CS5201: Advanced Artificial Intelligence

Prolog programming

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What is Prolog?

- Invented early seventies by Alain Colmerauer in France and Robert Kowalski in Britain
- Prolog = Programmation en Logique (Programming in Logic).
- Prolog is a declarative programming language unlike most common programming languages.
- In a declarative language
  - The programmer specifies a goal to be achieved
  - The Prolog system works out how to achieve it
- In purely declarative languages, the programmer only states what the problem is and leaves the rest to the language system

Relations

- Prolog programs specify relationships among objects and properties of objects
- When we say, ”Ayesha owns the pen”, we are declaring the ownership relationship between two objects: Ayesha and the pen.
- When we ask, ”Does Ayesha own the pen?” we are trying to find out about a relationship
- Relationships can also be rules such as:
  - Two people are sisters if – they are both female AND they have the same parents.
- In traditional programming relationship may be defined as
  - A and B are sisters if – A and B are both female AND they have the same father AND they have the same mother AND A is not the same as B
- A rule allows us to find out about a relationship even if the relationship is not explicitly stated as a fact

Programming prolog

- Declare facts describing explicit relationships between objects and properties objects might have (e.g. Subodh likes pizza, sky has_colour blue)
- Declare rules defining implicit relationships between objects and/or rules defining implicit object properties (e.g. X is a parent if there is a Y such that Y is a child of X).
- Then the system can be used by:
  - Asking questions above relationships between objects, and/or about object properties (e.g. does Subodh like pizza? is Ayesha a parent?)

Prolog & Predicate logic

- **Prolog is a programming language based on predicate logic.**
    - **A Prolog program attempts to prove a goal, such as brother(Barney,x), from a set of facts and rules.**
    - **In the process of proving the goal to be true, using substitution and the other rules of inference, Prolog substitutes values for the variables in the goal, thereby ”computing” an answer.**
- **How does Prolog know which facts and which rules to use in the proof?**
    - **Prolog uses unification to determine when two clauses can be made equivalent by a substitution of variables.**
    - **The unification procedure is used to instantiate the variables in a goal clause based on the facts and rules in the database.**

Tools

- **GNU prolog** - `gplc, gprolog`
- **SWI-Prolog** - `https://swi-prolog.org`
- **Online tool**
  - `https://swish.swi-prolog.org/`
  - `https://www.onlinegdb.com/online_prolog_compiler`

A simple Prolog program

```
male(albert).
male(edward).
female(alice).
female(victoria).
parent(albert,edward).
parent(victoria,edward).
father(X,Y) :- parent(X,Y), male(X).      % ∀X ∀Y ((parent(X, Y) ∧ male(X)) → father(X, Y))
mother(X,Y) :- parent(X,Y), female(X).    % ∀X ∀Y ((parent(X, Y) ∧ female(X)) → mother(X, Y))
```

- A fact/rule (statement) ends with "." and white space ignored
- Read ':-' after RULE HEAD as "if"
- Read comma in body as "and"
- Comment a line with % or use /* */ for multi-line comments

Facts & Rules

- **The Prolog environment maintains a set of facts and rules in its database.**
  - **Facts are axioms; relations between terms that are assumed to be true.**
  - **Rules are theorems that allow new inferences to be made.**

Facts & Rules

- The Prolog environment maintains a set of facts and rules in its database.
  - Facts are axioms; relations between terms that are assumed to be true.
  - Rules are theorems that allow new inferences to be made.
- Example facts & rules:
  - male(adam).
  - female(anne).
  - parent(adam,barney).
  - son(X,Y) :- parent(Y,X) , male(X).
  - daughter(X,Y) :- parent(Y,X) , female(X).
- The first rule is read as follows: for all X and Y, X is the son of Y if there exists X and Y such that Y is the parent of X and X is male. $\forall X \forall Y((parent(Y,X) \wedge male(X)) \to son(X,Y))$
- The second rule is read as follows: for all X and Y, X is the daughter of Y if there exists X and Y such that Y is the parent of X and X is female. $\forall X \forall Y((parent(Y,X) \wedge female(X)) \to daughter(X,Y))$

Horn clauses

- **To simplify the resolution process in Prolog, statements must be expressed in a simplified form, called Horn clauses.**
  - **Statements are constructed from terms.**
  - **Each statement (clause) has (at most) one term on the left hand side of an implication symbol (:-).**
  - **Each statement has a conjunction of zero or more terms on the right hand side.**
  - **Example:** $p_1 \wedge p_2 \wedge \ldots \rightarrow q \equiv \neg p_1 \vee \neg p_2 \vee \ldots \vee q$
- **Prolog has three kinds of statements, corresponding to the structure of the Horn clause used.**
  - **A fact is a clause with an empty right hand side.**
  - **A question (or goal) is a clause with an empty left hand side.**
  - **A rule is a clause with terms on both sides.**

Execution of Prolog program

```
male(albert).
male(edward).
female(alice).
female(victoria).
parent(albert,edward).
parent(victoria,edward).
father(X,Y) :- parent(X,Y), male(X).
mother(X,Y) :- parent(X,Y), female(X).
```

Query:

male(X). % $\exists X\, male(X)$

father(F,edward). % $\exists F\, father(F, edward)$

father(F,C). % $\exists F\, \exists C\, father(F, C)$

```
$> gplc family.pl
$> ./family
?- male(albert).
yes
?- male(victoria).
no
?- male(subodh).
no
?- male(X).
X = albert ? ;
X = edward
?- father(F,C).
F=albert, C=edward ? ;
no
```

Observation about Prolog rules

- **The implication is from right to left**
- **The scope of a variable is the clause in which it appears.**
- **Variables whose first appearance is on the left hand side of the clause have implicit universal quantifiers.**
- **Variables whose first appearance is on the right hand side of the clause have implicit existential quantifiers.**
    - **grandparent(X,Z) :- parent(X,Y), parent(Y,Z).**
    - $\forall X \forall Z \left(\left(parent(X, Y) \wedge parent(Y, Z) \right) \rightarrow grandparent(X, Z) \right)$

Basic syntax of Prolog: Terms

- Constants:
  - Identifiers - sequences of letters, digits, or ”_” that start with lower case letters.
  - Numbers - 1.001
  - Strings enclosed in single quotes
    - Can start with upper case letter or can be a number now treated as a string
- Variables:
  - Sequence of letters digits or ”_” that start with an upper case letter or the _
    - Underscore by itself is the special ”anonymous” variable
- Structures (like function applications)
  - $<$identifier$>$(Term-1,...,Term-k)
    - date(20,April,2020), point(X,Y,Z)
  - Definition can be recursive, so each term can itself be a structure
    - date(+(5,15),April,+(2000,$-$(140,120)))
  - Structures can be represented as tree

Arithmetic and logical operators

- Arithmetic operations: +, -, *, /
  - The ”is” operator forces arithmetic evaluation
  - ?- X is 3 + 8.
- Logical operators: >, <, >=, <=
  - x =:= y (equality check)
  - x =\= y (inequality check)

Syntax of Prolog: Lists

- **Lists are a very useful data structure in Prolog**
- **Lists are structured terms represented in a special way - [a, b, c, d]**
  - **This is actually structured term - [a | [b | [c | [d | []]]]]**
  - **In the above [] denotes empty list**
  - **Each list is thus of the form [<head> | <tail>]**
  - **<head> is an element of the list (not necessary a list itself)**
  - **<tail> is a list / sublist**
  - **Also, [a,b,c,d] = [a | [b,c,d]] = [a,b | [c,d]] = [a,b,c | [d]]**
- **This structure has important implications when it comes to matching variables against lists!**

Syntax of Prolog: Predicates

- Predicates are syntactically identical to structured items – <identifier>(Term-1,…,Term-k)
  - male(edward)
  - parent(edward,albert)
  - taller_than(subodh,shyam)
  - likes(X)
    - Note that X is a variable. X can take on any term as value so that this fact asserts
- Facts make assertion

Syntax of Prolog: Facts and Rules

- **Rules: PredicateH :- predicate-1, …, predicate-k.**
  - **First predicate is *rule head*. Terminated by a period**
  - **Rules encode ways of deriving or computing a new fact**
  - **animal(X) :- elephant(X). - X can be concluded to be animal if it shown that X is elephant**
  - **taller_than(X,Y) :- height(X,H1), height(Y,H2), H1 > H2.**
  - **father(X,Y) :- parent(X,Y), male(X).**

Operation of Prolog

- A query is a sequence of predicates: predicate-1, predicate-2, ..., predicate-k
- Prolog tries to prove that this sequence of predicates is true using the facts and rules in the Prolog Program.
- In proving the sequence it performs the computation you want.
- Example:
  - elephant(fred).
  - elephant(mary).
  - elephant(joe).
  - animal(fred) :- elephant(fred).
  - animal(mary) :- elephant(mary).
  - animal(joe) :- elephant(joe).
  - QUERY: animal(fred), animal(mary), animal(joe).

Operation

- **Starting with the first predicate P1 of the query Prolog examines the program from TOP to BOTTOM.**
- **It finds the first RULE HEAD or FACT that matches P1**
- **Then it replaces P1 with the RULE BODY.**
- **If P1 is matched a FACT, we can think of FACTs as having empty bodies (so P1 is simply removed).**
- **The result is a new query.**
- **Example**
    - **P1 :- Q1, Q2, Q3.**
    - **QUERY: P1, P2, P3.**
    - **P1 matches with the rule, therefore, new QUERY: Q1, Q2, Q3, P2, P3.**

Execution of Prolog program

```
elephant(fred).
elephant(mary).
elephant(joe).
animal(fred) :- elephant(fred).
animal(mary) :- elephant(mary).
animal(joe) :- elephant(joe).
QUERY: animal(fred), animal(mary), animal(joe).
```

```
1.  elephant(fred), animal(mary), animal(joe).
2.  animal(mary), animal(joe).
3.  elephant(mary), animal(joe).
4.  animal(joe).
5.  elephant(joe).
6.  EMPTY QUERY
```

Operation

- If this process reduces the query to the empty query, Prolog returns ”yes”.
- However, during this process each predicate in the query might match more than one fact or rule head
  - Prolog always choose the first match it finds. Then if the resulting query reduction did not succeed (i.e., we hit a predicate in the query that does not match any rule head of fact), Prolog backtracks and tries a new match.

Execution of Prolog program

ant_eater(fred).
animal(fred) :- elephant(fred).
animal(fred) :- ant_eater(fred).
QUERY: animal(fred)

```
1.  elephant(fred).
2.  FAIL BACKTRACK
3.  ant_eater(fred).
4.  EMPTY QUERY
```

Operation

- **Backtracking can occur at every stage as the query is processed**

```
p(1) :- a(1).
p(1) :- b(1).
a(1) :- c(1).
c(1) :- d(1).
c(1) :- d(2).
b(1) :- e(1).
e(1).
d(3).
QUERY: p(1)
```

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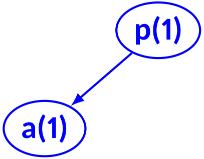

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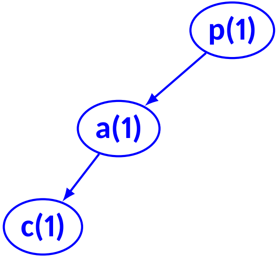

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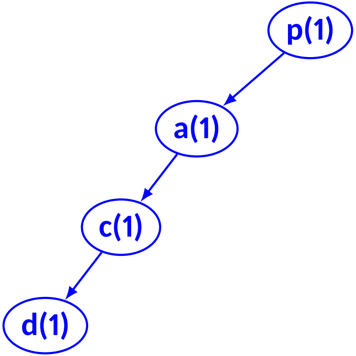

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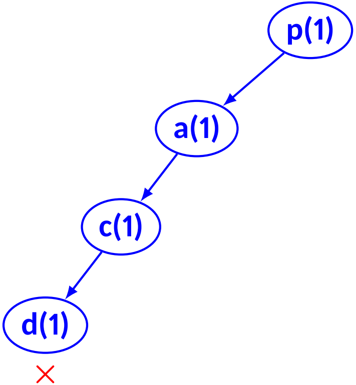

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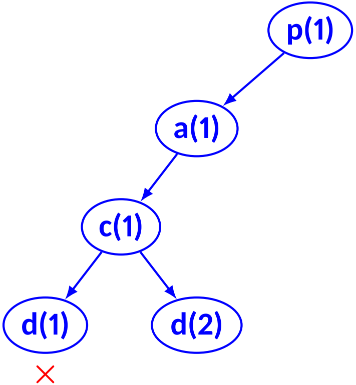

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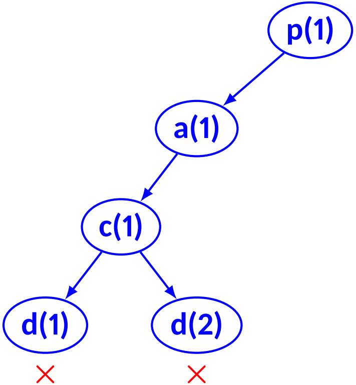

Operation

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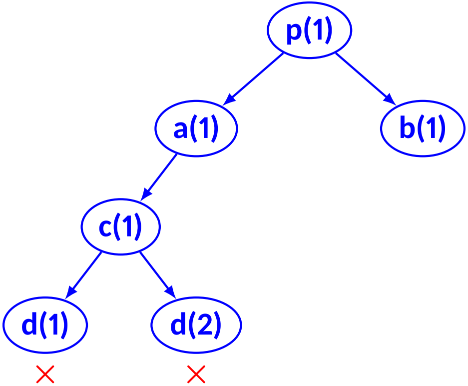

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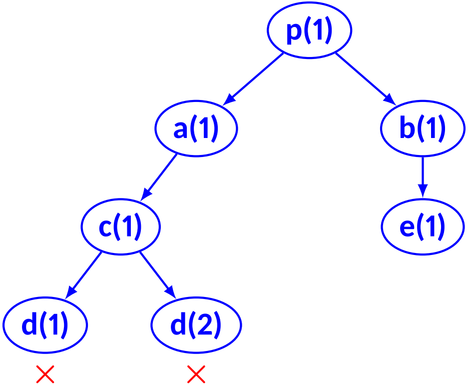

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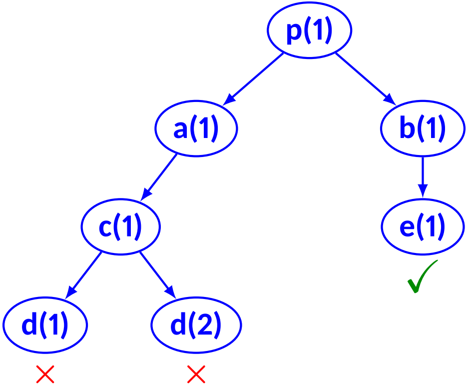

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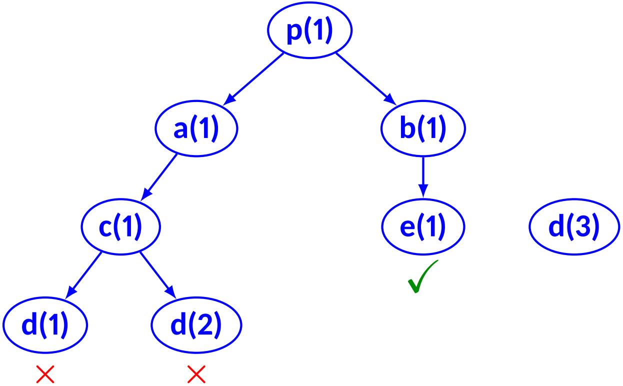
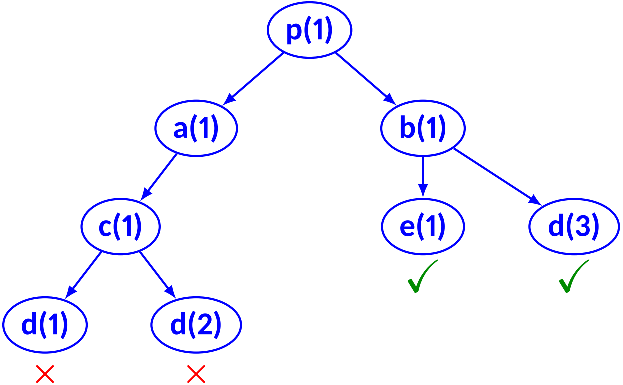
QUERY: p(1)



Operation

- **With backtracking we can get more answers by using ";"**

  p(1) :- a(1).
  p(1) :- b(1).
  a(1) :- c(1).
  c(1) :- d(1).
  c(1) :- d(2).
  b(1) :- e(1).
  b(1) :- d(3).
  e(1).
  d(3).
  QUERY: p(1)



Variables and Matching

- **Variables allow us to**
  - **Compute more than yes/no answer, compress the program**
- **Example:**
  - **elephant(fred).**
  - **elephant(mary).**
  - **elephant(joe).**
  - **animal(fred) :- elephant(fred).**
  - **animal(mary) :- elephant(mary).**
  - **animal(joe) :- elephant(joe).**
- **The three rules can be replaced by the single rule animal(X) :- elephant(X).**
- **When matching queries against rule heads (of facts) variables allow many additional matches.**

Example

```
elephant(fred).
elephant(mary).
elephant(joe).
animal(X) :- elephant(X).
QUERY: animal(fred), animal(mary), animal(joe)
```

1. X=fred, elephant(X), animal(mary), animal(joe)
2. animal(mary), animal(joe)
3. X=mary, elephant(X), animal(joe)
4. animal(joe)
5. X=joe, elephant(X)
6. EMPTY QUERY

Operation with Variables

- Queries are processed as before (via rule and fact matching and backtracking), but now we can use variables to help us match rule heads or facts.
- A query predicate matches a rule head or fact (either one with variables) if
  - The predicate name must match. So elephant(X) can match elephant(joe), but can never match ant_eater(joe).
  - Then, the arity of the predicates are matched (number of terms). So foo(X,Y) can match foo(joe,mary), but cannot match foo(joe) or foo(joe,mary,fred).
  - If the predicate names and arities match then each of the k-terms match. So for foo(t1, t2, t3) to match foo(s1, s2, s3) we must have that t1 matches s1, t2 matches s2, and t3 matches t3.
  - During this matching process we might have to ”bind” some of the variables to make the terms match.
  - These bindings are then passed on into the new query (consisting of the rule body and the left over query predicates).

Variable matching (Unification)

- **Two terms with variables match if :**
  - **If both are constants (identifiers, numbers, or strings) and are identical**
  - **If one or both are bound variables then they match if what the variables are bound to match**
    - **X and mary where X is bound to the value mary will match**
    - **X and Y where X is bound to mary and Y is bound to mary will match**
    - **X and ann where X is bound to mary will not match**
  - **If one of the terms is an unbound variable then they match and we bind the variable to the term**
    - **X and mary where X is unbound match and make X bound to mary.**
    - **X and Y where X is unbound and Y is bound to mary match and make X bound to mary.**
    - **X and Y where both X and Y are unbound match and make X bound to Y (or vice versa).**

Solving queries

- Prolog work as follows
  - Unification
  - Goal directed reasoning
  - Rule ordering
  - DFS and backtracking

List processing in Prolog

- **Much of prolog's computation is organized around lists. Two key things we do with a list is iterate over them and build new ones.**
- **Checking membership: member(X,Y) - X is a member of list Y**

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  - **member(X,[X|_]).**
  - **member(X,[_|T]) :- member(X,T).**

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  - **member(X,[_|T]) :- member(X,T).**
- **Building a list of integers in range [i,j] (build(from, to, NewList))**
  - **build(I,J,[]) :- I>J.**
  - **build(I,J,[I | Rest]) :- I =< J, N is I+1, build(N,J,Rest).**

List examples

- **Concatenation: concatenation(X, Y, Z)**

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  - **concatenation([], L, L).**
  - **concatenation([X|L1], L2, [X|L3]) :- concatenation(L1, L2, L3).**

List examples

- **Concatenation: concatenation(X, Y, Z)**
  - **concatenation([], L, L).**
  - **concatenation([X|L1], L2, [X|L3]) :- concatenation(L1, L2, L3).**
- **Example:**
  - **concatenation([a,b], [c,d], Y).**

List examples

- Concatenation: concatenation(X, Y, Z)
  - concatenation([], L, L).
  - concatenation([X|L1], L2, [X|L3]) :- concatenation(L1, L2, L3).
- Example:
  - concatenation([a,b], [c,d], Y).

  - X=a, concatenation([X|b], [c,d], [X|Y1]).
  - concatenation([b], [c,d], Y1).
  - X=b, concatenation([X|[]], [c,d], [X|Y2]).
  - concatenation([], [c,d], Y2).

List examples

- **Adding in front: add(X, Y, Z) – add X in front of Y and result into Z**
  - **add(X, L, [X|L]).**

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- **Deletion: del(X, Y, Z) – delete X from Y and store result in Z**

List examples

- **Adding in front: add(X, Y, Z) – add X in front of Y and result into Z**
  - `add(X, L, [X|L]).`
- **Deletion: del(X, Y, Z) – delete X from Y and store result in Z**
  - `del(X, [X|Tail], Tail).`
  - `del(X, [Y|Tail], [Y|Tail1]) :- del(X, Tail, Tail1).`

List examples

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- **Deletion: del(X, Y, Z) – delete X from Y and store result in Z**
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  - **del(X, [Y|Tail], [Y|Tail1]) :- del(X, Tail, Tail1).**

- **Sublist: sublist(S, L) – check whether S is sublist of L**

List examples

- Adding in front: add(X, Y, Z) – add X in front of Y and result into Z
  - add(X, L, [X|L]).

- Deletion: del(X, Y, Z) – delete X from Y and store result in Z
  - del(X, [X|Tail], Tail).
  - del(X, [Y|Tail], [Y|Tail1]) :- del(X, Tail, Tail1).

- Sublist: sublist(S, L) – check whether S is sublist of L
  - sublist(S, L) :- concatenation(L1, L2, L), concatenation(S, L3, L2).

List examples

- Adding in front: add(X, Y, Z) – add X in front of Y and result into Z
  - add(X, L, [X|L]).

- Deletion: del(X, Y, Z) – delete X from Y and store result in Z
  - del(X, [X|Tail], Tail).
  - del(X, [Y|Tail], [Y|Tail1]) :- del(X, Tail, Tail1).

- Sublist: sublist(S, L) – check whether S is sublist of L
  - sublist(S, L) :- concatenation(L1, L2, L), concatenation(S, L3, L2).

- GCD: gcd(X, Y, Z)
  - gcd(X,X,X).
  - gcd(X,Y,Z) :- X < Y, Y1 is Y-X, gcd(X,Y1,Z).

List examples

- **Permutation:**
  - permutation([], []).
  - permutation([X|L], P) :- permutation(L, L1), insert(X, L1, P).

8-queens

- **Solution:**
  - **solution(Queens):- permutation([1,2,3,4,5,6,7,8], Queens), safe(Queens).**

8-queens

- **Solution:**
  - **solution(Queens):- permutation([1,2,3,4,5,6,7,8], Queens), safe(Queens).**
- **Permutation:**
  - **permutation([],[]).**
  - **permutation([Head | Tail], PermList) :-**
    permutation(Tail, Permtail), del(Head, PermList, Permtail).

8-queens

- Solution:
  - solution(Queens):- permutation([1,2,3,4,5,6,7,8], Queens), safe(Queens).
- Permutation:
  - permutation([],[]).
  - permutation([Head | Tail], PermList) :-
    permutation(Tail, Permtail), del(Head, PermList, Permtail).
- Safe:
  - safe([]).
  - safe([Queen|Others]) :- safe(Others), noattack(Queen, Others, 1).

8-queens

- Solution:
  - solution(Queens):- permutation([1,2,3,4,5,6,7,8], Queens), safe(Queens).
- Permutation:
  - permutation([],[]).
  - permutation([Head | Tail], PermList) :-
    permutation(Tail, Permtail), del(Head, PermList, Permtail).
- Safe:
  - safe([]).
  - safe([Queen|Others]) :- safe(Others), noattack(Queen, Others, 1).
- No-attack:
  - noattack(_,[],_).
  - noattack(Y,[Y1|Ylist],Xdist) :-
    Y-Y1 =\= Xdist, Y1-Y=\= Xdist, Dist is Xdist+1, noattack(Y, Ylist, Dist).

Source of inefficiency

- f(X, normal) :- X < 3.
- f(X, alert1) :- 3 =< X, X < 6.
- f(X, alert2) :- 6 =< X.
- Query1: f(2, Y), Y = alert1.

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- f(X, normal) :- X < 3, !.
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- Query2: f(7, Y).

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- f(X, normal) :- X < 3, !.
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- f(X, alert2) :- 6 =< X.
- Query2: f(7, Y).

Efficient version

- f(X, normal) :- X < 3, !.
- f(X, alert1) :- X < 6, !.
- f(X, alert2) :- 6 =< X.

Cuts – controlled backtracking

```
C :- P, Q, R, !, S, T, U.
C :- V.
A :- B, C, D.
?- A.
```

- Backtracking within the goal list P, Q, R
- As soon as the cut is reached:
  - All alternatives of P, Q, R are suppressed
  - The clause C :- V will also be discarded
  - Backtracking is possible within S, T, U.
  - No effect for rule A, that is backtracking within B, C, D remains active.

Cuts

- del_duplicates([], []).
- del_duplicates([Head | Tail], Result) :-

  member(Head, Tail), !, del_duplicates(Tail, Result). % R1
- del_duplicates([Head | Tail], [Head | Result]) :- del_duplicates(Tail, Result). % R2

Cuts - Example

- If X >= Y then Max=X, otherwise Max=Y
  - max(X, Y, X) :- X>=Y, !.
  - max(X, Y, Y).
- Adding an element into a list without duplication
  - add(X, L, L) :- member(X,L), !.
  - add(X, L, [X|L]).

Thank you!