

CS5201: Advanced Artificial Intelligence

SAT Solvers



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Introduction

- SAT is one of the central problems in computer science community that has both theoretical as well as practical challenges
- This was the first NP-Complete problem
- Wide variety of application domains - formal verification, test pattern generation, planning, scheduling, time tabling, etc.
- Provides a generic framework for combinatorial reasoning and search platform
- It is based upon propositional logic (Boolean logic)
- CSP problems can be mapped to SAT
- There exists good open-source industrial strength SAT solvers

SAT problems

- **Propositions** - $\mathcal{P} = \{a, b, c, \dots\}$
- **Literals** - $\{a, \neg a, b, \neg b, \dots\}$
- **Clause** - $C_1 = (a \vee b \vee \neg c), C_2 = (\neg a \vee b \vee \neg d), \dots$
 - **Clause is disjunction of literals**
- **Formula** - $\mathcal{F} = C_1 \wedge C_2 \wedge \dots$
 - **Conjunctive normal form (CNF)**
- **Goal is to find an assignment (interpretation) to the propositions such that \mathcal{F} is true**
 - \mathcal{F} is **satisfiable** if there exists at least one valid interpretation
 - \mathcal{F} is **unsatisfiable** if there exists none

SAT tools

- Very good open-source SAT solvers are available
 - MiniSAT
 - zChaff
 - CaDiCaL
 - Glucose
 - Lingeling
 - PicoSAT
 - Cryptominisat
 - Rsat
 - Riss
 - many others
- `http://www.satcompetition.org/`

Input format - DIMACS

- There is standard format to specify clauses and its literals
- To specify comments

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```

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p cnf 3 4
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```
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```
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- To specify CNF

```
c list_of_literals 0
```

```
1 -2 3 0
```

```
2 4 0
```

```
-3 0
```

```
-1 2 3 -4 0
```

Output format

- Outputs from a SAT solver are - SATISFIABLE / UNSATISFIABLE, an assignment of Boolean variables
- Typically it will be as follows

SAT

-1 2 -3 4 0

- The last line needs to be interpreted as follows: $\neg a \wedge b \wedge \neg c \wedge d$
- There may be additional messages to provide information on resource usage, statistics, etc.

SAT modeling: Propositional logic - 1

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 - If \mathcal{M} is satisfiable then so is $\mathcal{F} \wedge \bar{\mathcal{G}}$

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```
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-1 2 0
1 0
-2 0
```

UNSATISFIABLE

SAT modeling: Propositional logic - 2

- If Rajat is the Director then Rajat is well known. Rajat is not the Director. So, Rajat is not well known.
- Propositions: a : Rajat is the Director, b : Rajat is well known.
- Formula (\mathcal{F}): $a \rightarrow b, \neg a$
- Goal (\mathcal{G}): $\neg b$

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SATISFIABLE

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- Target: Can we prove that the unicorn is mythical? Magical? Horned?
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 - $\neg a \rightarrow (b \wedge c)$

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p cnf 5 7

2 4 0

-1 -2 0

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-4 5 0

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-4 5 0

-1 0 // a - SAT

-5 0 // e - UNSAT

-4 0 // d - UNSAT

SAT modeling: Propositional logic - 4

- Holmes owns two suits: one black and one tweed. He always wears either a tweed suit or sandals. Whenever he wears his tweed suit and a purple shirt, he chooses to not wear a tie. He never wears the tweed suit unless he is also wearing either a purple shirt or sandals. Whenever he wears sandals, he also wears a purple shirt. Yesterday, Holmes wore a bow tie.
- Target: What else did he wear?
- Propositions: 1-*bs*-black suit, 2-*ts*-tweed suit, 3-*s*-sandals, 4-*ps*-purple shirt, 5-*t*-tie

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 - $ts \vee s$

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 - $\neg bs \vee \neg ts$
 - $ts \vee s$
 - $(ts \wedge ps) \rightarrow \neg t$

SAT modeling: Propositional logic - 4

- Holmes owns two suits: one black and one tweed. He always wears either a tweed suit or sandals. Whenever he wears his tweed suit and a purple shirt, he chooses to not wear a tie. He never wears the tweed suit unless he is also wearing either a purple shirt or sandals. Whenever he wears sandals, he also wears a purple shirt. Yesterday, Holmes wore a bow tie.
- Target: What else did he wear?
- Propositions: 1-*bs*-black suit, 2-*ts*-tweed suit, 3-*s*-sandals, 4-*ps*-purple shirt, 5-*t*-tie
- Formula (\mathcal{F}):
 - $\neg bs \vee \neg ts$
 - $ts \vee s$
 - $(ts \wedge ps) \rightarrow \neg t \equiv (\neg ts \vee \neg ps \vee \neg t)$

SAT modeling: Propositional logic - 4

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 - $ts \rightarrow (ps \vee s)$

SAT modeling: Propositional logic - 4

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SAT modeling: Propositional logic - 4

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 - $ts \rightarrow (ps \vee s) \equiv (\neg ts \vee ps \vee s)$
 - $s \rightarrow ps$

SAT modeling: Propositional logic - 4

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 - $s \rightarrow ps \equiv (\neg s \vee ps)$

SAT modeling: Propositional logic - 4

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 - $ts \rightarrow (ps \vee s) \equiv (\neg ts \vee ps \vee s)$ • $s \rightarrow ps \equiv (\neg s \vee ps)$ • t

SAT modeling: Propositional logic - 4

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 - $\neg bs \vee \neg ts$ • $ts \vee s$ • $(ts \wedge ps) \rightarrow \neg t \equiv (\neg ts \vee \neg ps \vee \neg t)$
 - $ts \rightarrow (ps \vee s) \equiv (\neg ts \vee ps \vee s)$ • $s \rightarrow ps \equiv (\neg s \vee ps)$ • t
- Goal (\mathcal{G}): All satisfying solutions

SAT modeling: Propositional logic - 4

- **Propositions:** 1-*bs*-black suit, 2-*ts*-tweed suit, 3-*s*-sandals, 4-*ps*-purple shirt, 5-*t*-tie
- **Formula (\mathcal{F}):**
 - $\neg bs \vee \neg ts$
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 - $s \rightarrow ps \equiv (\neg s \vee ps)$
 - t
- **Goal (\mathcal{G}):** All satisfying solutions
- **SAT modeling**

SAT modeling: Propositional logic - 4

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- $ts \rightarrow (ps \vee s) \equiv (\neg ts \vee ps \vee s)$
- $s \rightarrow ps \equiv (\neg s \vee ps)$
- t

- **Goal (\mathcal{G}):** All satisfying solutions

- **SAT modeling**

```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
```

SAT modeling: Propositional logic - 4

- **Propositions:** 1-*bs*-black suit, 2-*ts*-tweed suit, 3-*s*-sandals, 4-*ps*-purple shirt, 5-*t*-tie

- **Formula (\mathcal{F}):**

- $\neg bs \vee \neg ts$
- $ts \vee s$
- $(ts \wedge ps) \rightarrow \neg t \equiv (\neg ts \vee \neg ps \vee \neg t)$
- $ts \rightarrow (ps \vee s) \equiv (\neg ts \vee ps \vee s)$
- $s \rightarrow ps \equiv (\neg s \vee ps)$
- t

- **Goal (\mathcal{G}):** All satisfying solutions

- **SAT modeling**

```
p cnf 5 6
-1 -2 0
2 3 0
-2 -4 -5 0
-2 4 3 0
-3 4 0
5 0
```

SAT: -1 -2 3 4 5 0

SAT modeling: Propositional logic - 4

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- $(ts \wedge ps) \rightarrow \neg t \equiv (\neg ts \vee \neg ps \vee \neg t)$
- $ts \rightarrow (ps \vee s) \equiv (\neg ts \vee ps \vee s)$
- $s \rightarrow ps \equiv (\neg s \vee ps)$
- t

- Goal (\mathcal{G}): All satisfying solutions

- SAT modeling

p cnf 5 6

-1 -2 0

2 3 0

-2 -4 -5 0

-2 4 3 0

-3 4 0

5 0

SAT: -1 -2 3 4 5 0

Add: 1 2 -3 -4 -5 0

SAT modeling: Propositional logic - 4

- **Propositions:** 1-*bs*-black suit, 2-*ts*-tweed suit, 3-*s*-sandals, 4-*ps*-purple shirt, 5-*t*-tie

- **Formula (\mathcal{F}):**

- $\neg bs \vee \neg ts$
- $ts \vee s$
- $(ts \wedge ps) \rightarrow \neg t \equiv (\neg ts \vee \neg ps \vee \neg t)$
- $ts \rightarrow (ps \vee s) \equiv (\neg ts \vee ps \vee s)$
- $s \rightarrow ps \equiv (\neg s \vee ps)$
- t

- **Goal (\mathcal{G}):** All satisfying solutions

- **SAT modeling**

p cnf 5 6

-1 -2 0

2 3 0

-2 -4 -5 0

-2 4 3 0

-3 4 0

5 0

SAT: -1 -2 3 4 5 0

Add: 1 2 -3 -4 -5 0

SAT: 1 -2 3 4 5 0

SAT modeling: Propositional logic - 4

- Propositions: 1-*bs*-black suit, 2-*ts*-tweed suit, 3-*s*-sandals, 4-*ps*-purple shirt, 5-*t*-tie

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- $s \rightarrow ps \equiv (\neg s \vee ps)$
- t

- Goal (\mathcal{G}): All satisfying solutions

- SAT modeling

p cnf 5 6

-1 -2 0

2 3 0

-2 -4 -5 0

-2 4 3 0

-3 4 0

5 0

SAT: -1 -2 3 4 5 0

Add: 1 2 -3 -4 -5 0

SAT: 1 -2 3 4 5 0

Add: -1 2 -3 -4 -5 0

SAT modeling: Propositional logic - 4

- **Propositions:** 1-*bs*-black suit, 2-*ts*-tweed suit, 3-*s*-sandals, 4-*ps*-purple shirt, 5-*t*-tie

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- $\neg bs \vee \neg ts$
- $ts \vee s$
- $(ts \wedge ps) \rightarrow \neg t \equiv (\neg ts \vee \neg ps \vee \neg t)$
- $ts \rightarrow (ps \vee s) \equiv (\neg ts \vee ps \vee s)$
- $s \rightarrow ps \equiv (\neg s \vee ps)$
- t

- **Goal (\mathcal{G}):** All satisfying solutions

- **SAT modeling**

p cnf 5 6

-1 -2 0

2 3 0

-2 -4 -5 0

-2 4 3 0

-3 4 0

5 0

SAT: -1 -2 3 4 5 0

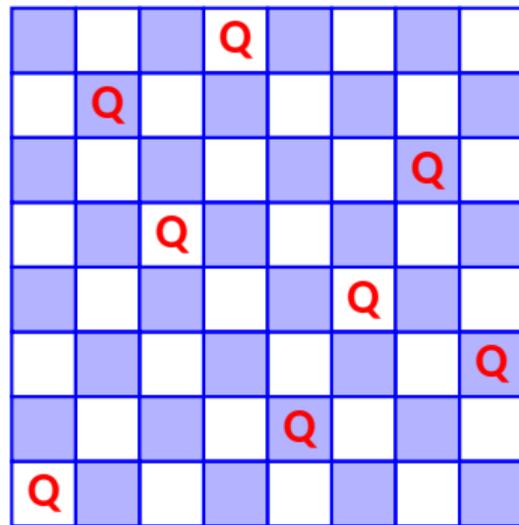
Add: 1 2 -3 -4 -5 0

SAT: 1 -2 3 4 5 0

Add: -1 2 -3 -4 -5 0

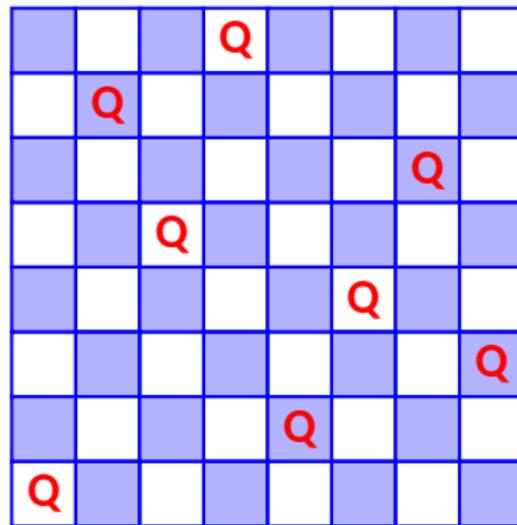
UNSAT

SAT modeling: 8-queens



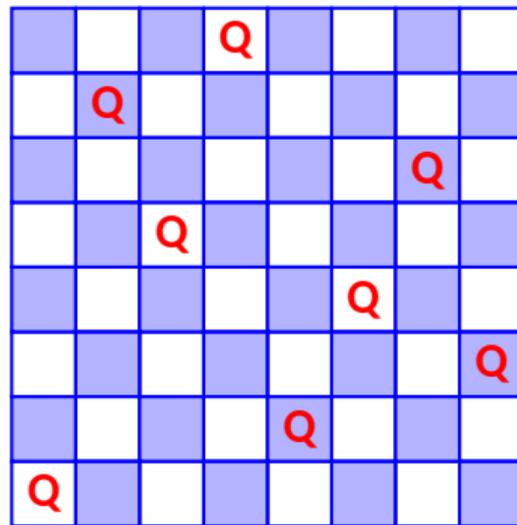
SAT modeling: 8-queens

- Define x_{ij} as 1 if (i, j) th cell contains a queen



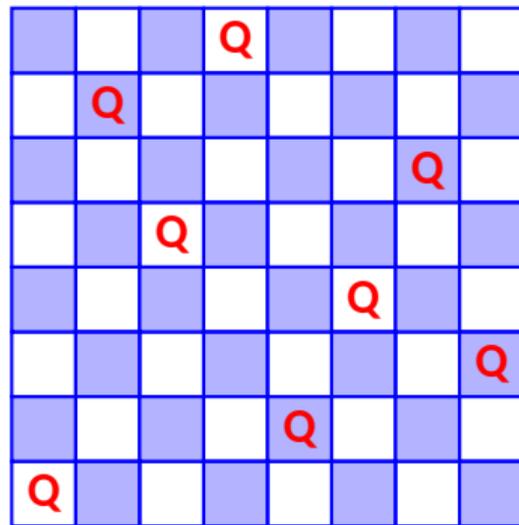
SAT modeling: 8-queens

- Define x_{ij} as 1 if (i, j) th cell contains a queen
- Constraints



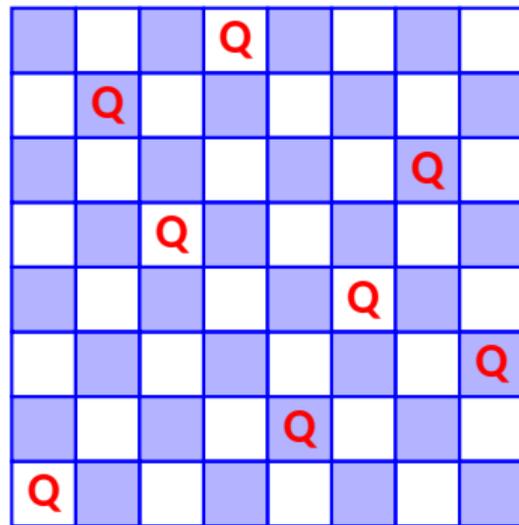
SAT modeling: 8-queens

- Define x_{ij} as 1 if (i, j) th cell contains a queen
- Constraints
 - $x_{ij} \rightarrow \neg x_{ij'}$ (row)
 - $x_{ij} \rightarrow \neg x_{i'j}$ (column)
 - $x_{ij} \rightarrow \neg x_{(i+k)(j+k)}$ (diagonal)
 - $x_{ij} \rightarrow \neg x_{(i+k)(j-k)}$ (diagonal)



SAT modeling: 8-queens

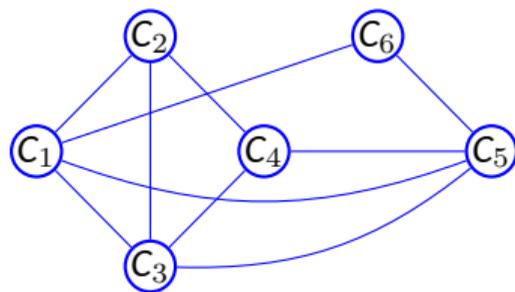
- Define x_{ij} as 1 if (i, j) th cell contains a queen
- Constraints
 - $x_{ij} \rightarrow \neg x_{ij'}$ (row)
 - $x_{ij} \rightarrow \neg x_{i'j}$ (column)
 - $x_{ij} \rightarrow \neg x_{(i+k)(j+k)}$ (diagonal)
 - $x_{ij} \rightarrow \neg x_{(i+k)(j-k)}$ (diagonal)
 - $\bigvee_i x_{ij}$ (column)
 - $\bigvee_j x_{ij}$ (row)



SAT modeling: Graph coloring

Student	Subjects
S_1	C_1, C_2, C_3
S_2	C_2, C_3, C_4
S_3	C_3, C_4
S_4	C_3, C_4, C_5
S_5	C_1, C_5, C_6

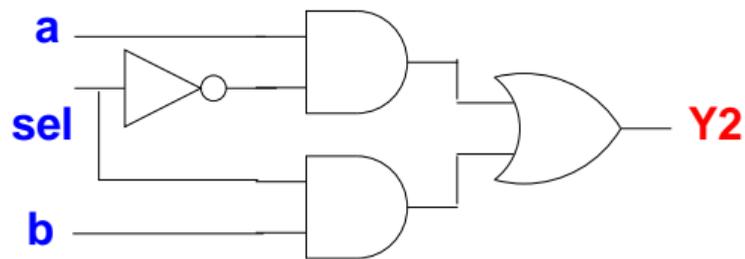
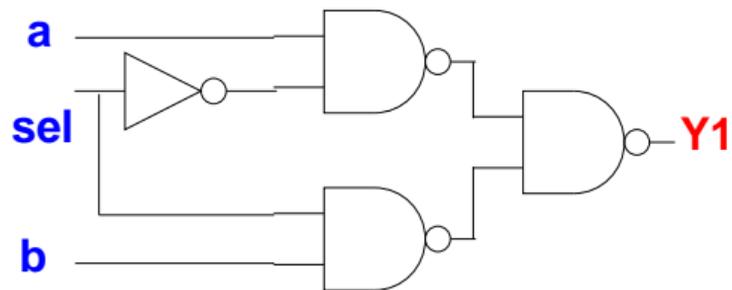
Is it possible to conduct all these exams in 3 days assuming one exam per day?



- Decision variables? Constraints?

SAT modeling: Circuit equivalence

- Are these two circuits equivalent?



SAT modeling: Sudoku

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- Define x_{ijk} as 1 if (i, j) th cell contains k

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- Define x_{ijk} as 1 if (i, j) th cell contains k
- Constraints:

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- Define x_{ijk} as 1 if (i, j) th cell contains k
- Constraints:
 - $x_{ijk} \rightarrow \neg x_{ij'k} \quad \forall i, k, j \neq j'$ (same row)

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

- Define x_{ijk} as 1 if (i, j) th cell contains k
- Constraints:
 - $x_{ijk} \rightarrow \neg x_{ij'k} \quad \forall i, k, j \neq j'$ (same row)
 - $x_{ijk} \rightarrow \neg x_{i'jk} \quad \forall j, k, i \neq i'$ (same column)

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

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 - $x_{ijk} \rightarrow \neg x_{i'jk} \quad \forall j, k, i \neq i'$ (same column)
 - $x_{ijk} \rightarrow \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ - every block

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

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 - $x_{ijk} \rightarrow \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ - every block

- $\bigvee_j x_{ijk} \quad \forall i, k$ (row)

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

SAT modeling: Sudoku

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 - $x_{ijk} \rightarrow \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ - every block

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

- $\bigvee_j x_{ijk} \quad \forall i, k$ (row)
- $\bigvee_i x_{ijk} \quad \forall j, k$ (column)

SAT modeling: Sudoku

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 - $x_{ijk} \rightarrow \neg x_{i'jk} \quad \forall j, k, i \neq i'$ (same column)
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		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

- $\bigvee_j x_{ijk} \quad \forall i, k$ (row)
- $\bigvee_i x_{ijk} \quad \forall j, k$ (column)
- $x_{ijk} \rightarrow \neg x_{ijk'} \quad \forall i, j, k \neq k'$ (same cell)

SAT modeling: Sudoku

- Define x_{ijk} as 1 if (i, j) th cell contains k

- Constraints:

- $x_{ijk} \rightarrow \neg x_{ij'k} \quad \forall i, k, j \neq j'$ (same row)
- $x_{ijk} \rightarrow \neg x_{i'jk} \quad \forall j, k, i \neq i'$ (same column)
- $x_{ijk} \rightarrow \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ - every block

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

- $\bigvee_j x_{ijk} \quad \forall i, k$ (row)

- $\bigvee_i x_{ijk} \quad \forall j, k$ (column)

- $x_{ijk} \rightarrow \neg x_{ij'k'} \quad \forall i, j, k \neq k'$ (same cell)

- $\bigvee_{i,j} x_{ijk} \quad \forall k$ every block

SAT modeling: Sudoku

- Define x_{ijk} as 1 if (i, j) th cell contains k

- Constraints:

- $x_{ijk} \rightarrow \neg x_{ij'k} \quad \forall i, k, j \neq j'$ (same row)
- $x_{ijk} \rightarrow \neg x_{i'jk} \quad \forall j, k, i \neq i'$ (same column)
- $x_{ijk} \rightarrow \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ - every block

		3		2		6		
9			3		5			1
		1	8		6	4		
		8	1		2	9		
7								8
		6	7		8	2		
		2	6		9	5		
8			2		3			9
		5		1		3		

- $\bigvee_j x_{ijk} \quad \forall i, k$ (row)

- $\bigvee_i x_{ijk} \quad \forall j, k$ (column)

- $x_{ijk} \rightarrow \neg x_{ijk'} \quad \forall i, j, k \neq k'$ (same cell)

- $\bigvee_{i,j} x_{ijk} \quad \forall k$ every block

- $\bigvee_k x_{ijk} \quad \forall i, j$ (every cell)

SAT modeling: Sudoku

- Define x_{ijk} as 1 if (i, j) th cell contains k

- Constraints:

- $x_{ijk} \rightarrow \neg x_{ij'k} \quad \forall i, k, j \neq j'$ (same row)
- $x_{ijk} \rightarrow \neg x_{i'jk} \quad \forall j, k, i \neq i'$ (same column)
- $x_{ijk} \rightarrow \neg x_{i'j'k} \quad \forall k, i \neq i', j \neq j', 1 \leq i, i', j, j' \leq 3$ - every block

	3	2	6		
9		3	5		1
	1	8	6	4	
	8	1	2	9	
7					8
	6	7	8	2	
	2	6	9	5	
8		2	3		9
	5	1	3		

- $\bigvee_j x_{ijk} \quad \forall i, k$ (row)

- $\bigvee_i x_{ijk} \quad \forall j, k$ (column)

- $x_{ijk} \rightarrow \neg x_{ij'k'} \quad \forall i, j, k \neq k'$ (same cell)

- $\bigvee_{i,j} x_{ijk} \quad \forall k$ every block

- $\bigvee_k x_{ijk} \quad \forall i, j$ (every cell)

- x_{133}, x_{176}, \dots

SAT modeling: Langford sequence

- Given the bag of numbers $\{1, 1, 2, 2, 3, 3, \dots, n, n\}$, can they be arranged in a sequence $L(n)$ such that for $1 \leq i \leq n$ there are i numbers between the two occurrences of i ?
 - $L(4) = 41312432$
 - $L(3) = ?$

SAT modeling: Langford sequence

- Given the bag of numbers $\{1, 1, 2, 2, 3, 3, \dots, n, n\}$, can they be arranged in a sequence $L(n)$ such that for $1 \leq i \leq n$ there are i numbers between the two occurrences of i ?
 - $L(4) = 41312432$
 - $L(3) = ?$

	1	2	3	4	5	6
x_1	1		1			
x_2		1		1		
x_3			1		1	
x_4				1		1
x_5	2			2		
x_6		2			2	
x_7			2			2
x_8	3				3	
x_9		3				3

SAT modeling: Langford sequence

- Given the bag of numbers $\{1, 1, 2, 2, 3, 3, \dots, n, n\}$, can they be arranged in a sequence $L(n)$ such that for $1 \leq i \leq n$ there are i numbers between the two occurrences of i ?
 - $L(4) = 41312432$
 - $L(3) = ?$
- Constraints:
 - $x_1 \vee x_2 \vee x_3 \vee x_4$
 - $x_k \rightarrow \neg x_{k'} \quad 1 \leq k < k' \leq 4$
 - Similarly for the other numbers
 - $x_1 \vee x_5 \vee x_8$
 - $x_1 \rightarrow \neg x_5, \dots$
 - Similarly for the other columns

	1	2	3	4	5	6
x_1	1		1			
x_2		1		1		
x_3			1		1	
x_4				1		1
x_5	2			2		
x_6		2			2	
x_7			2			2
x_8	3				3	
x_9		3				3

SAT modeling: Electric vehicle charge scheduling

- Consider a parking area that has the facility to charge electric vehicles. There are m number of ports to charge the vehicles. Each port can charge one vehicle at a time. Let us assume that there are n number of vehicles. Each vehicle has an arrival time (a_i) in the parking area and an departure time (d_i). While the vehicle is in the parking area, it needs to be charged uninterruptedly for a given duration (e_i). Given a set of vehicles and their arrival and departure time, does there exist a schedule such that each vehicle can be charged for its stipulated duration while it is in the parking area? A sample input will look as follows - $m = 10$ and

Vehicle	Arrival time	Departure time	Charging time
1	4	10	3
2	7	20	6
3	8	27	10
⋮	⋮	⋮	⋮

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- Can there be other encoding schemes to model the problem?

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- Develop a SAT based formulation to model the problem
- Can there be other encoding schemes to model the problem?
- Which encoding scheme is better?
- Explore the performance of different encoding schemes using various solvers.

Thank you!