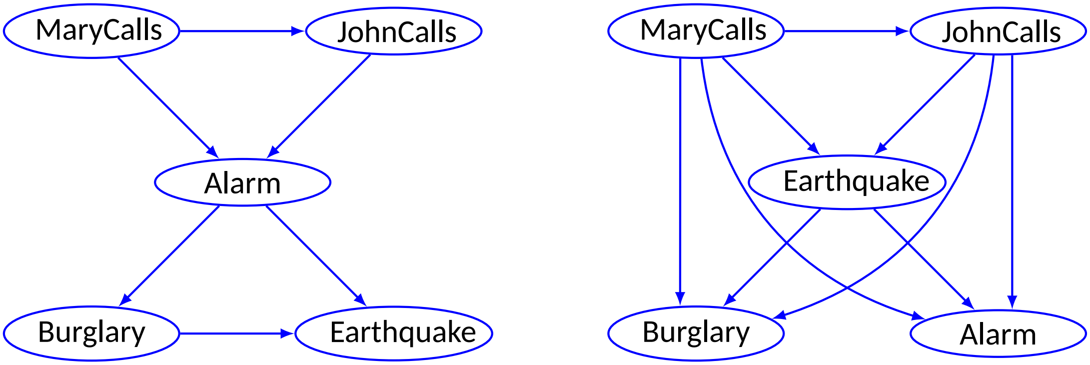
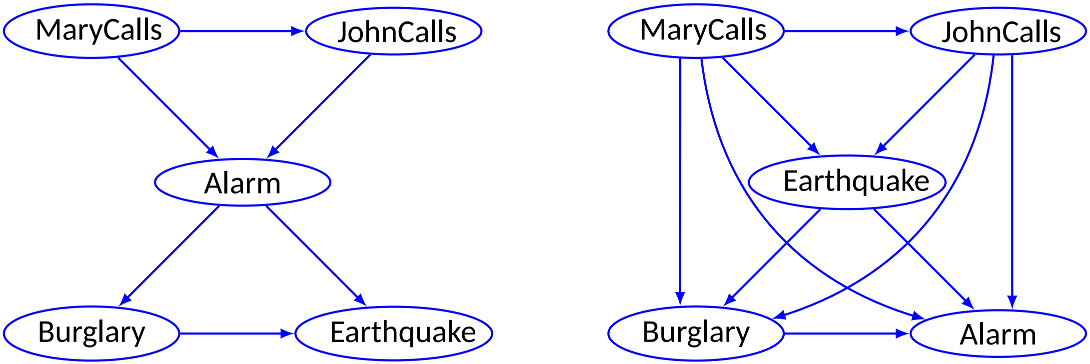
  - **MaryCalls, JohnCalls, Alarm, Burglary, Earthquake**
  - **MaryCalls, JohnCalls, Earthquake, Burglary, Alarm**

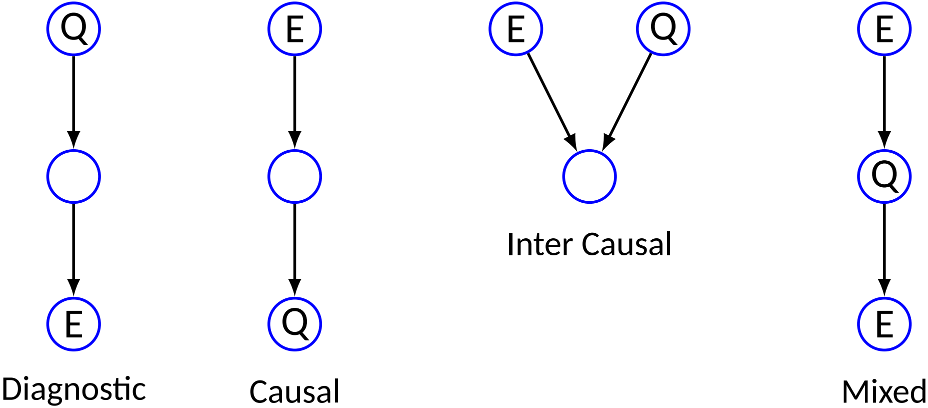


Burglar-Alarm

- **Variable ordering**
  - **MaryCalls, JohnCalls, Alarm, Burglary, Earthquake**
  - **MaryCalls, JohnCalls, Earthquake, Burglary, Alarm**

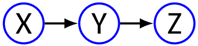


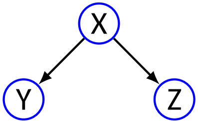
The four patterns

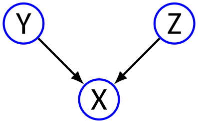


Conditional independence in DAG

- Given a DAG model, how to determine conditional independent relations among nodes/variables
- DAG has three basic graphical patterns, also known as canonical networks
  - Chain - three nodes connected in a line along the same direction
  - Fork - two children of a node
  - V-structure - two parents of a node





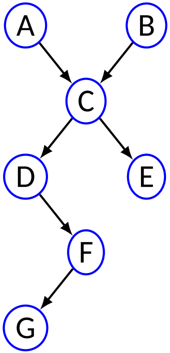


D-separation

- **Input: X, Y and Z**
- **Output: whether X is conditionally independent of Y given Z**

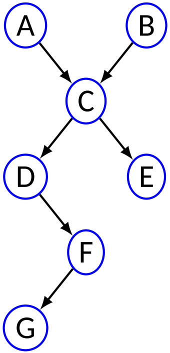
1. **[Ancestral graph] Consider ancestral subgraph using X, Y, Z and their ancestors**
2. **[Moralize] Add links between any unlinked pair of nodes that share a common child; now we have the so-called moral graph.**
3. **[Disorient] Replace all directed links by undirected links**
4. **[Delete givens] If Z blocks all paths between X and Y in the resulting graph, then Z d-separates X and Y. In that case, X is conditionally independent of Y, given Z.**

D-separation example

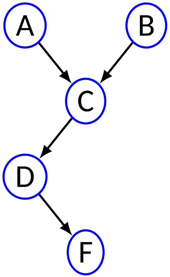


- **Are A and B conditionally independent, given D and F?**

D-separation example

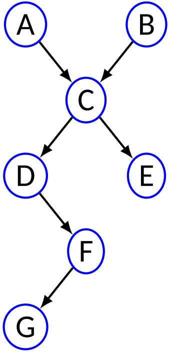


Ancestor graph

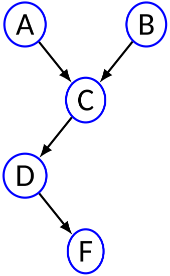


- **Are A and B conditionally independent, given D and F?**

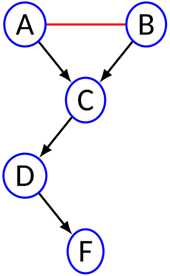
D-separation example



Ancestor graph

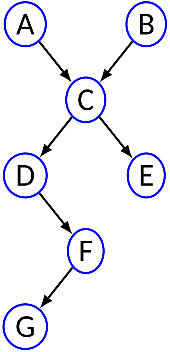


Moral graph

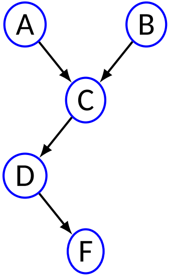


- **Are A and B conditionally independent, given D and F?**

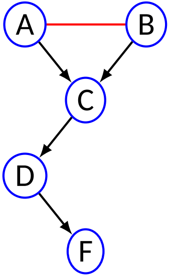
D-separation example



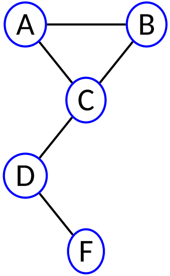
Ancestor graph



Moral graph

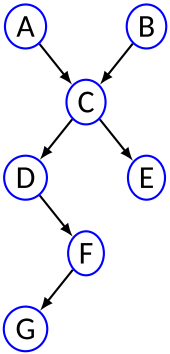


Disorient

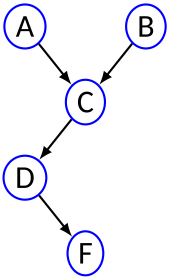


- **Are A and B conditionally independent, given D and F?**

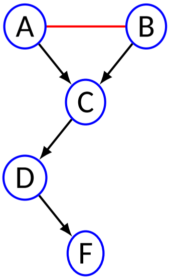
D-separation example



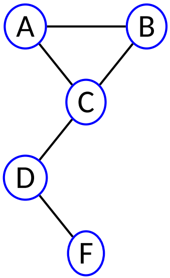
Ancestor graph



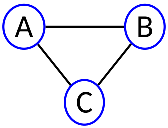
Moral graph



Disorient

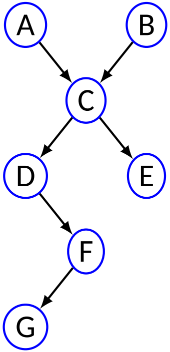


Delete givens



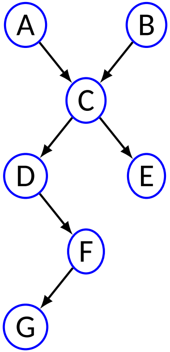
- **Are A and B conditionally independent, given D and F?**

D-separation example

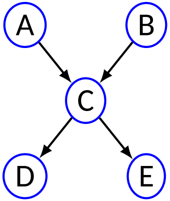


- **Are D and E conditionally independent, given C?**

D-separation example

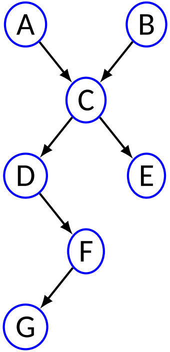


Ancestor graph

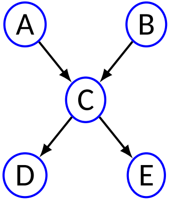


- **Are D and E conditionally independent, given C?**

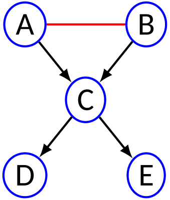
D-separation example



Ancestor graph

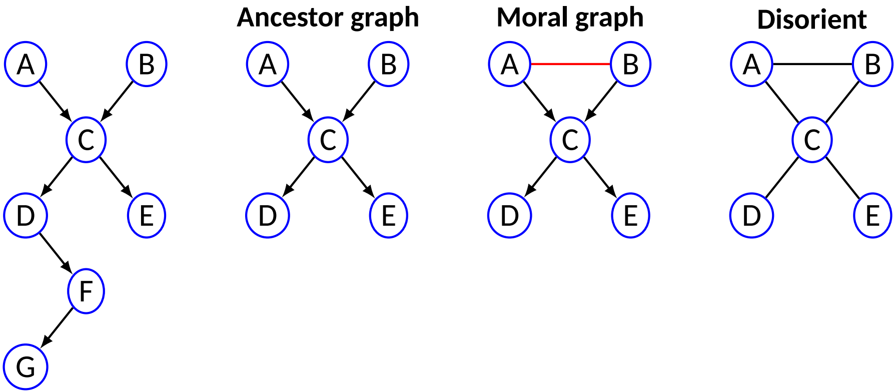


Moral graph



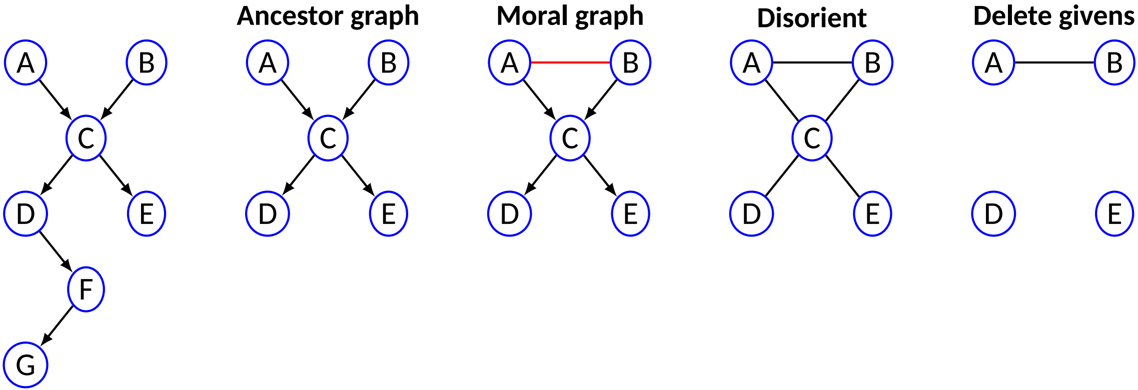
- **Are D and E conditionally independent, given C?**

D-separation example



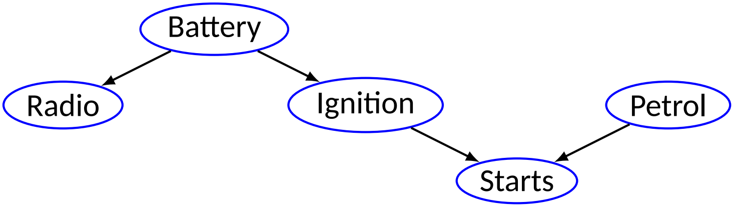
- **Are D and E conditionally independent, given C?**

D-separation example

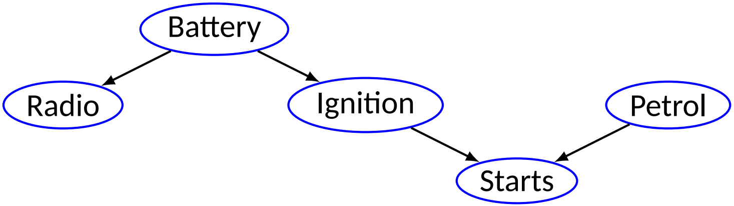


- **Are D and E conditionally independent, given C?**

Conditional independence in belief network

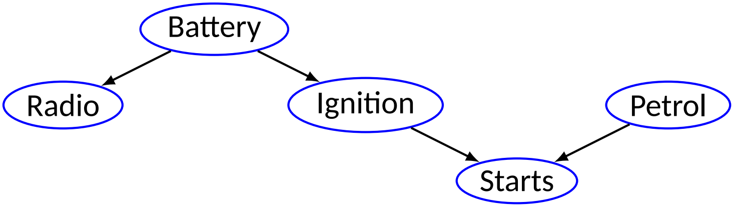


Conditional independence in belief network



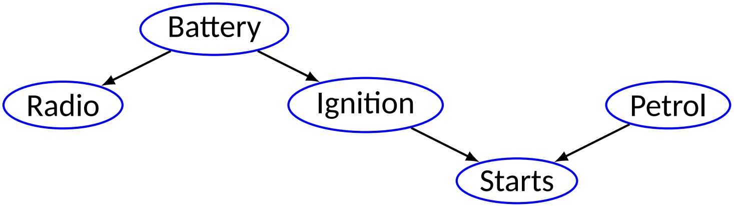
- **Whether there is petrol and whether the radio plays are independent given evidence about whether the ignition takes place**

Conditional independence in belief network



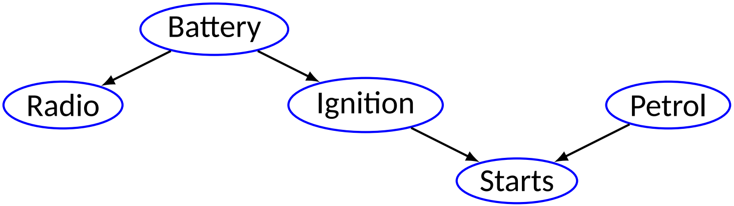
- **Whether there is petrol and whether the radio plays are independent given evidence about whether the ignition takes place**
- **Petrol and Radio are independent if it is known whether the battery works**

Conditional independence in belief network



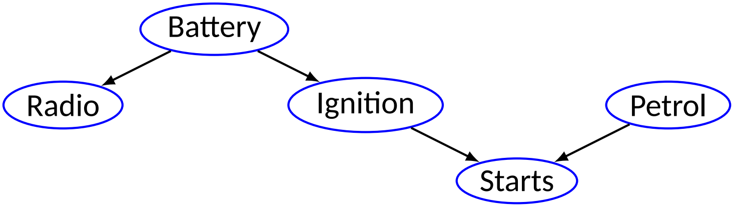
- Whether there is petrol and whether the radio plays are independent given evidence about whether the ignition takes place
- Petrol and Radio are independent if it is known whether the battery works
- Petrol and Radio are independent given no evidence at all.

Conditional independence in belief network



- Whether there is petrol and whether the radio plays are independent given evidence about whether the ignition takes place
- Petrol and Radio are independent if it is known whether the battery works
- Petrol and Radio are independent given no evidence at all.
- But they are dependent given evidence about whether the car starts.

Conditional independence in belief network

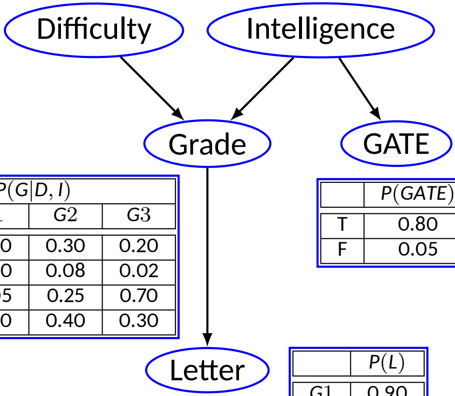


- Whether there is petrol and whether the radio plays are independent given evidence about whether the ignition takes place
- Petrol and Radio are independent if it is known whether the battery works
- Petrol and Radio are independent given no evidence at all.
- But they are dependent given evidence about whether the car starts.
- If the car does not start, then the radio playing is increased evidence that we are out of petrol.

Exercise: Grade

$P(D)$
0.40

$P(I)$
0.30



$P(G \mid D, I)$			
D, I	$G1$	$G2$	$G3$
T, T	0.50	0.30	0.20
T, F	0.90	0.08	0.02
F, T	0.05	0.25	0.70
F, F	0.30	0.40	0.30

	$P(GATE)$
T	0.80
F	0.05

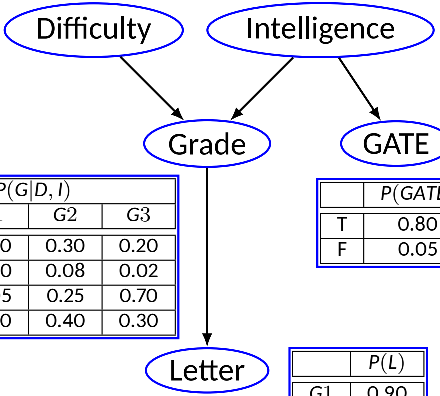
	$P(L)$
$G1$	0.90
$G2$	0.60
$G3$	0.01

Exercise: Grade

- **What is the probability of getting excellent recommendation letter given the semester question paper was very easy?**

$P(D)$
0.40

$P(I)$
0.30



$P(G \mid D, I)$			
D, I	$G1$	$G2$	$G3$
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F, F	0.30	0.40	0.30

	$P(GATE)$
T	0.80
F	0.05

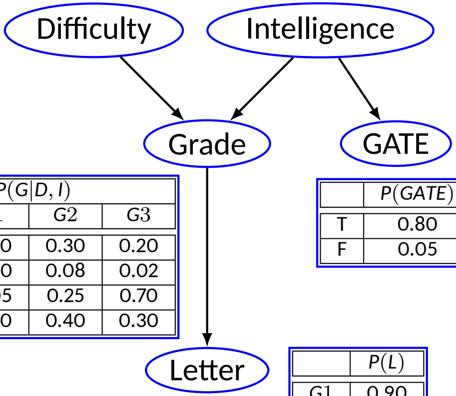
	$P(L)$
$G1$	0.90
$G2$	0.60
$G3$	0.01

Exercise: Grade

- **What is the probability of getting excellent recommendation letter given the semester question paper was very easy?**
- **What is the probability of a student being very intelligent given he gets a poor recommendation letter?**

$P(D)$
0.40

$P(I)$
0.30



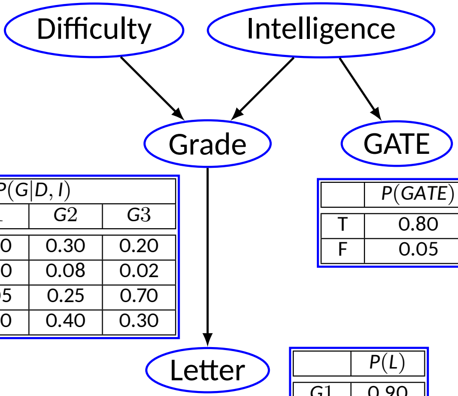
$P(G \mid D, I)$			
D, I	$G1$	$G2$	$G3$
T, T	0.50	0.30	0.20
T, F	0.90	0.08	0.02
F, T	0.05	0.25	0.70
F, F	0.30	0.40	0.30

	$P(GATE)$
T	0.80
F	0.05

	$P(L)$
$G1$	0.90
$G2$	0.60
$G3$	0.01

Exercise: Grade

- **What is the probability of getting excellent recommendation letter given the semester question paper was very easy?**
- **What is the probability of a student being very intelligent given he gets a poor recommendation letter?**
- **What is the probability of getting a poor GATE score given the semester grade was very good?**



$P(D)$
0.40

$P(I)$
0.30

$P(G \mid D, I)$			
D, I	$G1$	$G2$	$G3$
T, T	0.50	0.30	0.20
T, F	0.90	0.08	0.02
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F, F	0.30	0.40	0.30

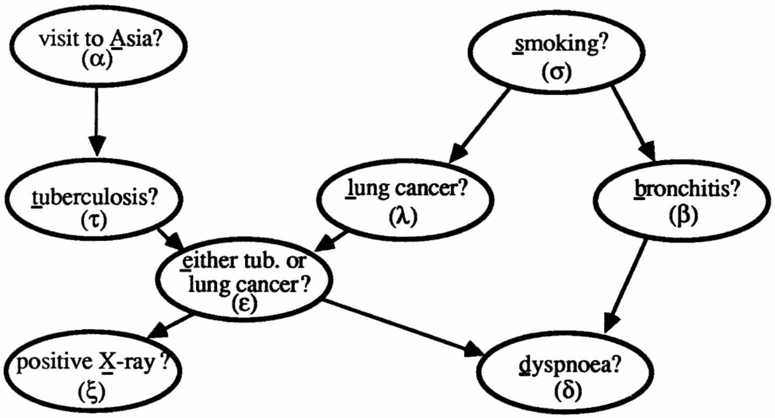
	$P(GATE)$
T	0.80
F	0.05

	$P(L)$
$G1$	0.90
$G2$	0.60
$G3$	0.01

Belief network: medical diagnosis

- **Construct a belief network for the following: [Lauritzen & Spiegelhalter, 1988]**
  - **Shortness-of-breath (dyspnoea) may be due to tuberculosis, lung cancer or bronchitis, or none of them, or more than one of them. A recent visit to Asia increases the chances of tuberculosis, while smoking is known to be a risk factor both lung cancer and bronchitis. The results of a single chest X-ray do not discriminate between lung cancer and tuberculosis, as neither does the presence or absence of dyspnoea.**

Belief network: medical diagnosis - BN



source: Lauritzen & Spiegelhalter, 1988

Belief network: medical diagnosis - CPT

α:	$p(a) = .01$		ε:	$p(e \mid l, t) = 1$	
				$p(e \mid l, \bar{t}) = 1$	
τ:	$p(t \mid a) = .05$			$p(e \mid \bar{l}, t) = 1$	
	$p(t \mid \bar{a}) = .01$			$p(e \mid \bar{l}, \bar{t}) = 0$	
σ:	$p(s) = .50$		ξ:	$p(x \mid e) = .98$	
				$p(x \mid \bar{e}) = .05$	
λ:	$p(l \mid s) = .10$				
	$p(l \mid \bar{s}) = .01$		δ:	$p(d \mid e, b) = .90$	
				$p(d \mid e, \bar{b}) = .70$	
β:	$p(b \mid s) = .60$			$p(d \mid \bar{e}, b) = .80$	
	$p(b \mid \bar{s}) = .30$			$p(d \mid \bar{e}, \bar{b}) = .10$	

- **A patient presents at a chest clinic with dyspnoea, and has recently visited Asia**
- **The doctor would like to know the chance that each of the diseases is present**
- **Suppose information on smoking has been provided now, repeat the above step**

source: Lauritzen & Spiegelhalter, 1988

Belief network: phd admission

- **Construct a belief network for the following**
  - **Let us assume that you are interested in pursing phd at IIT-Patna!!**
  - **To take admission in the phd program, you may need to provide at most two recommendation letters from the course instructors or to appear for PMRF interview.**
  - **A course instructor can provide at most one recommendation letter based on the your grade on that subject.**
  - **Moreover, the grade depends on the attendance, marks obtained in the assignments and in the final examination.**
  - **Intelligence of the student can also play a major role in the performance of a subject or in the interview.**
  - **You may feel happy if you perform well in the subjects or you get admission for the phd program.**

Thank you!