CS5201: Advanced Artificial Intelligence

Heuristic search

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State space modeling

- **State or configurations**
  - A set of variables which define a state or configuration
  - Domains for every variable and constraints among variables to define a valid configuration
- **State transformation rules or moves**
  - A set of rules which define which are the valid set of next state of a given state
  - It also indicates who can make these moves (OR Nodes, AND nodes, etc)
- **State space or implicit graph**
  - The Complete Graph produced out of the state transformation rules.
  - Typically too large to store. Could be Infinite.
- **Start and goal states**
- **Solutions, costs**
  - Depending on the problem formulation, it can be a PATH from Start to Goal or a Sub-graph of And-ed Nodes
- **Search algorithms**
  - Intelligently explore the Implicit Graph or State Space by examining only a small sub-set to find the solution
  - To use Domain Knowledge or HEURISTICS to try and reach Goals faster

Example problems

image source: Wikipedia
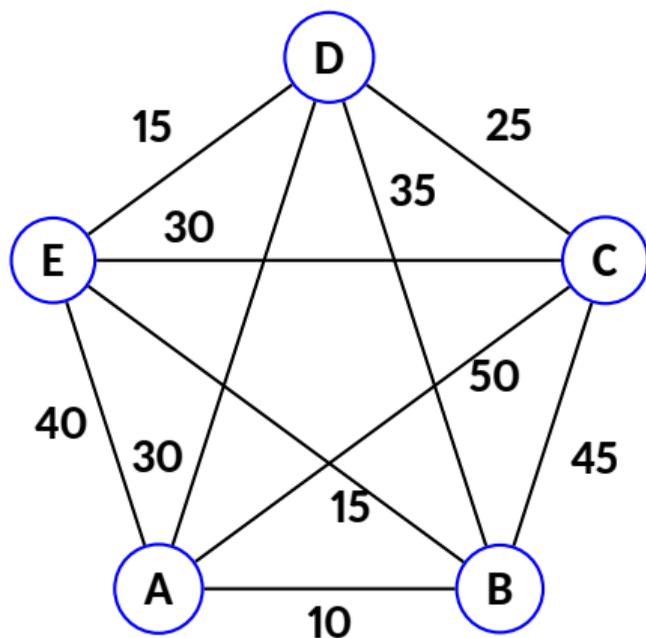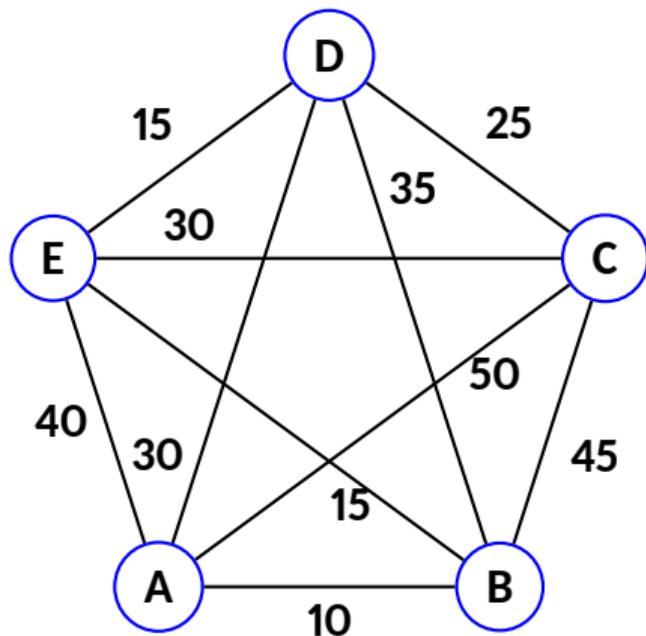
Example problems

Sliding puzzle

1	3	4
2		7
5	8	6

Start state

1	2	3
4	5	6
7	8	

Goal state

image source: Wikipedia

Example problems
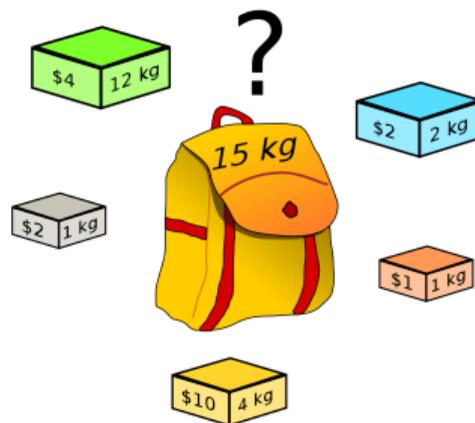
Searching implicit graph: Algorithms

- **Basic algorithms** — Depth-First (DFS), Breadth-First (BFS), Iterative deepening (IDS)
- **Cost-based algorithms** — Depth-First Branch-and-Bound, Best First Search, Best-First Iterative deepening
- **Widely used algorithms** — A* and IDA* (OR graphs), AO* (AND/OR graphs), Alpha-beta pruning (Game-trees)

Searching state space graph
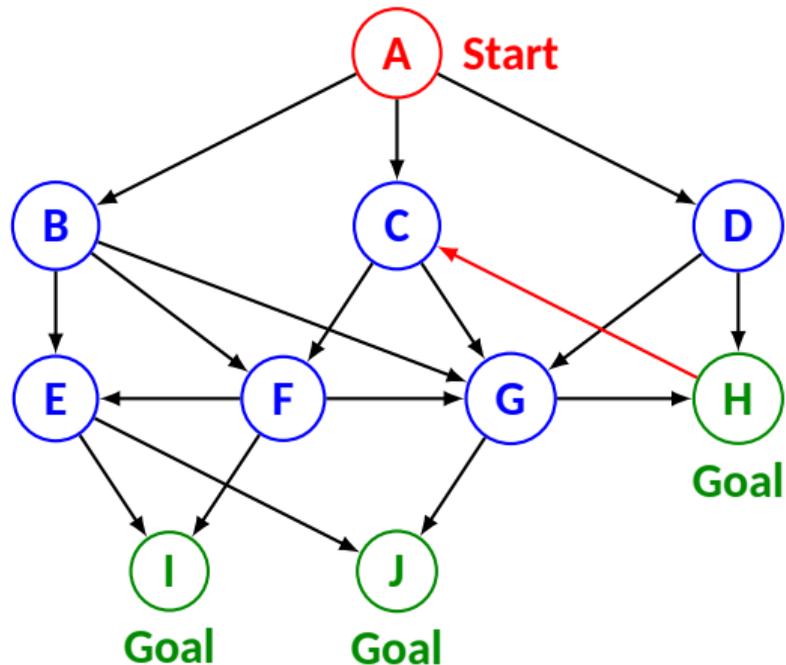
- Depth-first search
- Breadth-first search
- Iterative deepening search

Basic algorithms

1. [Initialize] Initially the OPEN List contains the Start Node s. CLOSED List is Empty.

2. [Select] Select the first Node n on the OPEN List. If OPEN is empty, Terminate

3. [Goal Test] If n is Goal, then decide on Termination or Continuation / Cost Updation

4. [Expand]
   a. Generate the successors $n_1, n_2, \ldots, n_k$, of node n, based on the State Transformation Rules
   b. Put n in LIST CLOSED
   c. For each n_i, not already in OPEN or CLOSED List, put n_i in the FRONT (for DFS) / END (for BFS) of OPEN List
   d. For each n_i already in OPEN or CLOSED decide based on cost of the paths

5. [Continue] Go to Step 2

Algorithm IDS Performs DFS level by level iteratively (DFS(1), DFS(2), ...and so on)

Searching state space graph with edge cost
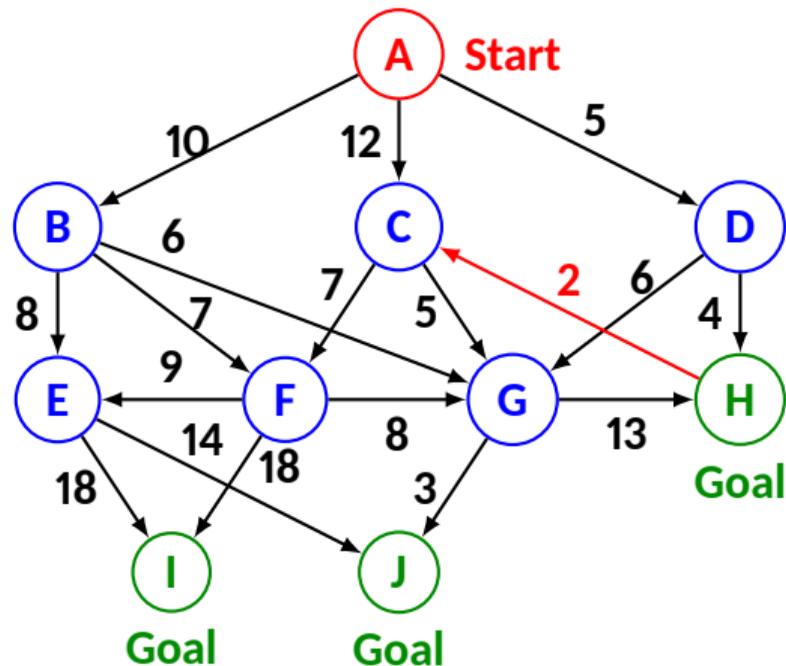
Cost ordered search:
- DFBB
- Best first search
- Best first IDS
- Use of heuristic estimates: A*, AO*

Heuristic search

- State or configurations
- State transformation rules or moves
- State space or implicit graph
- Start and goal states
- Solutions, costs

Heuristic search

- **State or configurations**
- **State transformation rules or moves**
- **State space or implicit graph**
- **Start and goal states**
- **Solutions, costs**
- **Heuristics**
  - **Estimates of cost from a given state to goal. This, along with the current cost of the path from start till now is used to guide the search**
- **Heuristic search algorithms**
  - **Algorithm A*, Depth-First Branch & Bound, IDA*, AO*, Alpha-Beta, etc.**
- $g(n)$ – **current cost of the node from start**
- $h(n)$ – **estimated cost of the best path from n to goal**
- $f(n) = g(n) + h(n)$ – **heuristic estimate**

Example of heuristics: 8 puzzle problem

1	3	4
2		7
5	8	6

Start state

1	2	3
4	5	6
7	8	

Goal state

Example of heuristics: 8 puzzle problem

● **Heuristic 1: Number of misplaced tiles**

1	3	4
2		7
5	8	6

Start state

1	2	3
4	5	6
7	8	

Goal state

Example of heuristics: 8 puzzle problem

- **Heuristic 1: Number of misplaced tiles**
- **Heuristic 2: Sum of tile distance from goal**

1	3	4
2		7
5	8	6

Start state

1	2	3
4	5	6
7	8	

Goal state

OR Graph: Travelling salesperson problem
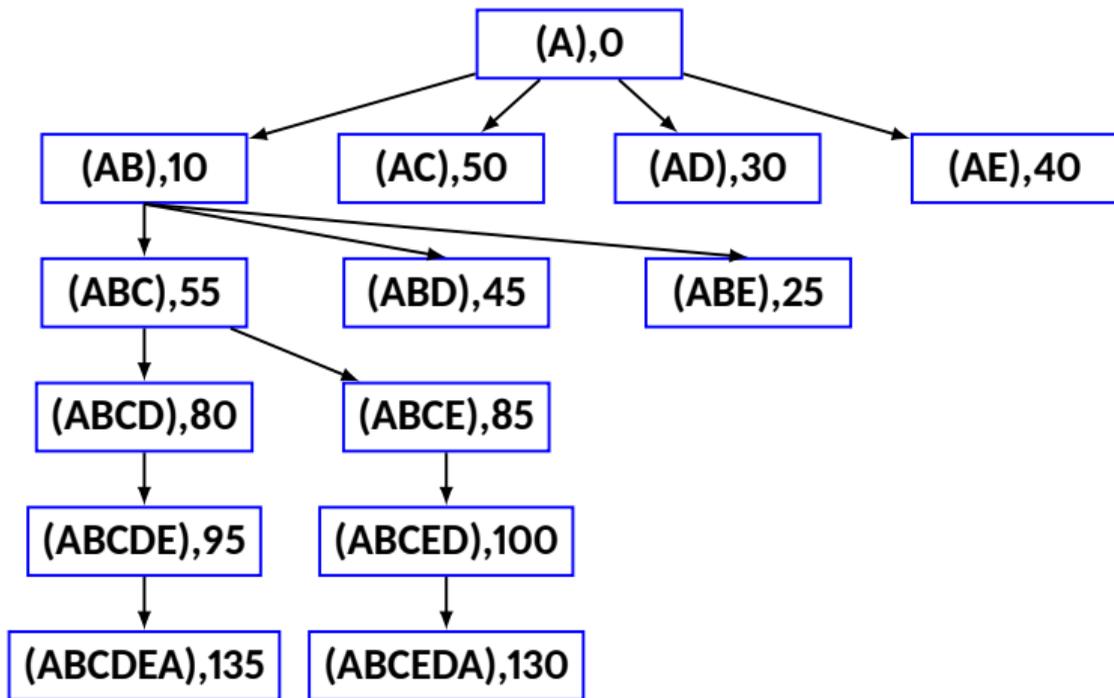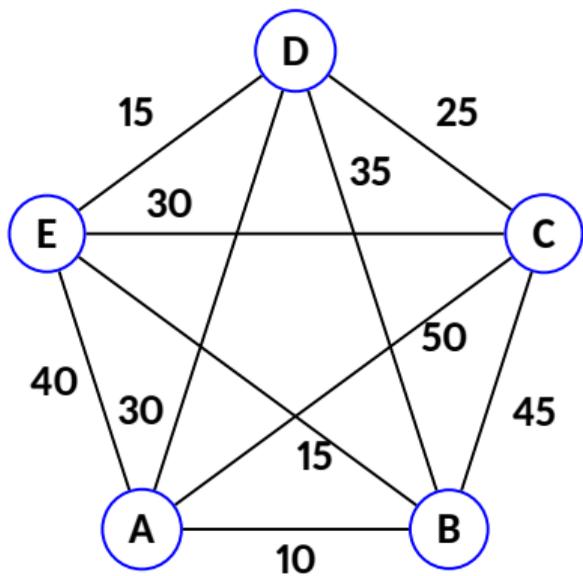
OR Graph: Travelling salesperson problem
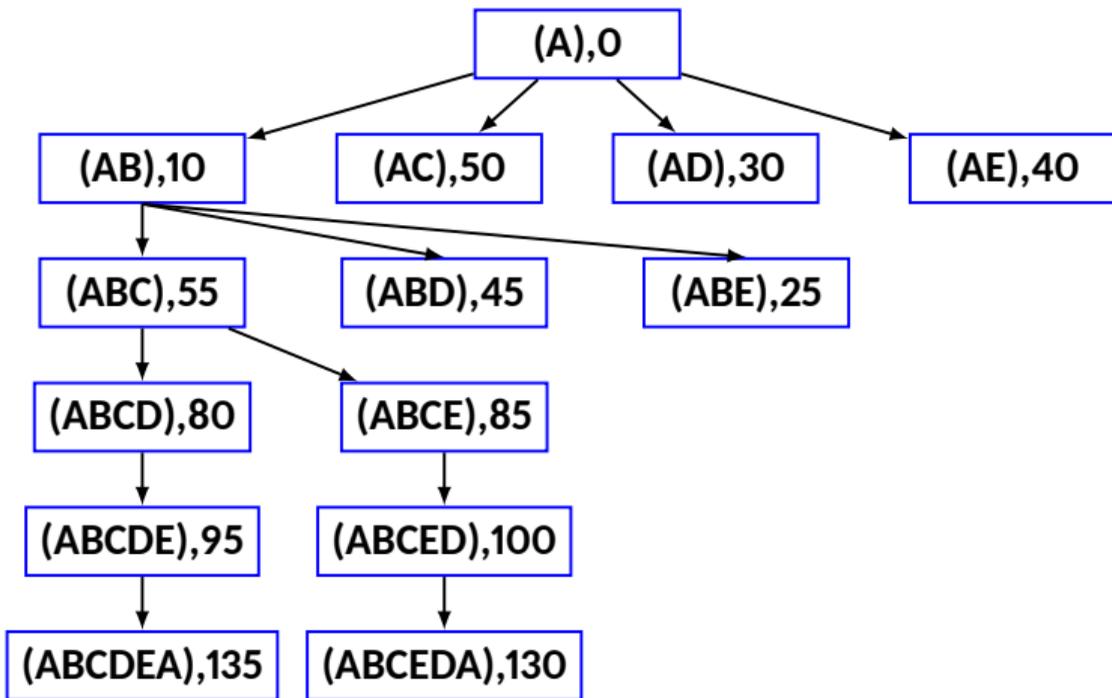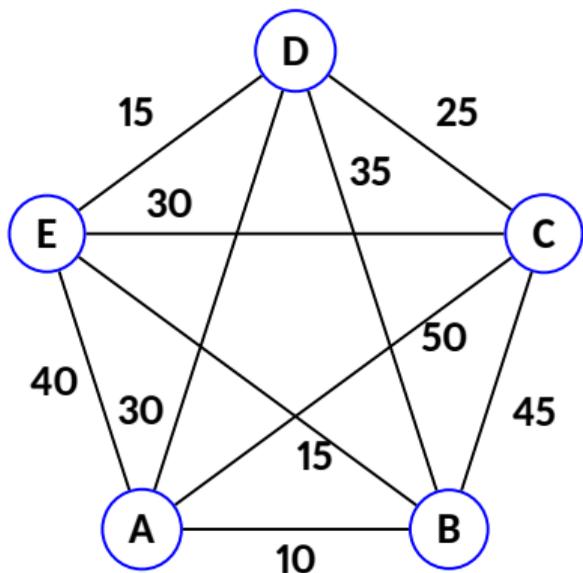
- Heuristic 1: Shortest path

OR Graph: Travelling salesperson problem
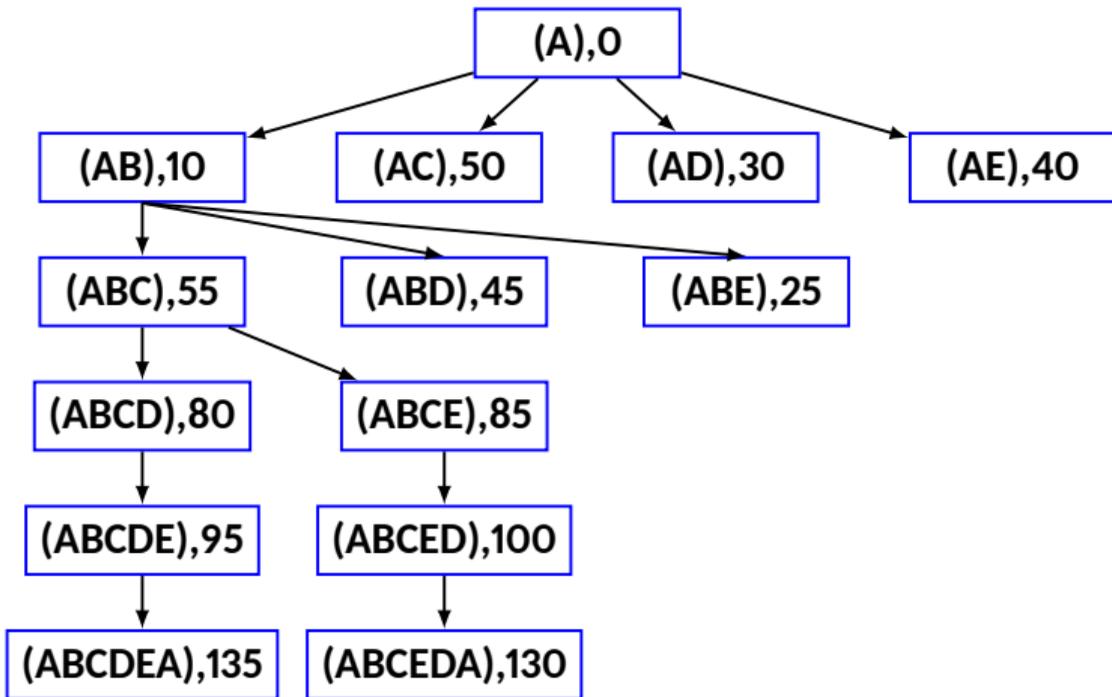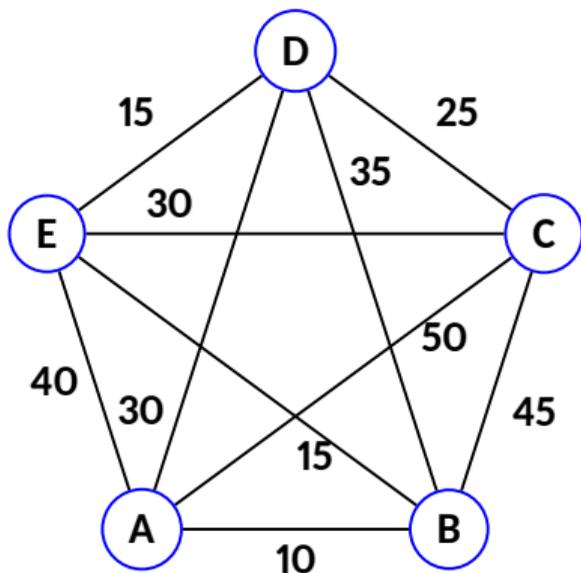
- Heuristic 1: Shortest path
- Heuristic 2: Minimum spanning tree

Searching implicit graph: Edge & Heuristic costs
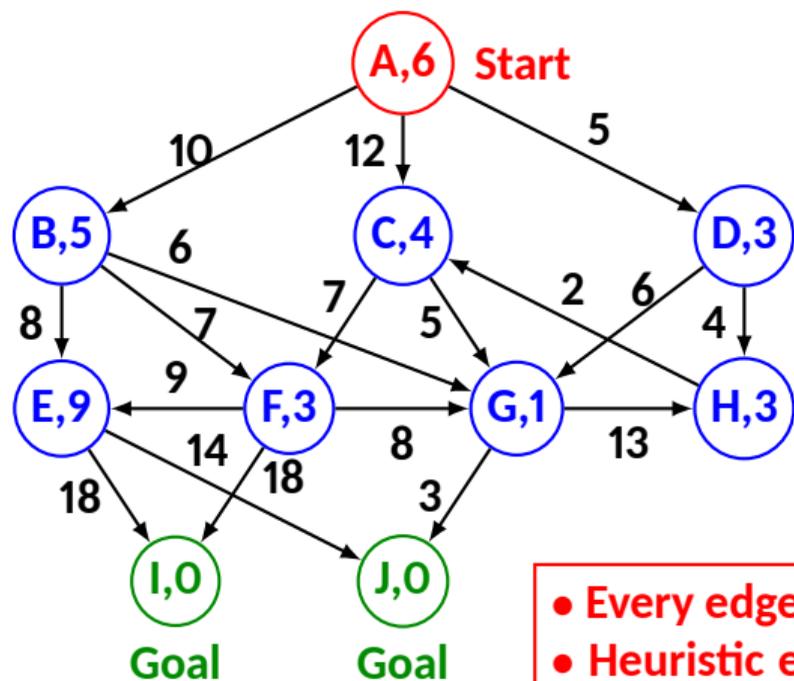
Cost ordered search:
- DFBB
- A* - Best first search
- Best first IDS - IDA*
- Use of heuristic estimates: A*, AO*

- Every edge (n, m) has a cost $c(n, m) > 0$
- Heuristic estimate: $h(n) \geq 0$ at every node is the estimated cost of the minimum cost path from node n to goal

Algorithm A*

- **Each Node n in the algorithm has a cost $g(n)$ and a heuristic estimate $h(n)$, $f(n) = g(n) + h(n)$. Assume all $c(n, m) > 0$**

1. **[Initialize] Initially the OPEN List contains the Start Node s. $g(s) = 0$, $f(s) = h(s)$. CLOSED List is Empty.**
2. **[Select] Select the Node n on the OPEN List with minimum $f(n)$. If OPEN is empty, Terminate with Failure**
3. **[Goal Test, Terminate] If n is Goal, then Terminate with Success and path from s to n.**
4. **[Expand]**
    a. **Generate the successors $n_1, n_2, \ldots, n_k$, of node n, based on the State Transformation Rules**
    b. **Put n in CLOSED List**
    c. **For each n_i, not already in OPEN or CLOSED List, compute $g(n_i) = g(n) + c(n, n_i)$, $f(n_i) = g(n_i) + h(n_i)$, Put n_i in the OPEN List**
    d. **For each n_i already in OPEN, if $g(n_i) > g(n) + c(n, n_i)$, then revise costs as: $g(n_i) = g(n) + c(n, n_i)$, $f(n_i) = g(n_i) + h(n_i)$**
5. **[Continue] Go to Step 2**

Execution of A*
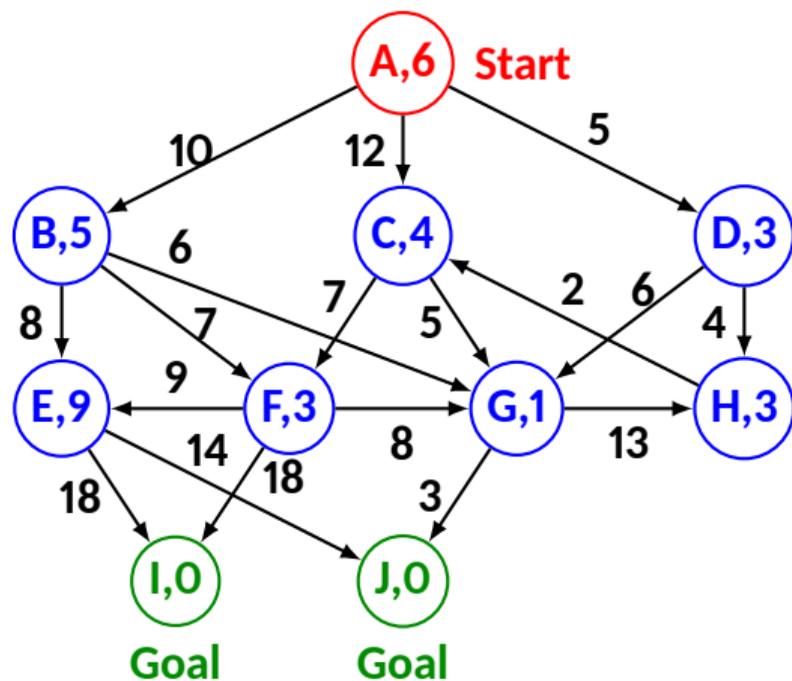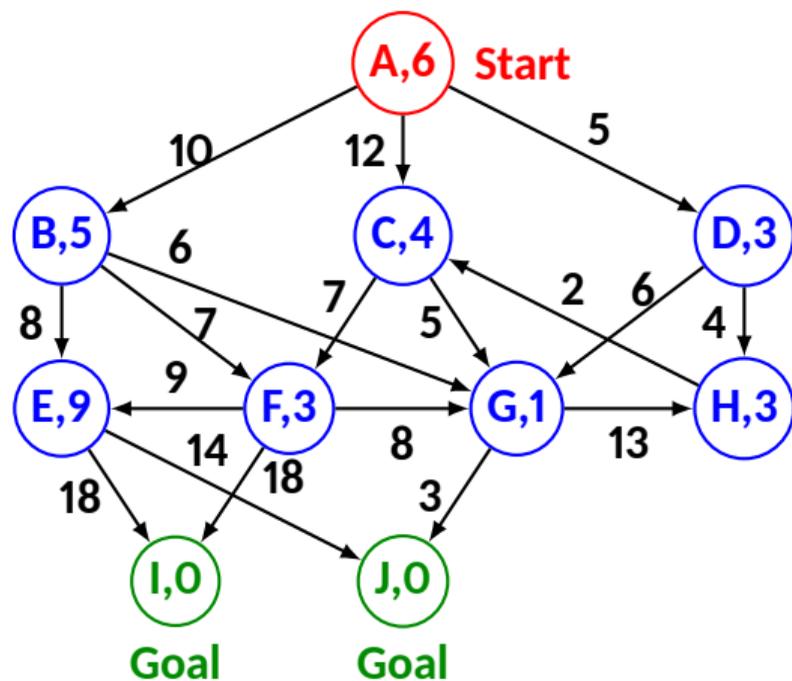
Execution of A*

(node, $g(n), h(n), f(n)$), O–OPEN, C–CLOSED

St.	Outcome
1	O={(A,0,6,6)}, C={}

Execution of A*
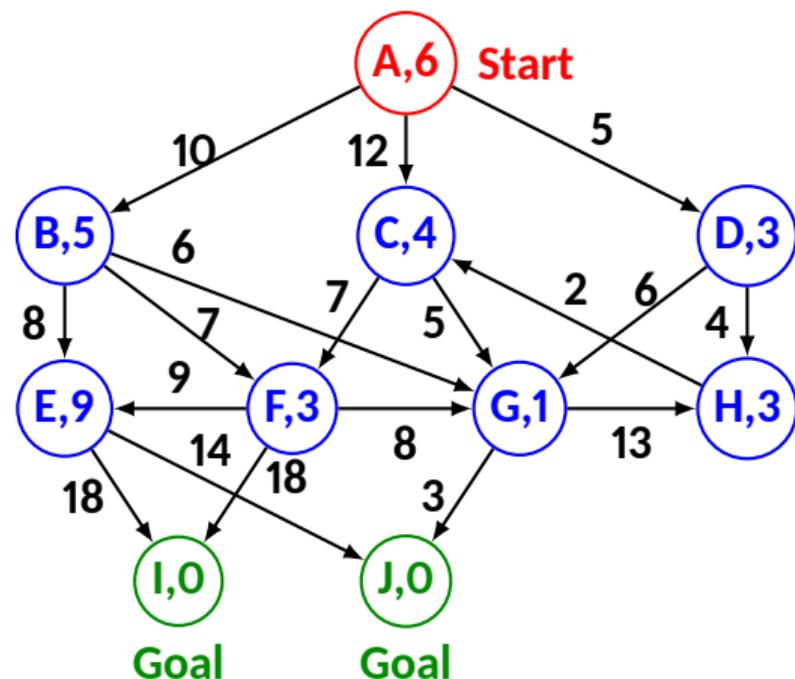
Execution of A*

Execution of A*

Execution of A*

(node, $g(n)$, $h(n)$, $f(n)$), O–OPEN, C–CLOSED

St.	Outcome
1	O={(A,0,6,6)}, C={}
2	O={(B,10,5,15), (C,12,4,16), (D,5,3,8)}, C={A}
3	O={(B,10,5,15), (C,12,4,16), (G,11,1,12), (H,9,3,12)}, C={A,D[A]}
4	O={(B,10,5,15), (C,11,4,15), (G,11,1,12)}, C={A,D[A], H[D]}
5	O={(B,10,5,15), (C,11,4,15), (J,14,0,14)}, C={A,D[A], H[D], G[D]}

Execution of A*
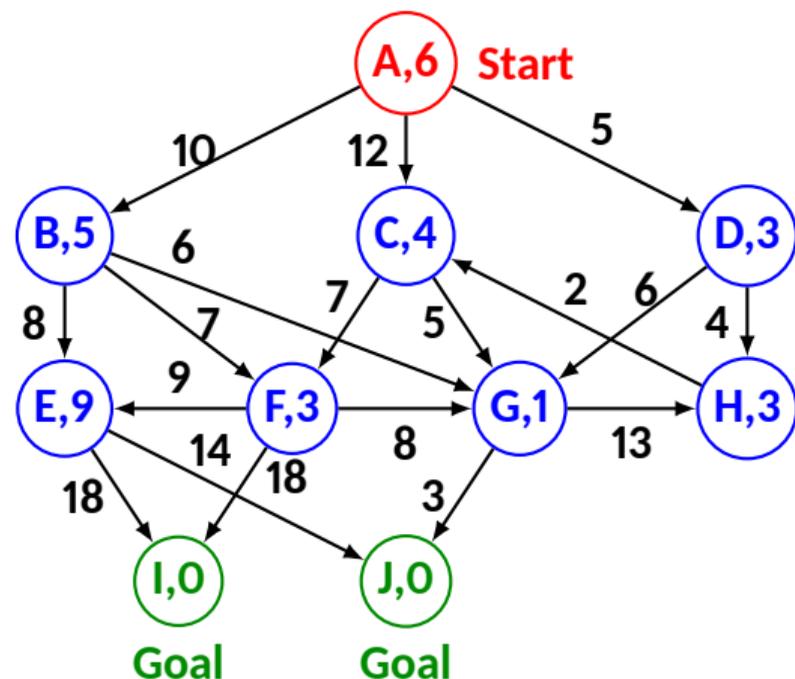
(node, $g(n)$, $h(n)$, $f(n)$), O–OPEN, C–CLOSED

St.	Outcome
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2	O={(B,10,5,15), (C,12,4,16), (D,5,3,8)}, C={A}
3	O={(B,10,5,15), (C,12,4,16), (G,11,1,12), (H,9,3,12)}, C={A,D[A]}
4	O={(B,10,5,15), (C,11,4,15), (G,11,1,12)}, C={A,D[A], H[D]}
5	O={(B,10,5,15), (C,11,4,15), (J,14,0,14)}, C={A,D[A], H[D], G[D]}
6	Goal J found. Terminate with cost 14, path – A, D, G, J

Properties of A*
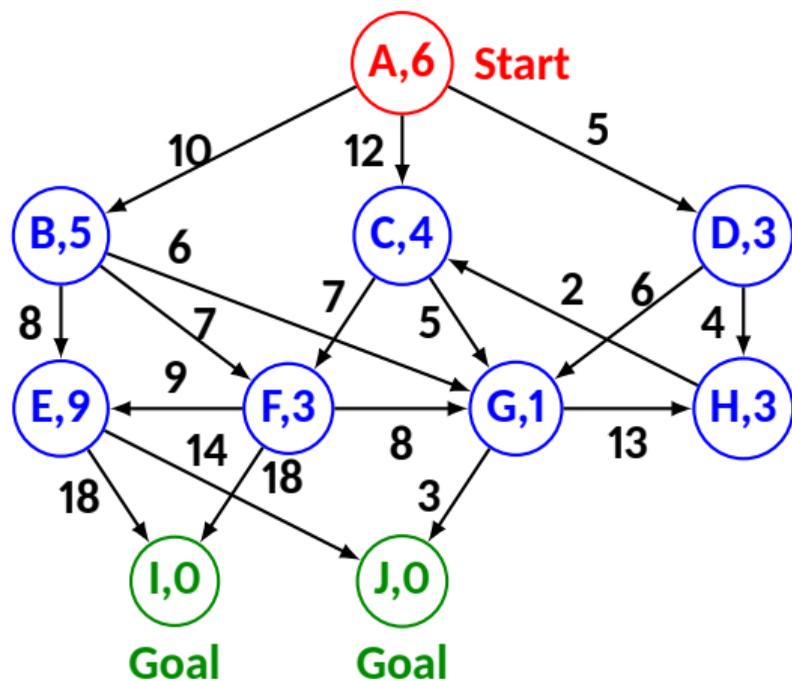
If lower heuristic bounds estimates and edge costs are non-negative:

- First solution is optimal
- No node in closed is ever reopened
- Whenever a node is removed from open its minimum cost from start is found
- Every node n with $f(n)$ less than optimal cost is expanded
- If heuristics are more accurate then search is less

Heuristic properties

- An **admissible heuristic** is one that never overestimates the cost to reach the goal
  - Since $g(n)$ is the actual cost to reach n, therefore $f(n) = g(n) + h(n)$ never overestimates the true cost of a goal
- A heuristic $h(n)$ is **consistent** if, for every node n and every successor n' of n generated by any action a, the estimated cost of reaching the goal from n is no greater than the step cost of getting to n' plus the estimated cost of reaching the goal from n' : $h(n) \leq c(n, a, n') + h(n')$

A* search is optimal

- **With an admissible heuristic, A* is cost-optimal**

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  - **Suppose the optimal path cost is C^* but the algorithm returns a path with cost $C > C^*$**

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  - **Assume, $g^*(n)$ to denote the cost of optimal path from the start to n**
  - **Assume, $h^*(n)$ to denote the cost of optimal path from n to goal node**

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  - **We know, $f(n) = g(n) + h(n)$ hence, $f(n) = g^*(n) + h(n)$ as n is on optimal path**

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  - **$f(n) \leq g^*(n) + h^*(n)$ as $h(n) < h^*(n)$ (admissibility)**

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- **With an admissible heuristic, A* is cost-optimal**
  - **Suppose the optimal path cost is C^* but the algorithm returns a path with cost $C > C^*$**
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  - **We know, $f(n) = g(n) + h(n)$ hence, $f(n) = g^*(n) + h(n)$ as n is on optimal path**
  - $f(n) \leq g^*(n) + h^*(n)$ **as $h(n) < h^*(n)$ (admissibility)**
  - $f(n) \leq C^* = g^*(n) + h^*(n)$ **– contradiction**

Depth First Branch-and-Bound algorithm

1. **Initialize Best-Cost to INFINITY**
2. **Perform DFS with costs and Backtrack from any node n whose $f(n) \geq$ Best-Cost**
3. **On reaching a Goal Node, update Best-Cost to the current best**
4. **Continue till OPEN becomes empty**

Execution of DFBB
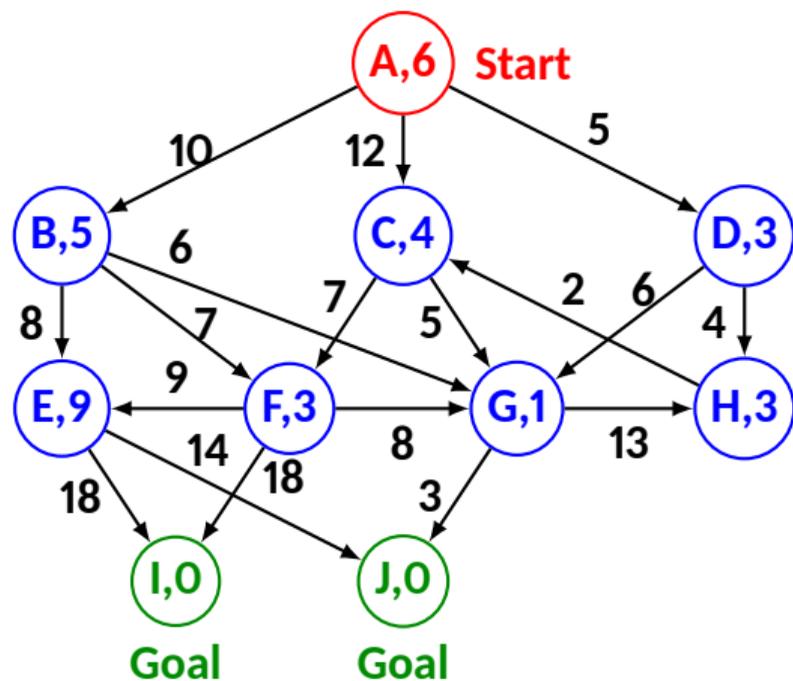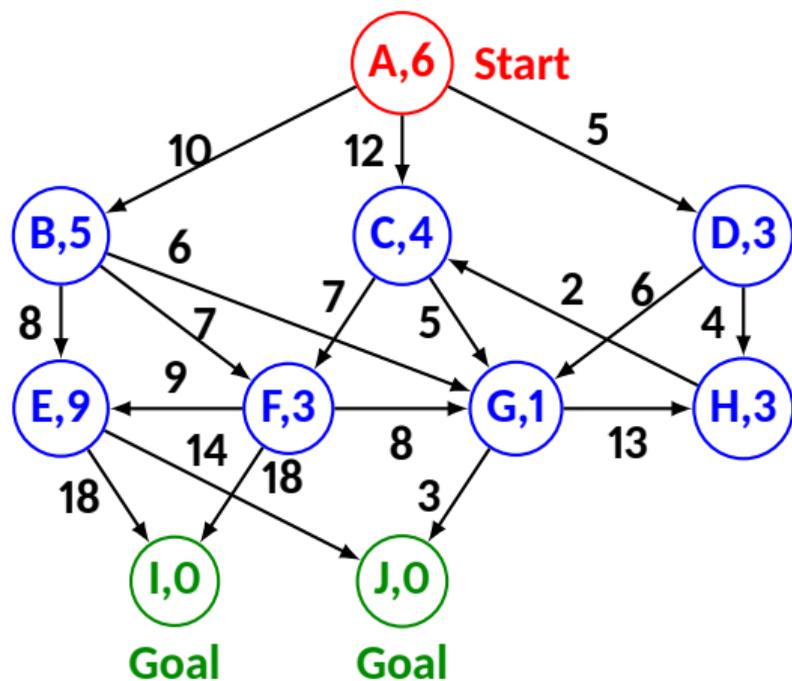
Execution of DFBB

Step	OPEN	CLOSED
1	A	{}

Execution of DFBB
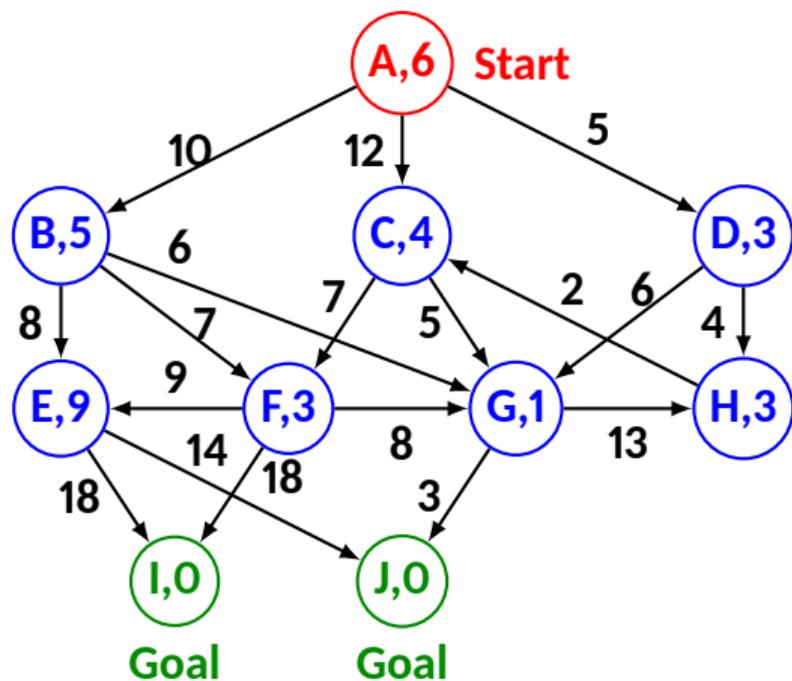
Step	OPEN	CLOSED
1	A	{}
2	(B,15), (C,16), (D,8), CB=∞	A

Execution of DFBB
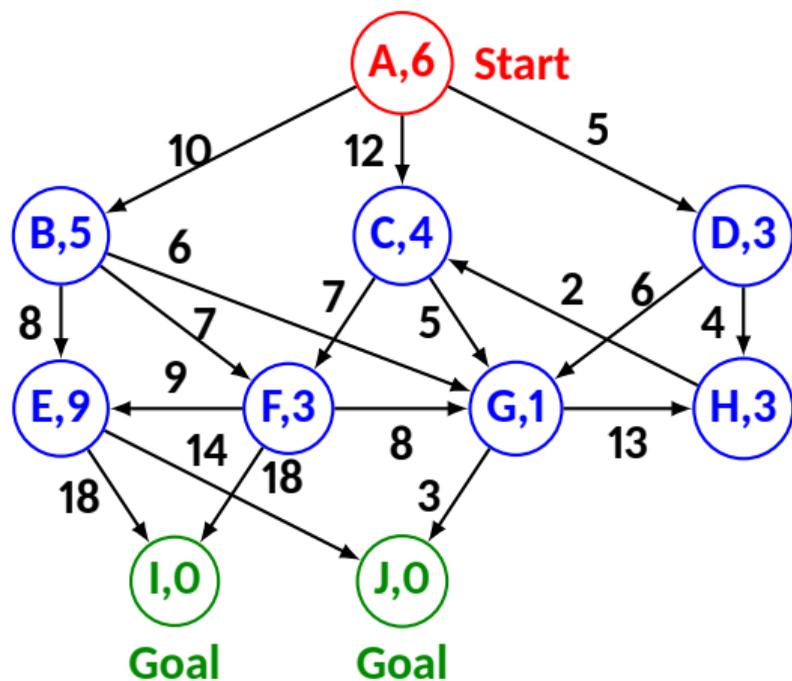
Step	OPEN	CLOSED
1	A	{}
2	(B,15), (C,16), (D,8), CB=∞	A
3	(E,27), (F,20), (G,17), (C,16), (D,8), CB=∞	B

Execution of DFBB
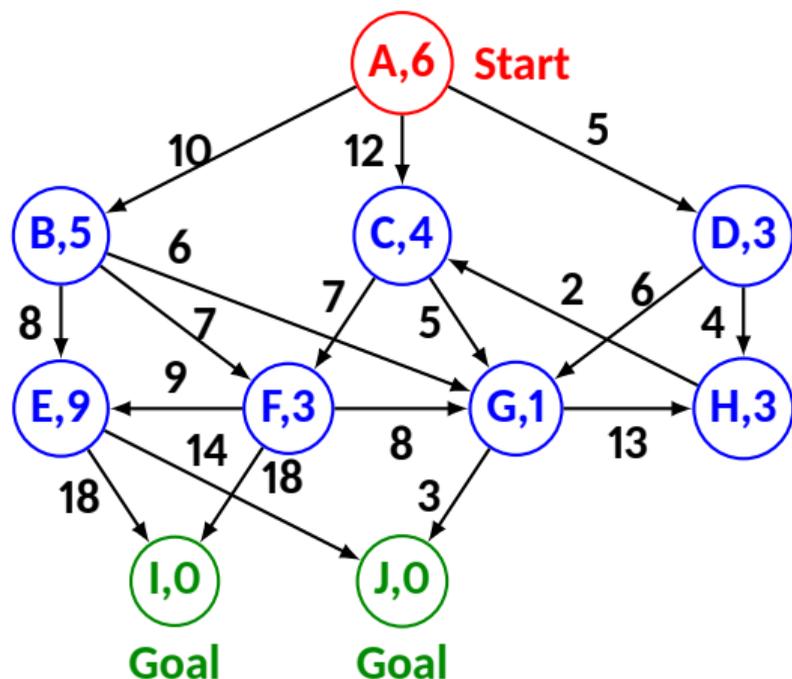
Step	OPEN	CLOSED
1	A	{}
2	(B,15), (C,16), (D,8), CB=∞	A
3	(E,27), (F,20), (G,17), (C,16), (D,8), CB=∞	B
4	(I,36), (J,32), (F,20), (G,17), (C,16), (D,8), CB=∞	E

Execution of DFBB
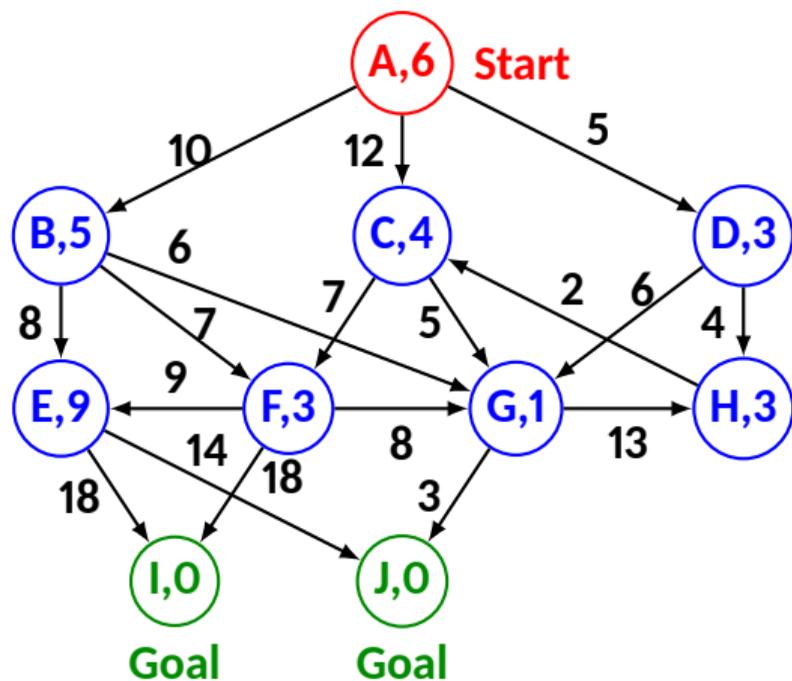
Step	OPEN	CLOSED
1	A	{}
2	(B,15), (C,16), (D,8), CB=∞	A
3	(E,27), (F,20), (G,17), (C,16), (D,8), CB=∞	B
4	(I,36), (J,32), (F,20), (G,17), (C,16), (D,8), CB=∞	E
5	(J,32), (F,20), (G,17), (C,16), (D,8), CB=36	I

Execution of DFBB

Step	OPEN	CLOSED
1	A	{}
2	(B,15), (C,16), (D,8), CB=∞	A
3	(E,27), (F,20), (G,17), (C,16), (D,8), CB=∞	B
4	(I,36), (J,32), (F,20), (G,17), (C,16), (D,8), CB=∞	E
5	(J,32), (F,20), (G,17), (C,16), (D,8), CB=36	I
6	(F,20), (G,17), (C,16), (D,8), CB=32	J

Execution of DFBB

Step	OPEN	CLOSED
1	A	{}
2	(B,15), (C,16), (D,8), CB=∞	A
3	(E,27), (F,20), (G,17), (C,16), (D,8), CB=∞	B
4	(I,36), (J,32), (F,20), (G,17), (C,16), (D,8), CB=∞	E
5	(J,32), (F,20), (G,17), (C,16), (D,8), CB=36	I
6	(F,20), (G,17), (C,16), (D,8), CB=32	J
7	(G,17), (C,16), (D,8), CB=32	F

Execution of DFBB

Step	OPEN	CLOSED
1	A	{}
2	(B,15), (C,16), (D,8), CB=∞	A
3	(E,27), (F,20), (G,17), (C,16), (D,8), CB=∞	B
4	(I,36), (J,32), (F,20), (G,17), (C,16), (D,8), CB=∞	E
5	(J,32), (F,20), (G,17), (C,16), (D,8), CB=36	I
6	(F,20), (G,17), (C,16), (D,8), CB=32	J
7	(G,17), (C,16), (D,8), CB=32	F
8	(J,20), (H,33), (C,16), (D,8), CB=32	G

Execution of DFBB

Step	OPEN	CLOSED
1	A	{}
2	(B,15), (C,16), (D,8), **CB=∞**	A
3	(E,27), (F,20), (G,17), (C,16), (D,8), **CB=∞**	B
4	(I,36), (J,32), (F,20), (G,17), (C,16), (D,8), **CB=∞**	E
5	(J,32), (F,20), (G,17), (C,16), (D,8), **CB=36**	I
6	(F,20), (G,17), (C,16), (D,8), **CB=32**	J
7	(G,17), (C,16), (D,8), **CB=32**	F
8	(J,20), (H,33), (C,16), (D,8), **CB=32**	G
...	..., **CB=20**	J

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

Coin change: DFBB

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→ Coin selection

→ No coin selection

CB = ∞

15,0 -- {1,2,4,7,8,10}

-- 10

{1,2,4,7,8}

-- 8

{1,2,4,7}

-- 7

{1,2,4}

-- 4

{1,2}

-- 2

{1}

-- 1

--- {}

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15,0 - {1,2,4,7,8,10}

- **10**

5,1 ← {1,2,4,7,8}

- **8**

{1,2,4,7}

- **7**

{1,2,4}

- **4**

{1,2}

- **2**

{1}

- **1**

- - - **{}**

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- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

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→ **No coin selection**

$CB = \infty$

15,0 - {1,2,4,7,8,10}

- 10

5,1 ← {1,2,4,7,8}

- 8

-3,2 {1,2,4,7}

- 7

{1,2,4}

- - - - - - - - - - - - - - - - - - - 4

{1,2}

- - - - - - - - - - - - - - - - - - 2

{1}

- - - - - - - - - - - - - - - - - 1

- - - {}

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→ No coin selection

CB = ∞

15,0 - {1,2,4,7,8,10}

- 10

5,1   {1,2,4,7,8}

- 8

-3,2   5,1   {1,2,4,7}

- 7

{1,2,4}

- 4

{1,2}

- 2

{1}

- 1

- - - {}

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CB = ∞

15,0 -- {1,2,4,7,8,10}
-- 10
5,1   {1,2,4,7,8}
-- 8
-3,2   5,1 -- {1,2,4,7}
-- 7
-2,2   {1,2,4}
-- 4
                                                       {1,2}
-- 2
                                                       {1}
-- 1
--- {}

Coin change: DFBB

- **Given a set of coins** C**, what is the minimum number coins required to provide sum** S**?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

Coin selection → (green)
No coin selection → (blue)

CB = ∞

15,0 — — — — — {1,2,4,7,8,10}
— — — — 10
5,1 {1,2,4,7,8}
— — — 8
-3,2   5,1 {1,2,4,7}
— — — 7
-2,2   5,1 {1,2,4}
— — — 4
{1,2}
— — — 2
{1}
— — — 1
— — {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ **Coin selection**

→ **No coin selection**

CB = ∞

15,0 - {1,2,4,7,8,10}

- 10

5,1

- {1,2,4,7,8}

- 8

-3,2   **5,1**

- {1,2,4,7}

- 7

-2,2   **5,1**

- {1,2,4}

- 4

1,2

- {1,2}

- 2

- {1}

- 1

- - - {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ **Coin selection**
→ **No coin selection**

CB = ∞

15,0 - {1,2,4,7,8,10}
- 10
5,1 {1,2,4,7,8}
- 8
-3,2   5,1 {1,2,4,7}
- 7
-2,2   5,1 {1,2,4}
- 4
1,2 {1,2}
- 2
-1,3 {1}
- 1
- - - {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ **Coin selection**

→ **No coin selection**

CB = ∞   15,0 - {1,2,4,7,8,10}

- **10**

5,1   {1,2,4,7,8}

- **8**

-3,2   5,1   {1,2,4,7}

- **7**

-2,2   5,1   {1,2,4}

- **4**

1,2   {1,2}

- **2**

-1,3   1,2   {1}

- **1**

- - - **{}**

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ **Coin selection**
→ **No coin selection**

CB = 3

15,0 - {1,2,4,7,8,10}

- 10

5,1 - {1,2,4,7,8}

- 8

-3,2   5,1 - {1,2,4,7}

- 7

-2,2   5,1 - {1,2,4}

- 4

1,2 - {1,2}

- 2

-1,3   1,2 - {1}

- 1

0,3 - - - {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ **Coin selection**
→ **No coin selection**

CB = 3

15,0 --------------------------------- {1,2,4,7,8,10}
--- 10
5,1   {1,2,4,7,8}
--- 8
-3,2   5,1   {1,2,4,7}
--- 7
-2,2   5,1   {1,2,4}
--- 4
1,2   {1,2}
--- 2
-1,3   1,2   {1}
--- 1
0,3   1,2   {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ Coin selection

→ No coin selection

CB = 3

15,0 — {1,2,4,7,8,10}

— 10

5,1   {1,2,4,7,8}

— 8

-3,2   5,1   {1,2,4,7}

— 7

-2,2   5,1   {1,2,4}

— 4

1,2   5,1   {1,2}

— 2

-1,3   1,2   {1}

— 1

0,3   1,2   {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

- → **Coin selection**
- → **No coin selection**

CB = 3

15,0 -- {1,2,4,7,8,10}
-- 10
5,1   {1,2,4,7,8}
--- 8
-3,2   5,1   {1,2,4,7}
--- 7
-2,2   5,1   {1,2,4}
--- 4
1,2   5,1   {1,2}
--- 2
-1,3   1,2   3,2   {1}
--- 1
0,3   1,2   ---- {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ **Coin selection**

→ **No coin selection**

CB = 3

15,0 --- {1,2,4,7,8,10}

-- 10

5,1 -- {1,2,4,7,8}

-- 8

-3,2   5,1 --- {1,2,4,7}

--- 7

-2,2   5,1 --- {1,2,4}

--- 4

1,2   5,1 -------------------------------------- {1,2}

--- 2

-1,3   1,2   3,2 ------------------------------------ {1}

-- 1
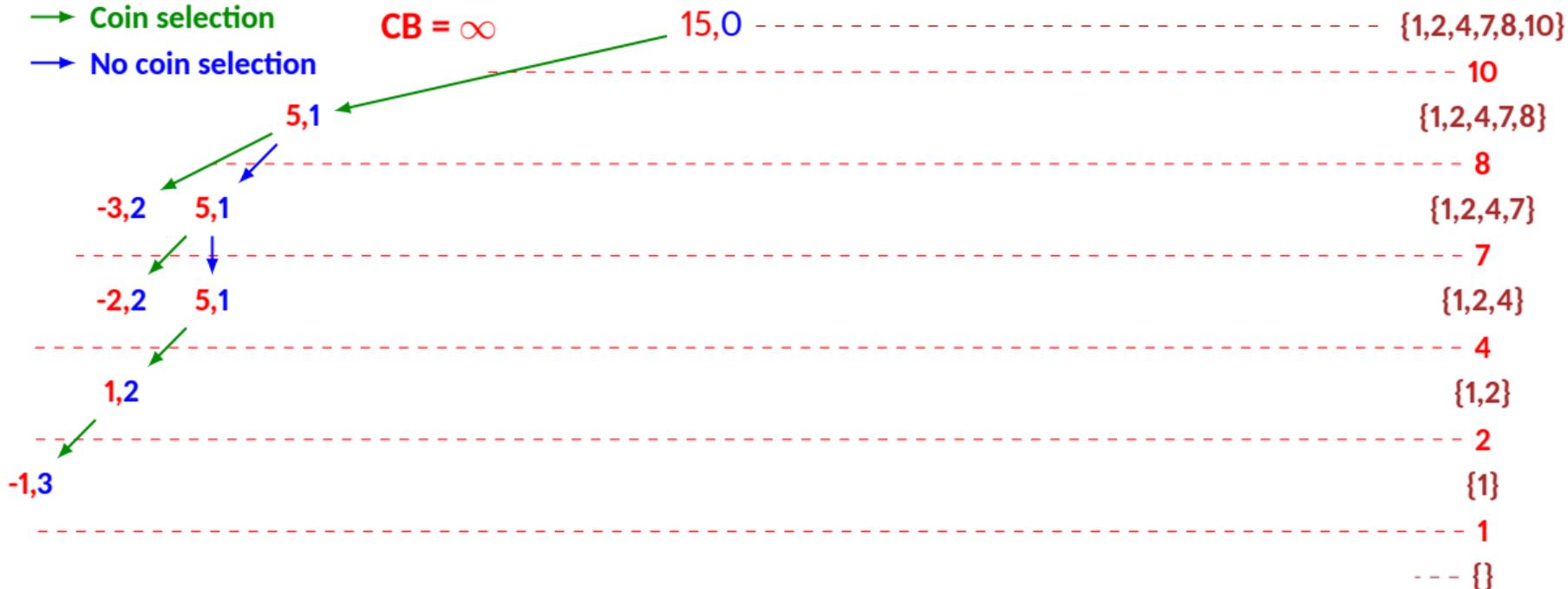
0,3   1,2 2,3 ----------------------------------- {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ Coin selection
→ No coin selection

CB = 3

15,0 --- {1,2,4,7,8,10}
-- 10
5,1   {1,2,4,7,8}
-- 8
-3,2   5,1   {1,2,4,7}
-- 7
-2,2   5,1   {1,2,4}
-- 4
1,2   5,1   {1,2}
-- 2
-1,3   1,2   3,2   {1}
-- 1
0,3 1,2   2,3 3,2   {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ **Coin selection**

→ **No coin selection**

CB = 3

15,0 --- {1,2,4,7,8,10}

-- 10

5,1 --- {1,2,4,7,8}

-- 8

-3,2   5,1 ----------------------------------- {1,2,4,7}

-- 7

-2,2   5,1 ----------------------------------- {1,2,4}

-- 4

1,2   5,1 ----------------------------------- {1,2}

-- 2

-1,3   1,2   3,2   5,1 --------------------------- {1}

-- 1

0,3   1,2   2,3   3,2 ----------------------------- {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ Coin selection
→ No coin selection

CB = 3

15,0   {1,2,4,7,8,10}
       10
5,1   {1,2,4,7,8}
       8
-3,2   5,1   {1,2,4,7}
       7
-2,2   5,1   {1,2,4}
       4
1,2   5,1   {1,2}
       2
-1,3   1,2   3,2   5,1   {1}
       1
0,3   1,2   2,3   3,2   4,2   {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ **Coin selection**
→ **No coin selection**

CB = 3

15,0 - {1,2,4,7,8,10}
- 10

5,1
                                                          {1,2,4,7,8}
- 8

-3,2   5,1
                                                          {1,2,4,7}
- 7

-2,2   5,1
                                                          {1,2,4}
- 4

1,2   5,1
                                                          {1,2}
- 2

-1,3 1,2   3,2   5,1
                                                          {1}
- 1

0,3 1,2 2,3 4,2 5,1
         3,2   {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ **Coin selection**

→ **No coin selection**

CB = 3

15,0 --------------------------------- {1,2,4,7,8,10}

--------------------------------- 10

5,1   15,0   {1,2,4,7,8}

--------------------------------- 8

-3,2   5,1   {1,2,4,7}

--------------------------------- 7

-2,2   5,1   {1,2,4}

--------------------------------- 4

1,2   5,1   {1,2}

--------------------------------- 2

-1,3   1,2   3,2   5,1   {1}

--------------------------------- 1

0,3 1,2 2,3 4,2 5,1   - - - {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ **Coin selection**

→ **No coin selection**

CB = 3

15,0 --- {1,2,4,7,8,10}

-- 10

15,0 ------------------------------ {1,2,4,7,8}

5,1 --- 8

7,1 --------------- {1,2,4,7}

-3,2   5,1

-- 7

-2,2   5,1 -- {1,2,4}

-- 4

1,2   5,1 -- {1,2}

-- 2

-1,3   1,2   3,2   5,1 --------------------------------- {1}

-- 1
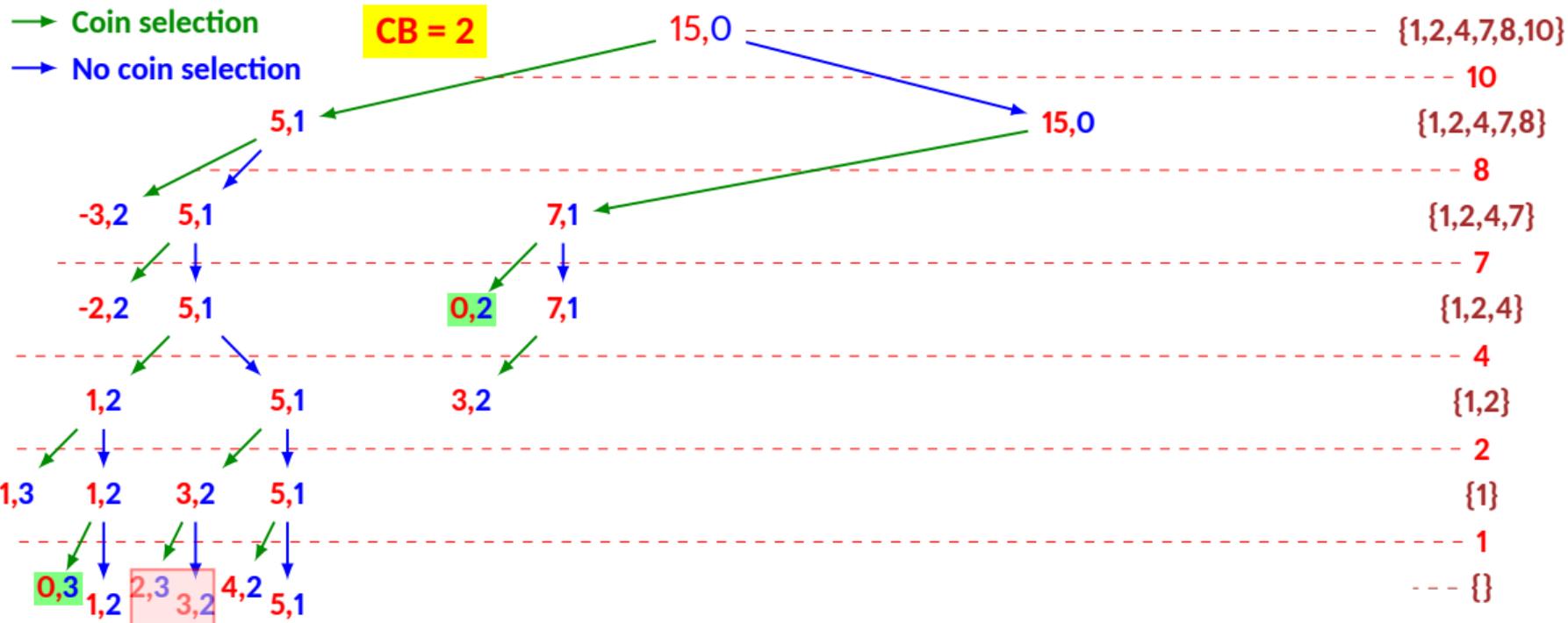
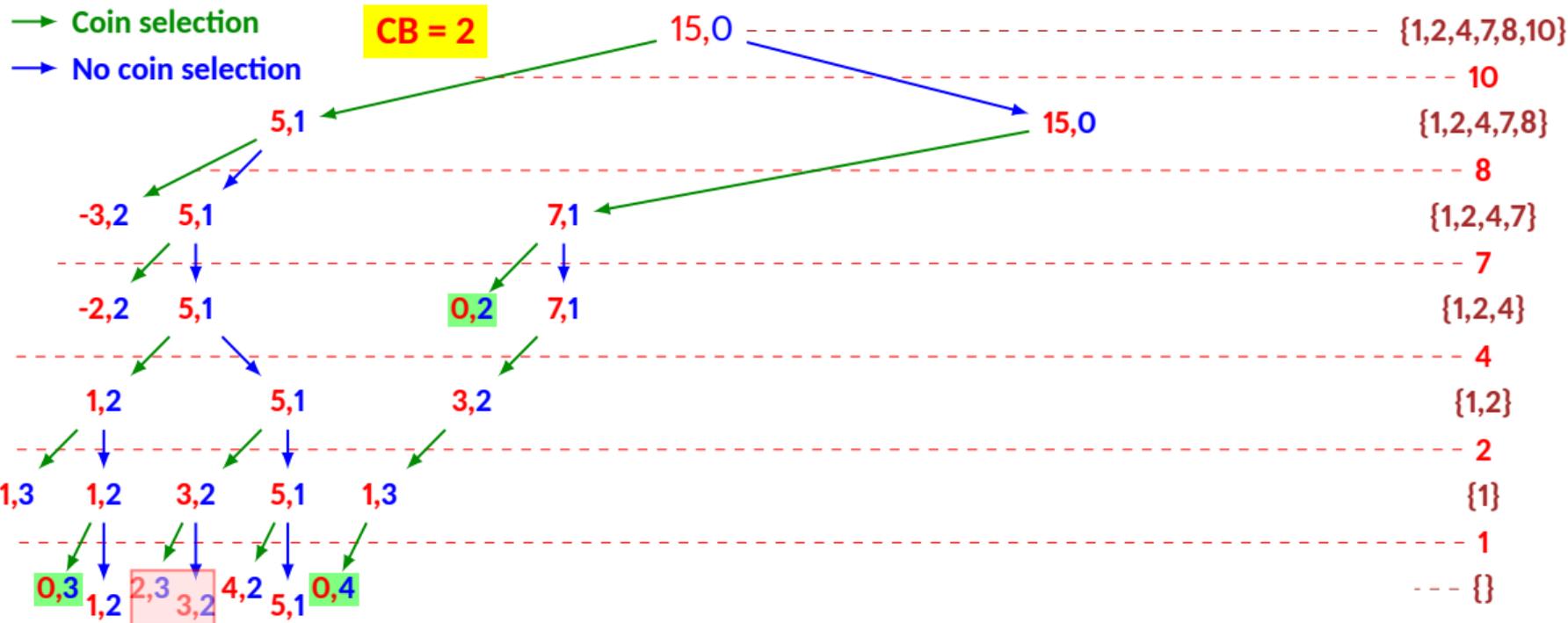0,3   1,2   2,3   3,2   4,2   5,1 --------------------- {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ Coin selection
→ No coin selection

CB = 2

15,0 -- {1,2,4,7,8,10}
--- 10
                    15,0   {1,2,4,7,8}
5,1 -- 8
-3,2   5,1   7,1   {1,2,4,7}
--- 7
-2,2   5,1   0,2   {1,2,4}
--- 4
1,2   5,1   {1,2}
--- 2
-1,3   1,2   3,2   5,1   {1}
--- 1
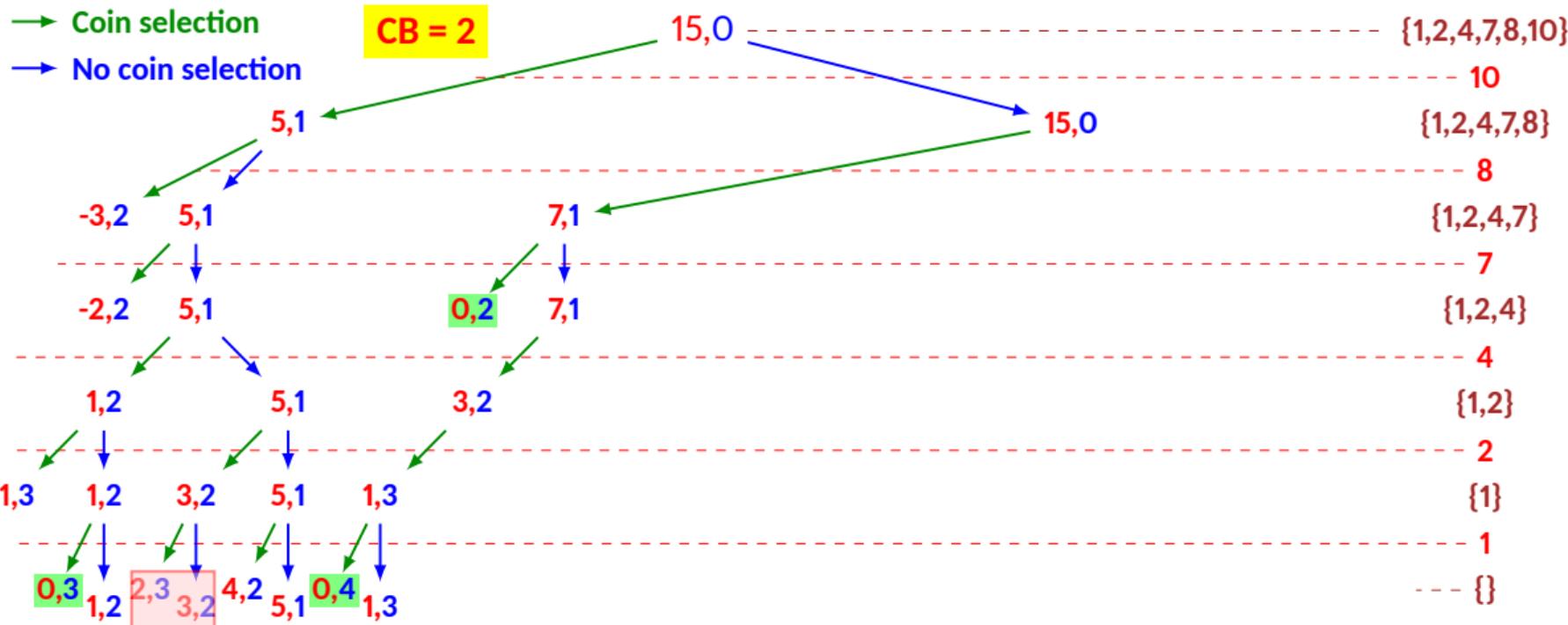0,3   1,2   2,3   4,2   - - - {}
            3,2   5,1

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ **Coin selection**

→ **No coin selection**

CB = 2

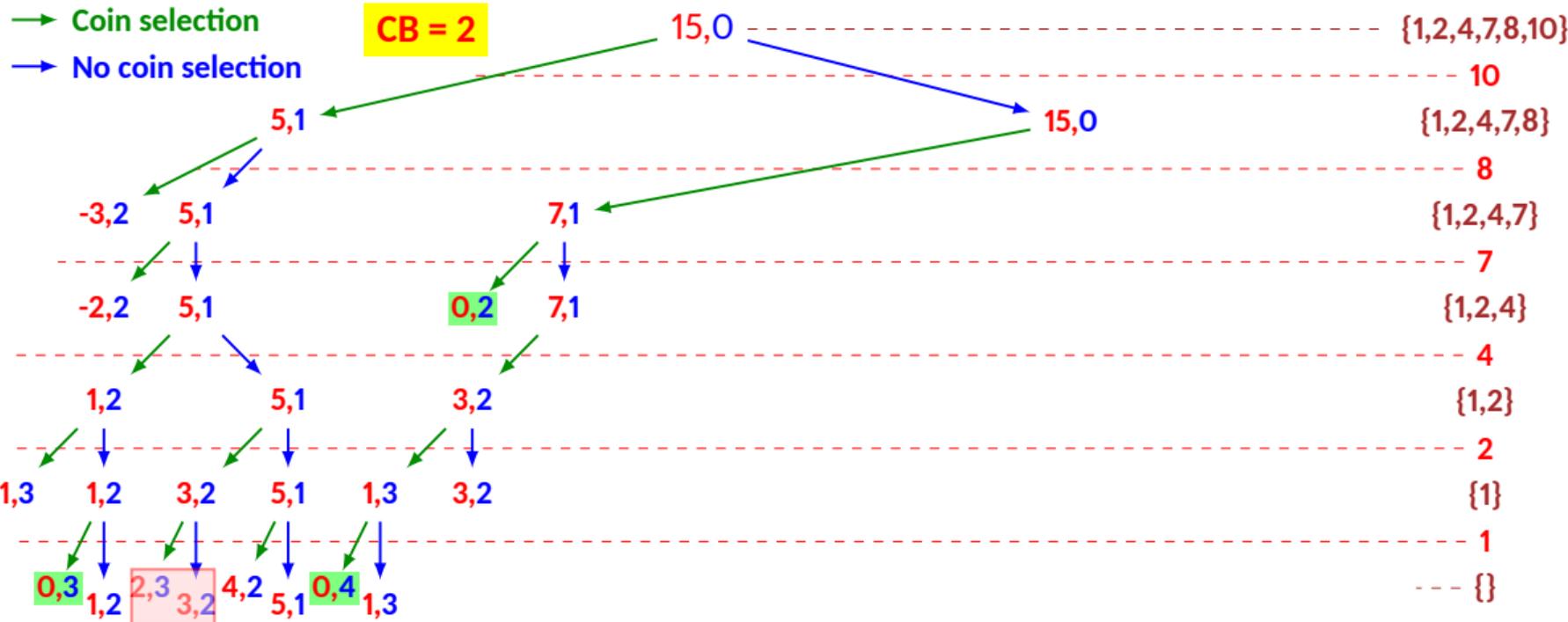|	
15,0	{1,2,4,7,8,10}
	10
15,0	{1,2,4,7,8}
	8
5,1	
7,1	{1,2,4,7}
	7
-3,2 5,1	
0,2 7,1	{1,2,4}
	4
-2,2 5,1	
1,2 5,1	{1,2}
	2
-1,3 1,2 3,2 5,1	{1}
	1
0,3 1,2 2,3 4,2 5,1	{}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ Coin selection
→ No coin selection

CB = 2

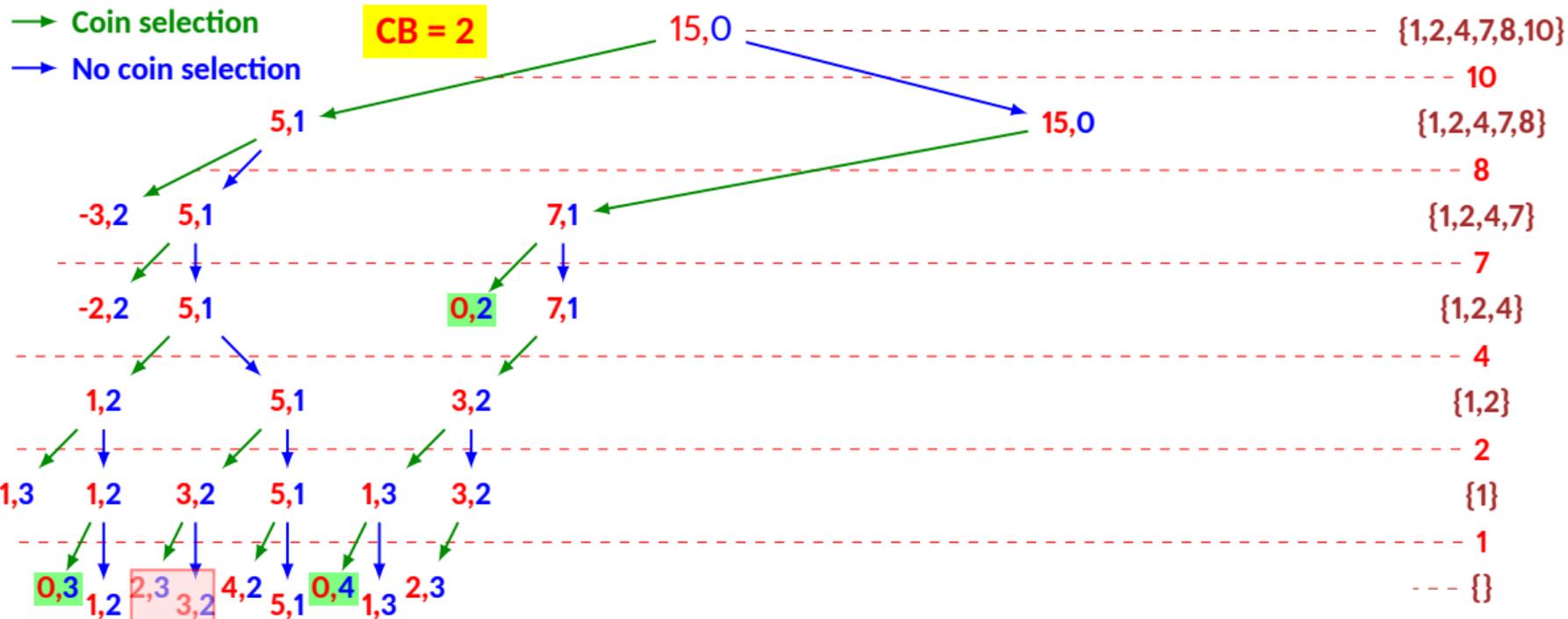| value	set
15,0	{1,2,4,7,8,10}
	10
15,0	{1,2,4,7,8}
	8
5,1 ... 7,1	{1,2,4,7}
	7
-3,2 5,1 ... 0,2 7,1	{1,2,4}
	4
-2,2 5,1 ... 3,2	{1,2}
	2
1,2 5,1	{1}
-1,3 1,2 3,2 5,1	1
0,3 1,2 2,3 3,2 4,2 5,1	{}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

- → **Coin selection**
- → **No coin selection**

CB = 2

15,0 --------------------------------- {1,2,4,7,8,10}

-------------------------------------- 10

5,1   15,0   {1,2,4,7,8}

-------------------------------------- 8

-3,2 5,1   7,1   {1,2,4,7}

-------------------------------------- 7

-2,2 5,1   0,2 7,1   {1,2,4}

-------------------------------------- 4

1,2   5,1   3,2   {1,2}

-------------------------------------- 2

-1,3 1,2 3,2 5,1 1,3   {1}

-------------------------------------- 1
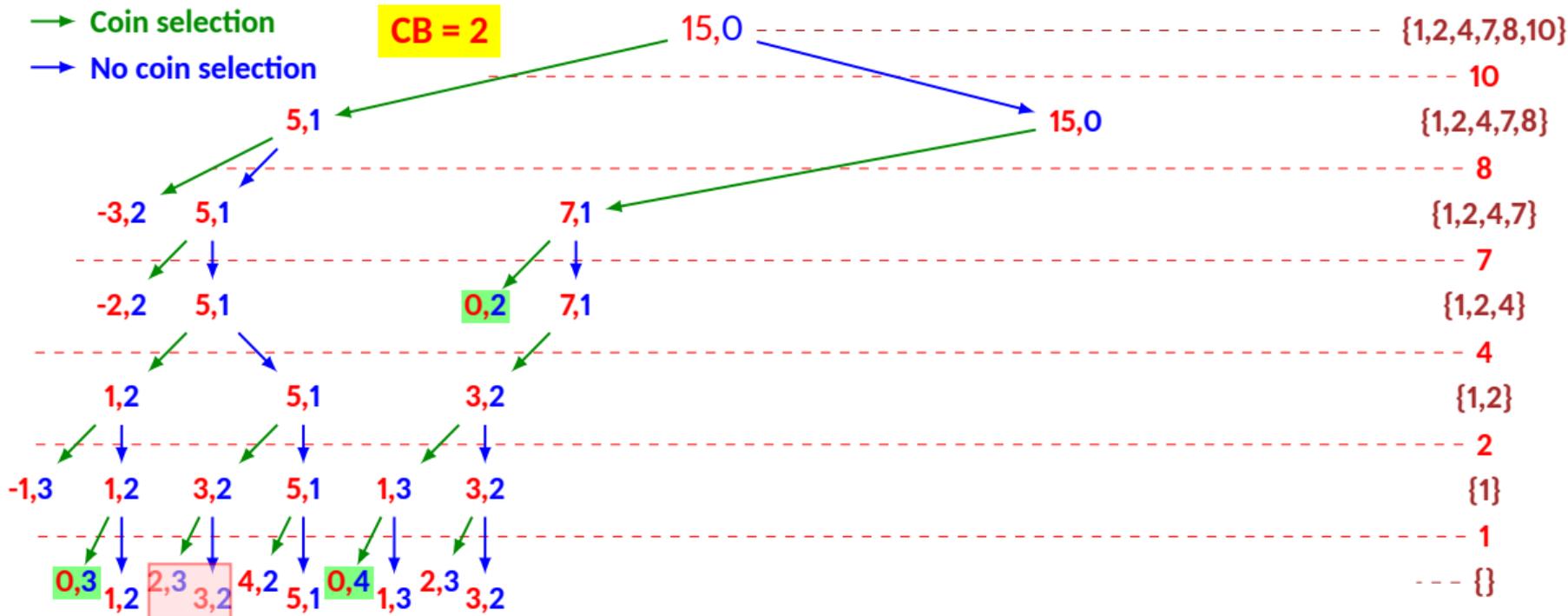
0,3 1,2 3,2 4,2 5,1   {}

2,3 3,2

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ Coin selection
→ No coin selection

CB = 2

| | 15,0 | - - - - - - - - - - - - {1,2,4,7,8,10} |

15,0 → {1,2,4,7,8}

5,1 → 15,0

-3,2 5,1   7,1 → {1,2,4,7}

-2,2 5,1   0,2 7,1 → {1,2,4}

1,2   5,1   3,2 → {1,2}

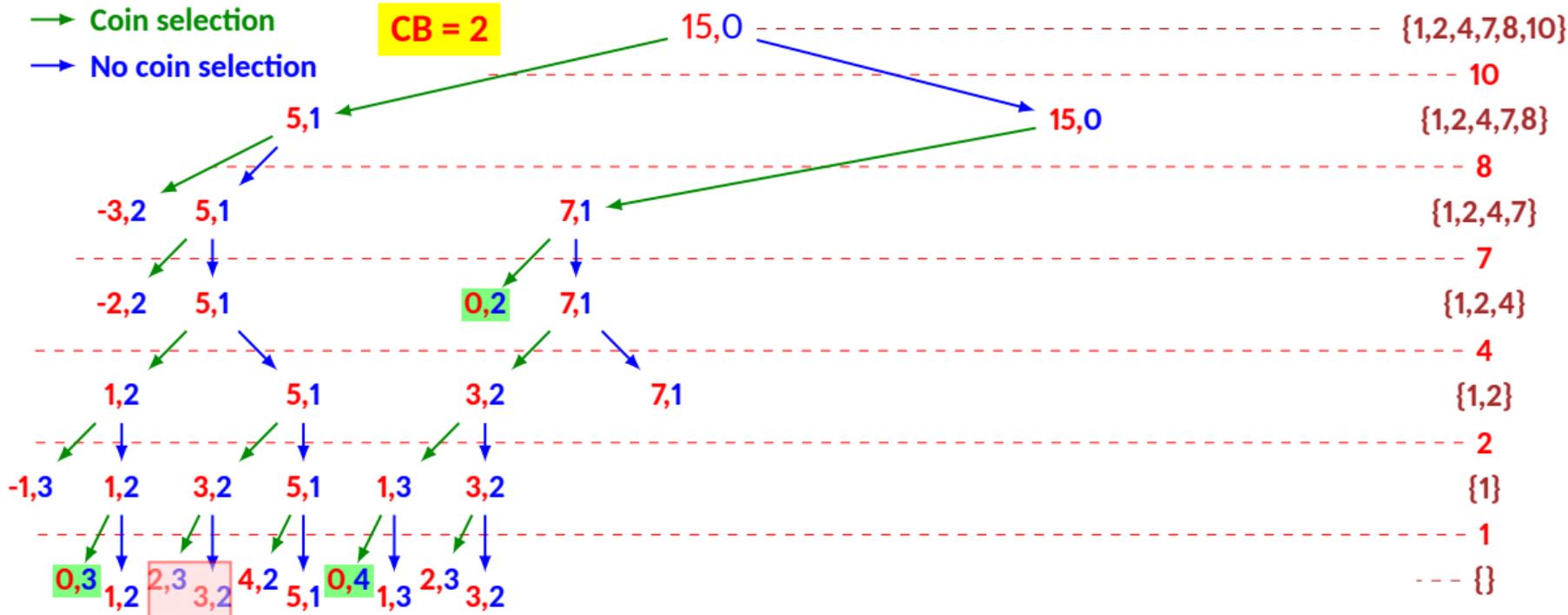-1,3 1,2 3,2 5,1 1,3 → {1}

0,3 1,2 2,3 3,2 4,2 5,1 0,4 → {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

- → Coin selection
- → No coin selection

CB = 2

15,0	{1,2,4,7,8,10}
	10
15,0	{1,2,4,7,8}
	8
5,1	
	7,1 {1,2,4,7}
-3,2 5,1	7
-2,2 5,1	0,2 7,1 {1,2,4}
	4
1,2 5,1 3,2	{1,2}
	2
-1,3 1,2 3,2 5,1 1,3	{1}
	1
0,3 1,2 2,3 4,2 0,4	{}
3,2 5,1 1,3	

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

CB = 2

Coin selection
No coin selection

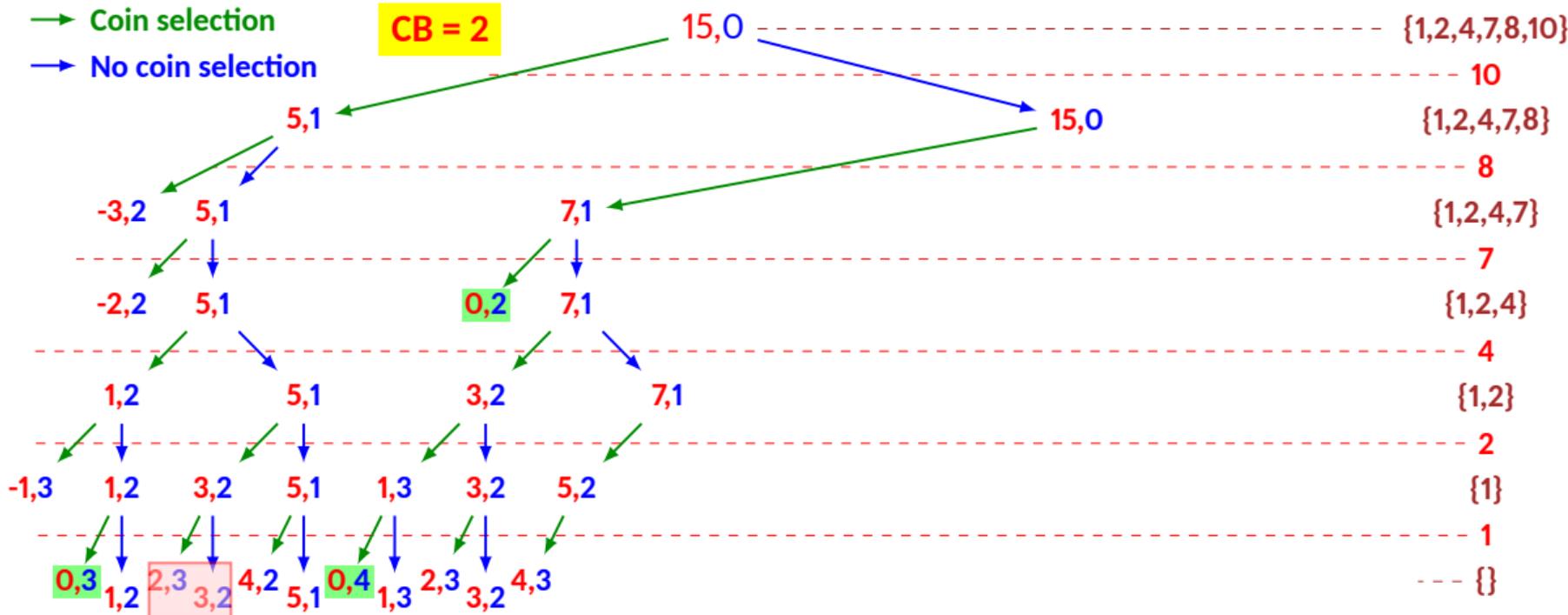| | | 15,0 | | {1,2,4,7,8,10} |
				10
	5,1		15,0	{1,2,4,7,8}
				8
-3,2	5,1	7,1		{1,2,4,7}
				7
-2,2	5,1	0,2	7,1	{1,2,4}
				4
1,2	5,1	3,2		{1,2}
				2
-1,3	1,2	3,2	5,1	1,3	3,2	{1}
				1
0,3	1,2	2,3	3,2	4,2	5,1	0,4	1,3	{}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ Coin selection
→ No coin selection

CB = 2

15,0 — — — — — — — — — — — — {1,2,4,7,8,10}
— — — 10
15,0 — — — — {1,2,4,7,8}
5,1
— — — 8
-3,2   5,1   7,1 — — — {1,2,4,7}
— — — 7
-2,2   5,1   0,2   7,1 — — — {1,2,4}
— — — 4
1,2   5,1   3,2 — — — {1,2}
— — — 2
-1,3 1,2 3,2 5,1 1,3 3,2 — — — {1}
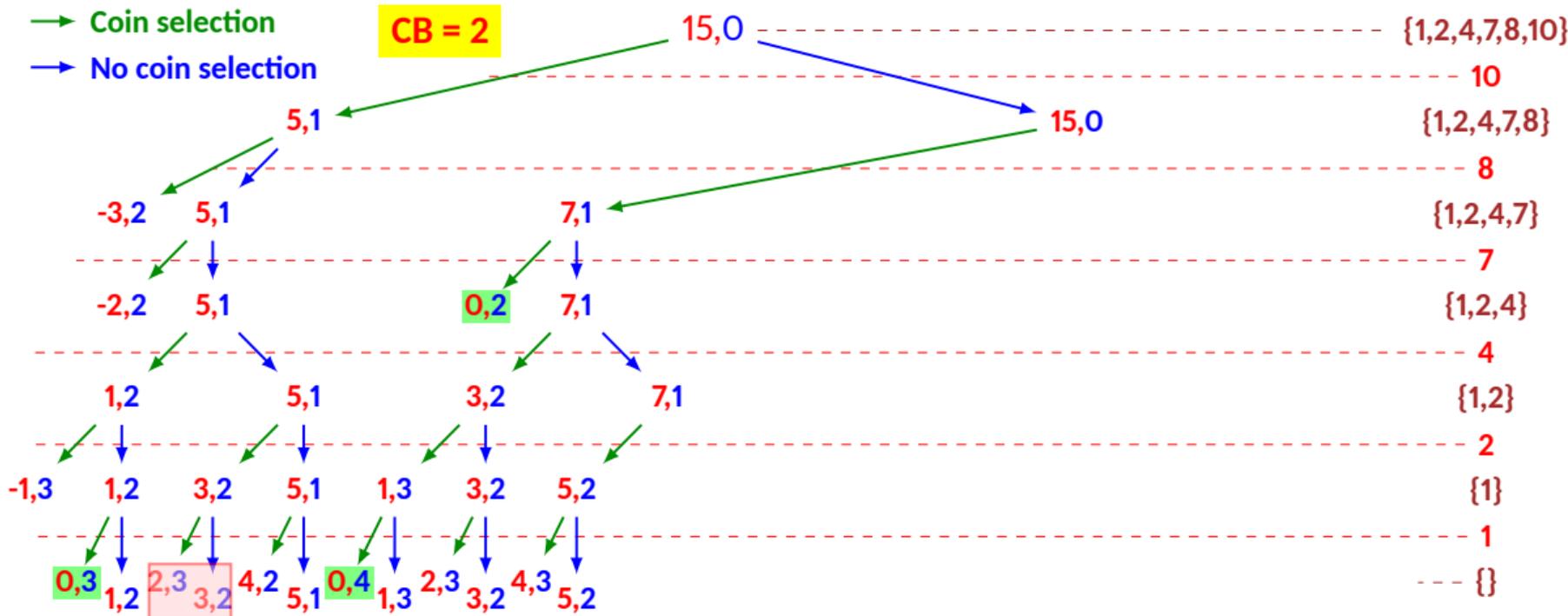— — — 1
0,3 1,2 2,3 3,2 4,2 5,1 0,4 1,3 2,3 — — — {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ **Coin selection**
→ **No coin selection**

CB = 2

15,0 — — — — — — — — — — — — — — — — {1,2,4,7,8,10}

— — — — — — — 10

5,1   15,0   {1,2,4,7,8}

— — — 8

-3,2   5,1   7,1   {1,2,4,7}

— — 7

-2,2   5,1   0,2   7,1   {1,2,4}

— — 4

1,2   5,1   3,2   {1,2}

— — 2

-1,3 1,2   3,2   5,1   1,3 3,2   {1}

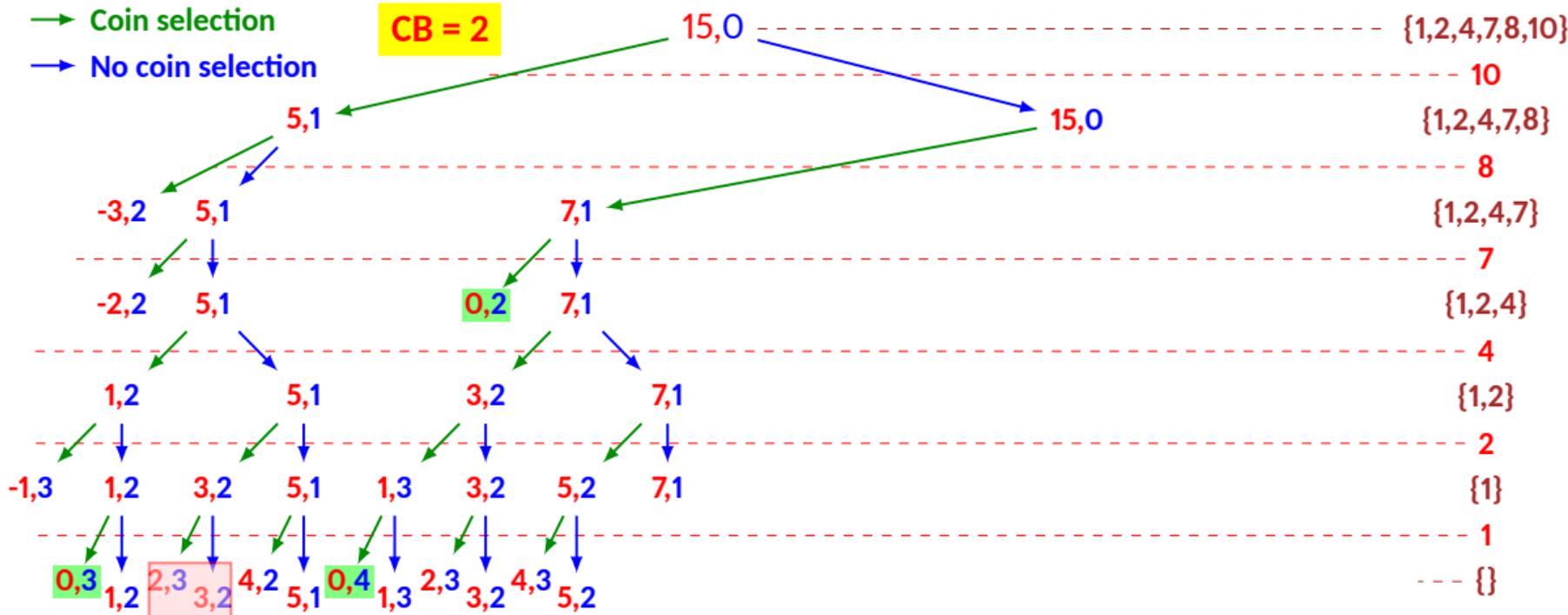— — 1

0,3 1,2 2,3 3,2 4,2 5,1 0,4 1,3 2,3 3,2   — — {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ Coin selection
→ No coin selection

CB = 2

15,0 --------------------------------- {1,2,4,7,8,10}
--- 10
15,0 --------------------------------- {1,2,4,7,8}
--- 8
5,1
-3,2   5,1   7,1 --------------- {1,2,4,7}
--- 7
-2,2   5,1   0,2   7,1 ---------------- {1,2,4}
--- 4
1,2   5,1   3,2   7,1 ----------- {1,2}
--- 2
-1,3 1,2 3,2 5,1 1,3 3,2 --------- {1}
--- 1
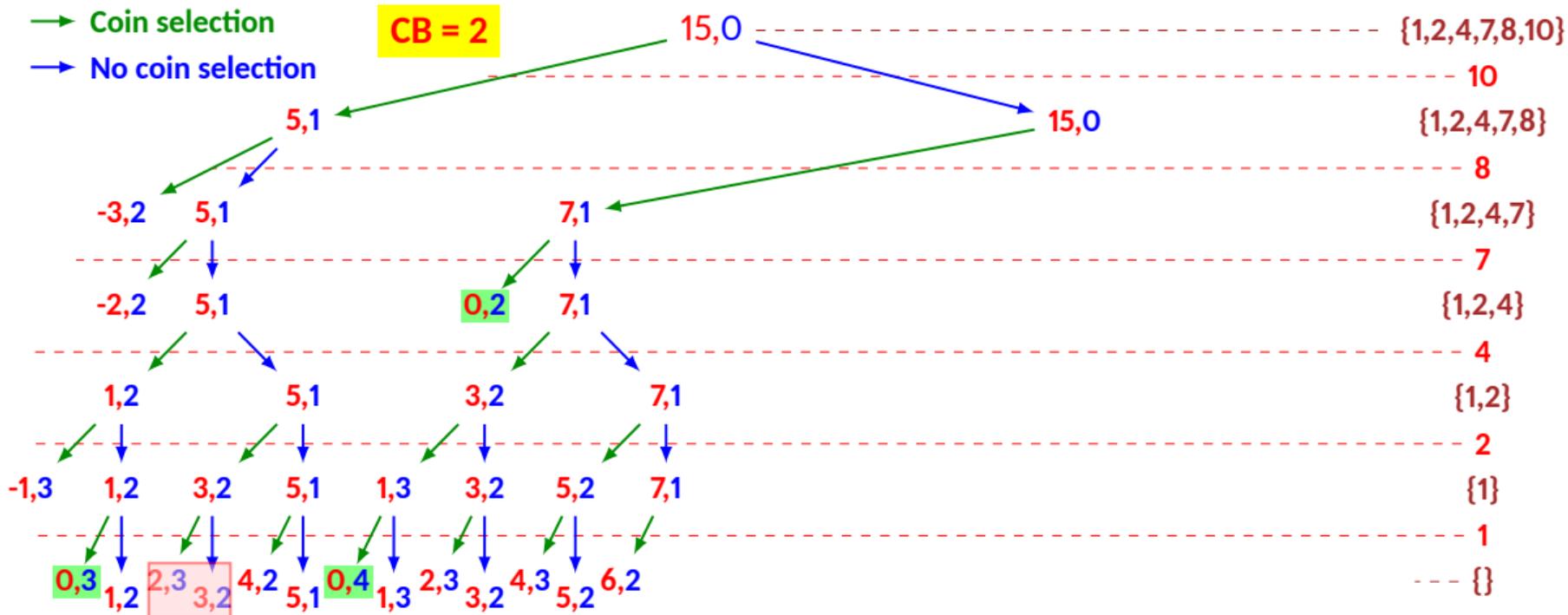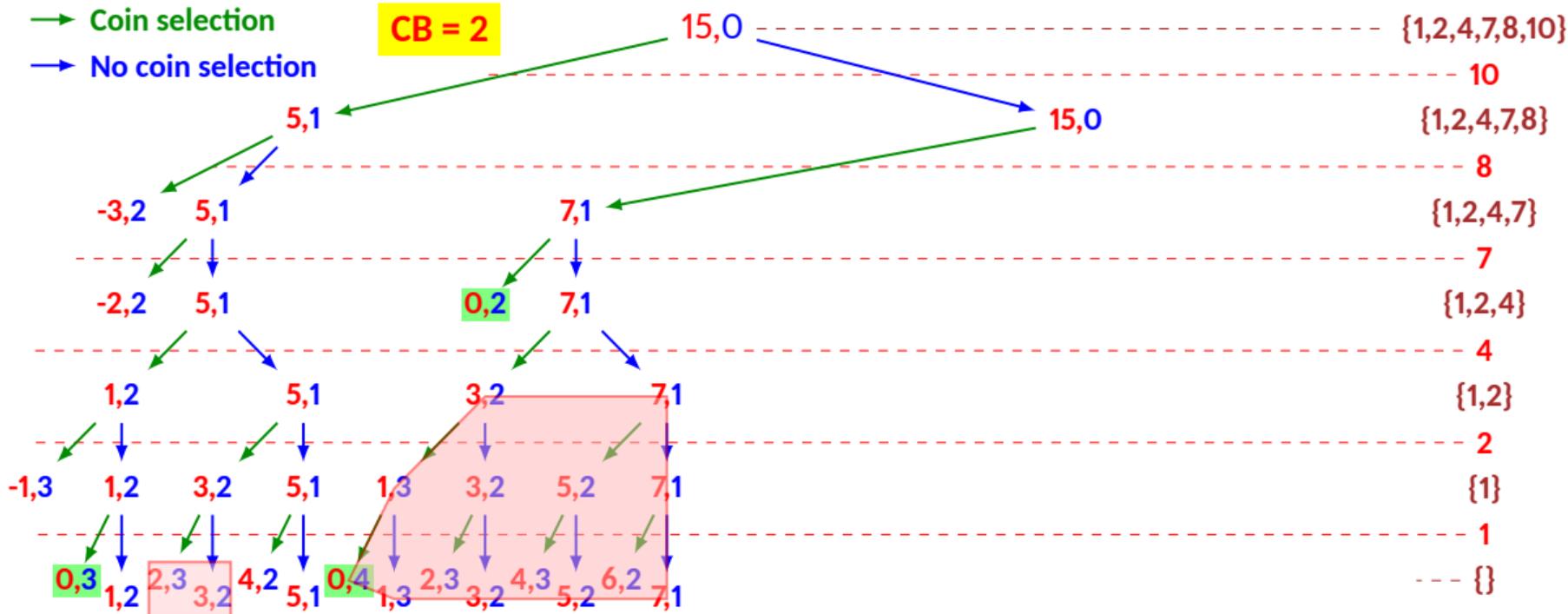0,3 1,2 3,2 4,2 5,1 0,4 1,3 2,3 3,2 - {}
      2,3 3,2

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

Coin selection (green arrow)
No coin selection (blue arrow)

CB = 2

15,0 → {1,2,4,7,8,10}
→ 10
5,1   15,0 → {1,2,4,7,8}
→ 8
-3,2   5,1   7,1 → {1,2,4,7}
→ 7
-2,2   5,1   0,2   7,1 → {1,2,4}
→ 4
1,2   5,1   3,2   7,1 → {1,2}
→ 2
-1,3 1,2 3,2 5,1 1,3 3,2 5,2 → {1}
→ 1
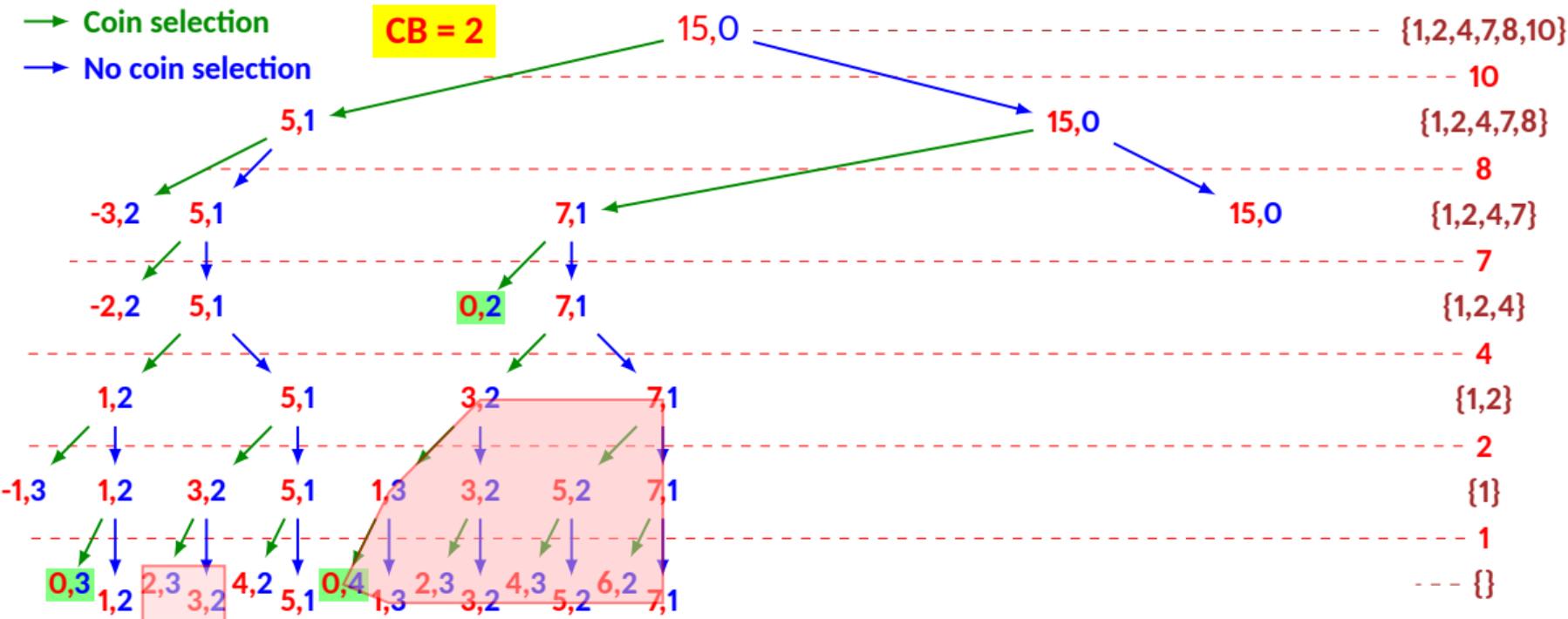0,3 1,2 2,3 3,2 4,2 5,1 0,4 1,3 2,3 3,2 → {}

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

CB = 2

→ Coin selection
→ No coin selection

15,0 ----------------------------- {1,2,4,7,8,10}
----------------------------------- 10
5,1   15,0   {1,2,4,7,8}
----------------------------------- 8
-3,2   5,1   7,1   {1,2,4,7}
----------------------------------- 7
-2,2   5,1   0,2   7,1   {1,2,4}
----------------------------------- 4
1,2   5,1   3,2   7,1   {1,2}
----------------------------------- 2
-1,3 1,2 3,2 5,1 1,3 3,2 5,2   {1}
----------------------------------- 1
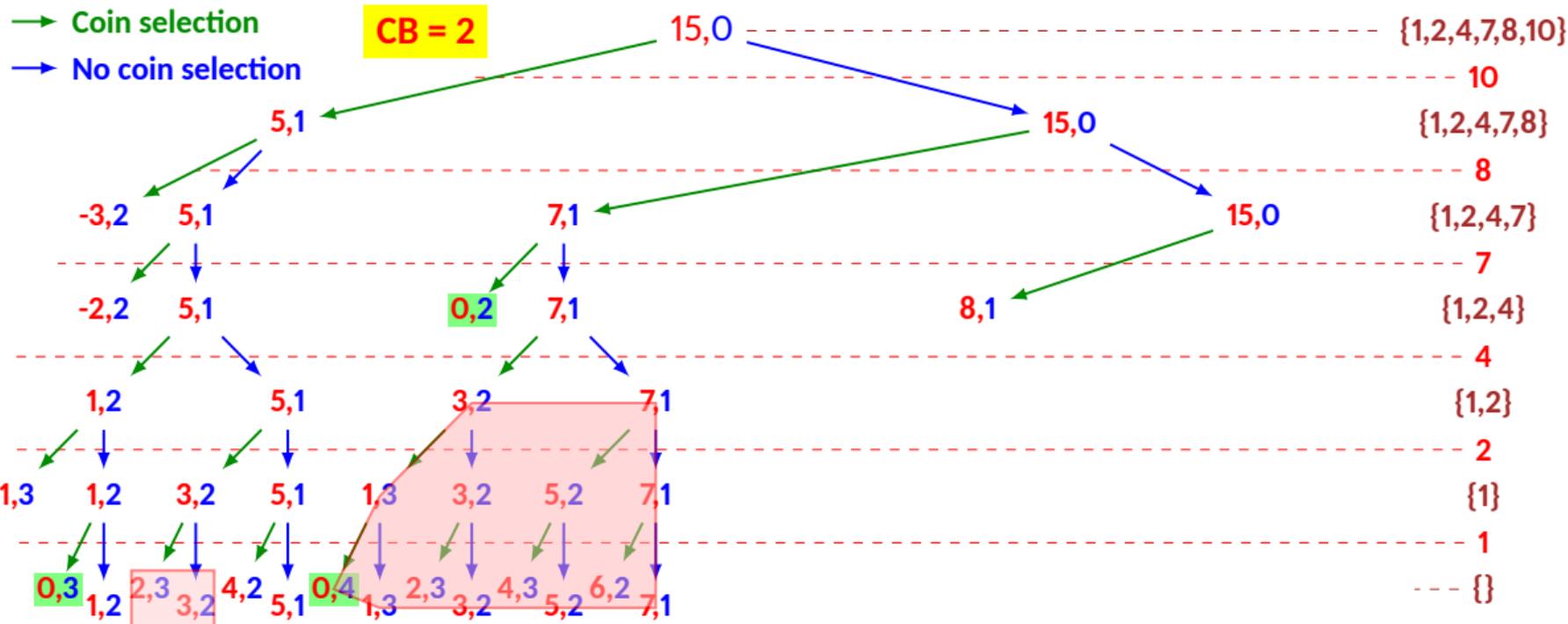0,3 1,2 3,2 4,2 0,4 2,3 4,3   {}
     2,3 3,2 5,1 1,3 3,2

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ Coin selection
→ No coin selection

CB = 2

15,0 ─────────────────────────── {1,2,4,7,8,10}
─────────────────────────────────── 10
5,1   15,0   {1,2,4,7,8}
──────────────────────────────────── 8
-3,2   5,1   7,1   {1,2,4,7}
──────────────────────────────────── 7
-2,2   5,1   0,2   7,1   {1,2,4}
──────────────────────────────────── 4
1,2   5,1   3,2   7,1   {1,2}
──────────────────────────────────── 2
-1,3 1,2 3,2 5,1 1,3 3,2 5,2   {1}
──────────────────────────────────── 1
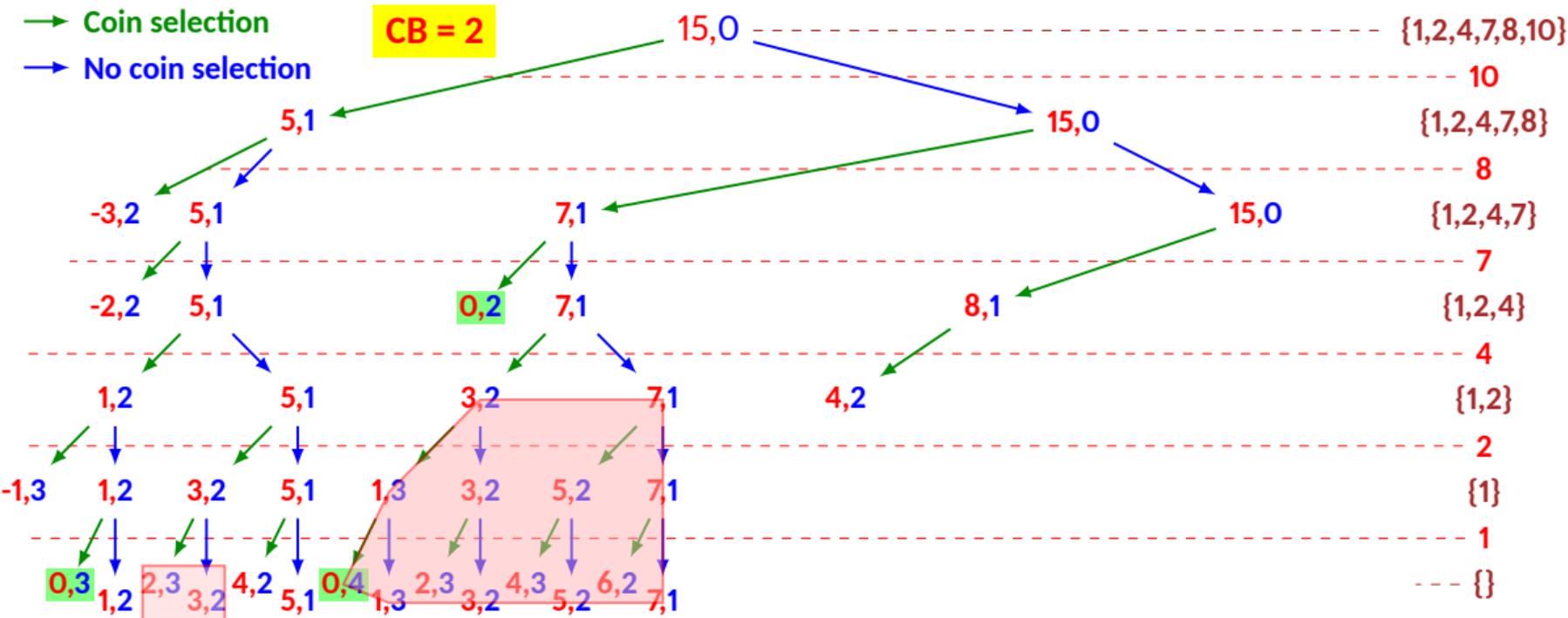0,3 1,2 2,3 4,2 0,4 2,3 4,3 5,2   {}
    3,2   5,1 1,3 3,2

Coin change: DFBB

- **Given a set of coins C, what is the minimum number coins required to provide sum S?**
- **Example:** $C = \{1, 2, 4, 7, 8, 10\}, S = 15$

→ Coin selection
→ No coin selection

CB = 2

15,0	{1,2,4,7,8,10}
	10
15,0	{1,2,4,7,8}
	8
7,1	{1,2,4,7}
	7
0,2 7,1	{1,2,4}
	4
3,2 7,1	{1,2}
	2
1,3 3,2 5,2 7,1	{1}
	1
0,4 2,3 3,2 4,3 5,2	{}

5,1
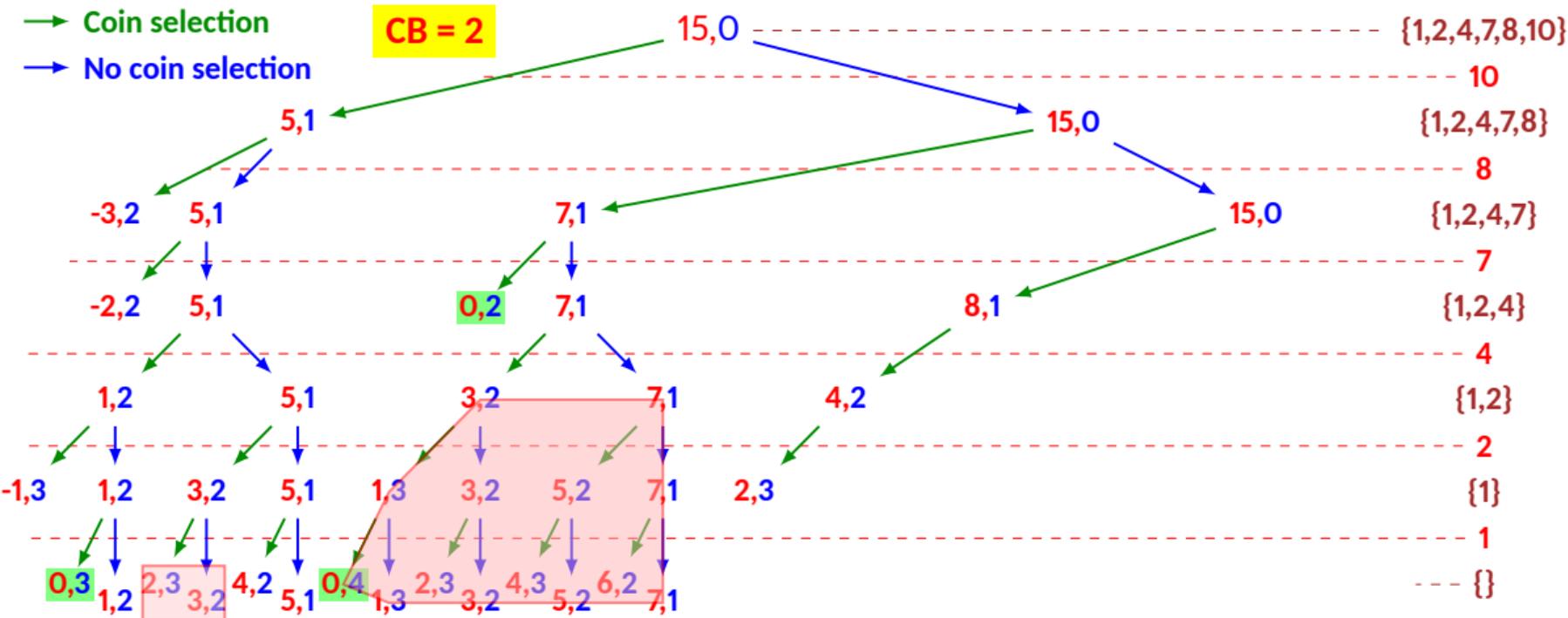
-3,2 5,1

-2,2 5,1

1,2 5,1

-1,3 1,2 3,2 5,1

0,3 1,2 3,2

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CB = 2

15,0 — — — — — — — — — — — — — — — — — — — {1,2,4,7,8,10}
— — — — — — — — — — — — — — 10
15,0 {1,2,4,7,8}
5,1 — — — — — — — — — 8
-3,2   5,1   7,1 {1,2,4,7}
— — — — — — — — 7
-2,2   5,1   0,2   7,1 {1,2,4}
— — — — — — — — 4
1,2   5,1   3,2   7,1 {1,2}
— — — — — — — — 2
-1,3   1,2   3,2   5,1   1,3   3,2   5,2   7,1 {1}
— — — — — — — — 1
0,3   1,2   2,3   3,2   4,2   5,1   0,4   1,3   2,3   3,2   4,3   5,2   6,2 - - - {}

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Coin selection
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15,0 — — — — — — — — — — — — — — — {1,2,4,7,8,10}
— — — — — — — — — — 10
15,0 {1,2,4,7,8}
5,1 — — — — — — — — — — 8
-3,2   5,1   7,1 {1,2,4,7}
— — — — — — — — — — 7
-2,2   5,1   0,2   7,1 {1,2,4}
— — — — — — — — — — 4
1,2   5,1   3,2   7,1 {1,2}
— — — — — — — — — — 2
-1,3 1,2 3,2 5,1   1,3 3,2 5,2 7,1 {1}
— — — — — — — — — — 1
0,3 1,2 2,3 3,2 4,2 5,1   0,4 1,3 2,3 3,2 4,3 5,2 6,2 7,1 {}

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15,0 — — — — — — — — — — — — — — {1,2,4,7,8,10}

— — — — 10

15,0 — — {1,2,4,7,8}

5,1

— — 8

-3,2   5,1   7,1   15,0 — {1,2,4,7}

— — 7

-2,2 5,1   0,2   7,1   8,1   {1,2,4}

— — 4

1,2   5,1   3,2   7,1   {1,2}

— — 2

-1,3 1,2 3,2 5,1   1,3 3,2 5,2 7,1   {1}

— — 1

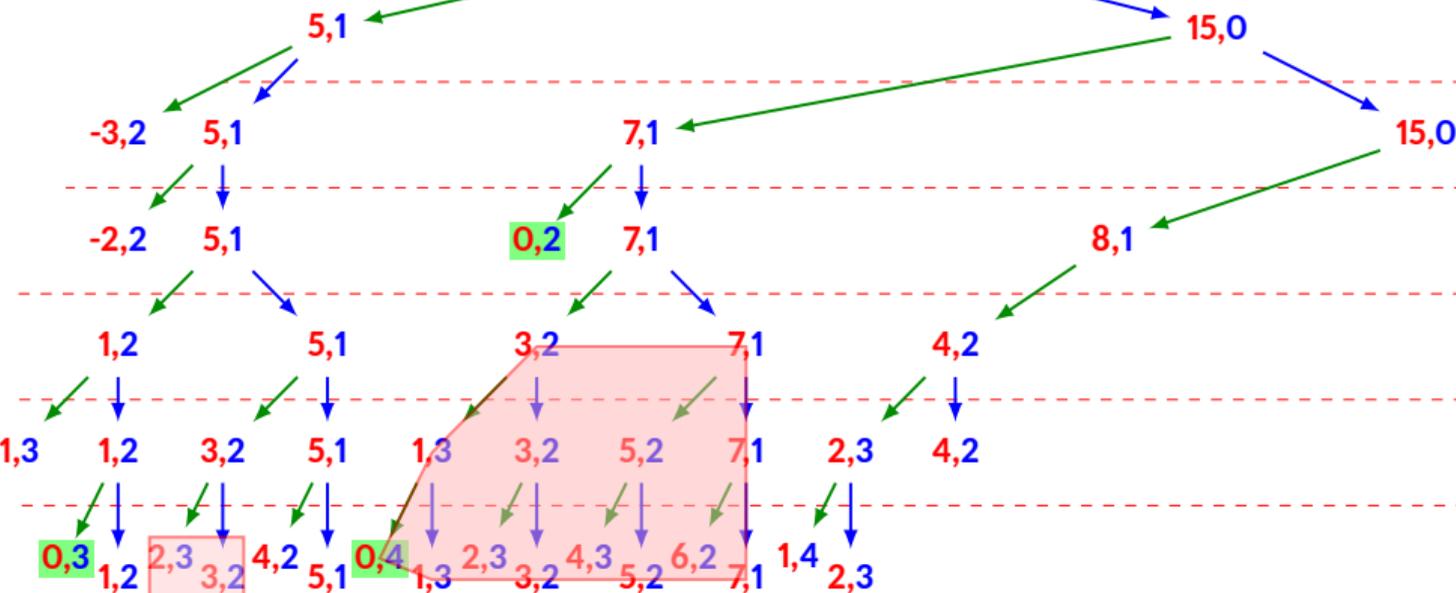0,3 1,2 2,3 3,2 4,2 5,1 0,4 1,3 2,3 3,2 4,3 5,2 6,2 7,1   {}

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Coin selection →
No coin selection →

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CB = 2

→ Coin selection
→ No coin selection

15,0 — — — — — — — — — — — — — — — — — — — {1,2,4,7,8,10}
— — — 10
5,1   15,0   {1,2,4,7,8}
— — — 8
-3,2   5,1   7,1   15,0   {1,2,4,7}
— — — 7
-2,2   5,1   0,2   7,1   8,1   {1,2,4}
— — — 4
1,2   5,1   3,2   7,1   4,2   {1,2}
— — — 2
-1,3   1,2   3,2   5,1   1,3   3,2   5,2   7,1   2,3   {1}
— — — 1
0,3   1,2   2,3   3,2   4,2   5,1   0,4   1,3   2,3   3,2   4,3   5,2   6,2   7,1   {}
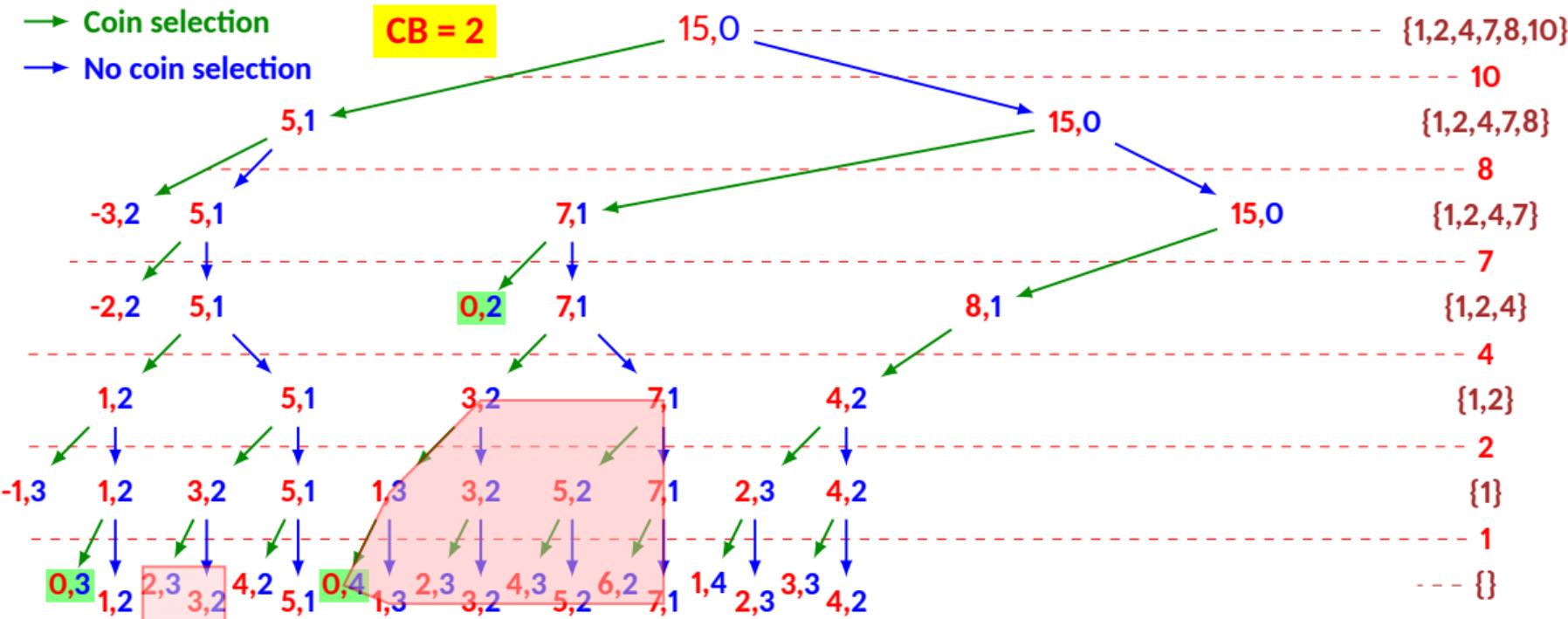
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CB = 2

→ Coin selection
→ No coin selection

15,0	{1,2,4,7,8,10}
	10
15,0	{1,2,4,7,8}
	8
15,0	{1,2,4,7}
	7
	{1,2,4}
	4
	{1,2}
	2
	{1}
	1
	{}

The diagram contains many node labels: 5,1 -3,2 5,1 7,1 15,0 -2,2 5,1 0,2 7,1 8,1 1,2 5,1 3,2 7,1 4,2 -1,3 1,2 3,2 5,1 1,3 3,2 5,2 7,1 2,3 0,3 1,2 2,3 3,2 4,2 5,1 0,4 1,3 2,3 3,2 4,3 5,2 6,2 7,1 1,4 2,3
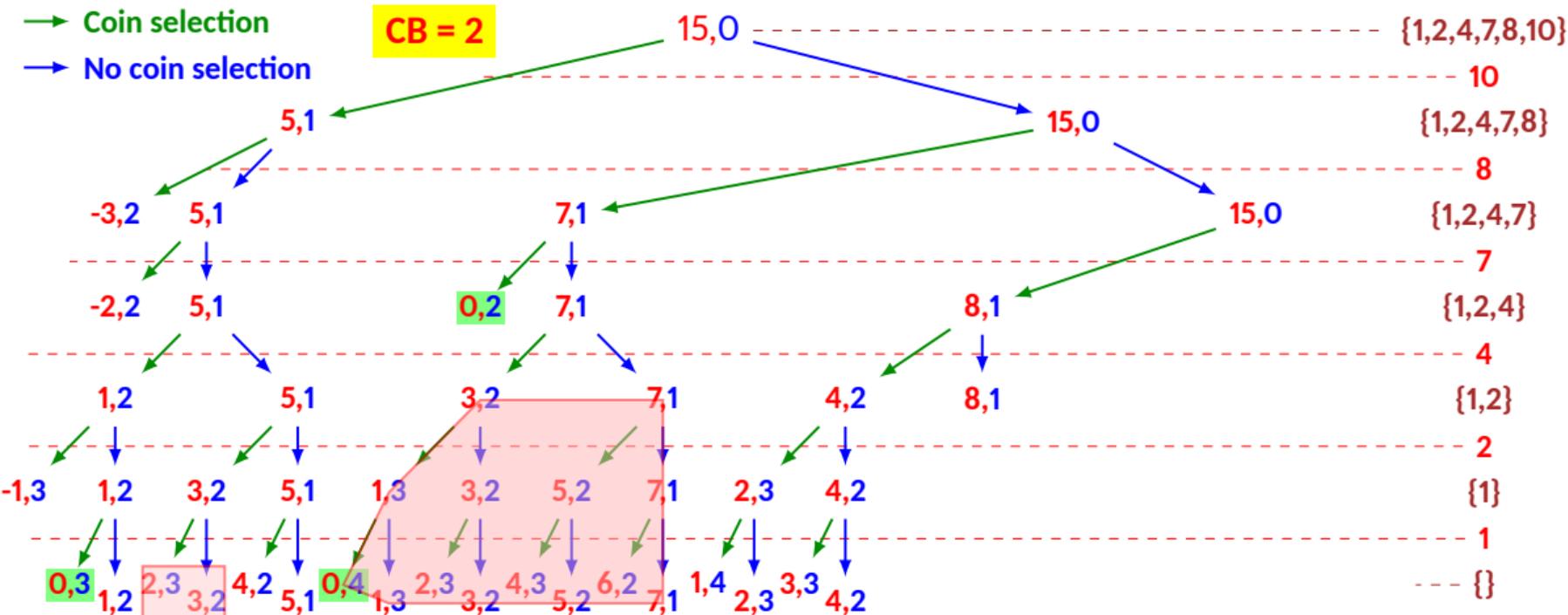
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→ Coin selection
→ No coin selection

CB = 2

15,0 — — — — — — — — — {1,2,4,7,8,10}

— — — — — — — — 10

5,1   15,0 {1,2,4,7,8}

— — — — — — — 8

-3,2   5,1   7,1   15,0 {1,2,4,7}

— — — — — — 7

-2,2   5,1   0,2   7,1   8,1 {1,2,4}

— — — — — 4

1,2   5,1   3,2   7,1   4,2 {1,2}

— — — — — 2

-1,3   1,2   3,2   5,1   1,3   3,2   5,2   7,1   2,3   4,2 {1}

— — — — 1

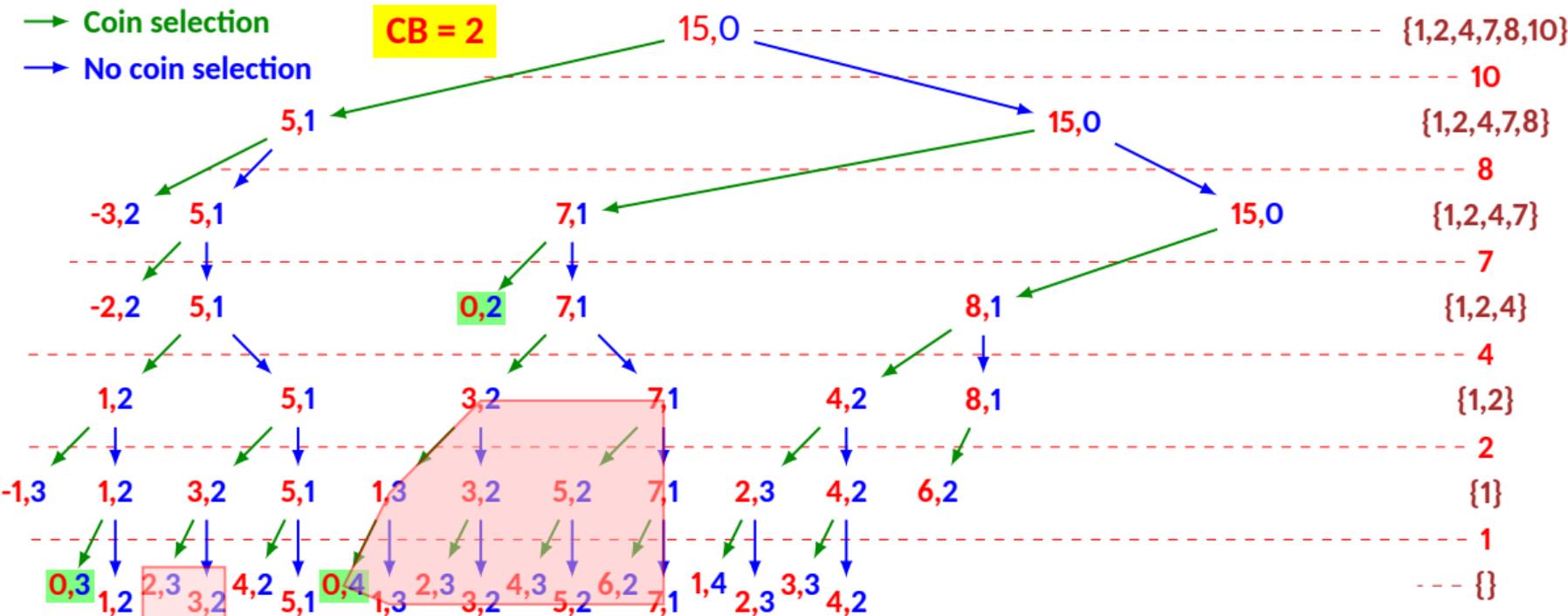0,3   1,2   2,3   3,2   4,2   5,1   0,4   1,3   2,3   3,2   4,3   5,2   6,2   7,1   1,4   2,3 — — {}

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→ No coin selection

CB = 2

15,0 — — — — — — — — — — — — — — — {1,2,4,7,8,10}
— — — — — — — — — — — — — — — 10
5,1   15,0   {1,2,4,7,8}
— — — — — — — — — — — — — — — 8
-3,2   5,1   7,1   15,0   {1,2,4,7}
— — — — — — — — — — — — — — — 7
-2,2   5,1   0,2   7,1   8,1   {1,2,4}
— — — — — — — — — — — — — — — 4
1,2   5,1   3,2   7,1   4,2   {1,2}
— — — — — — — — — — — — — — — 2
-1,3 1,2 3,2 5,1   1,3 3,2 5,2 7,1   2,3 4,2   {1}
— — — — — — — — — — — — — — — 1
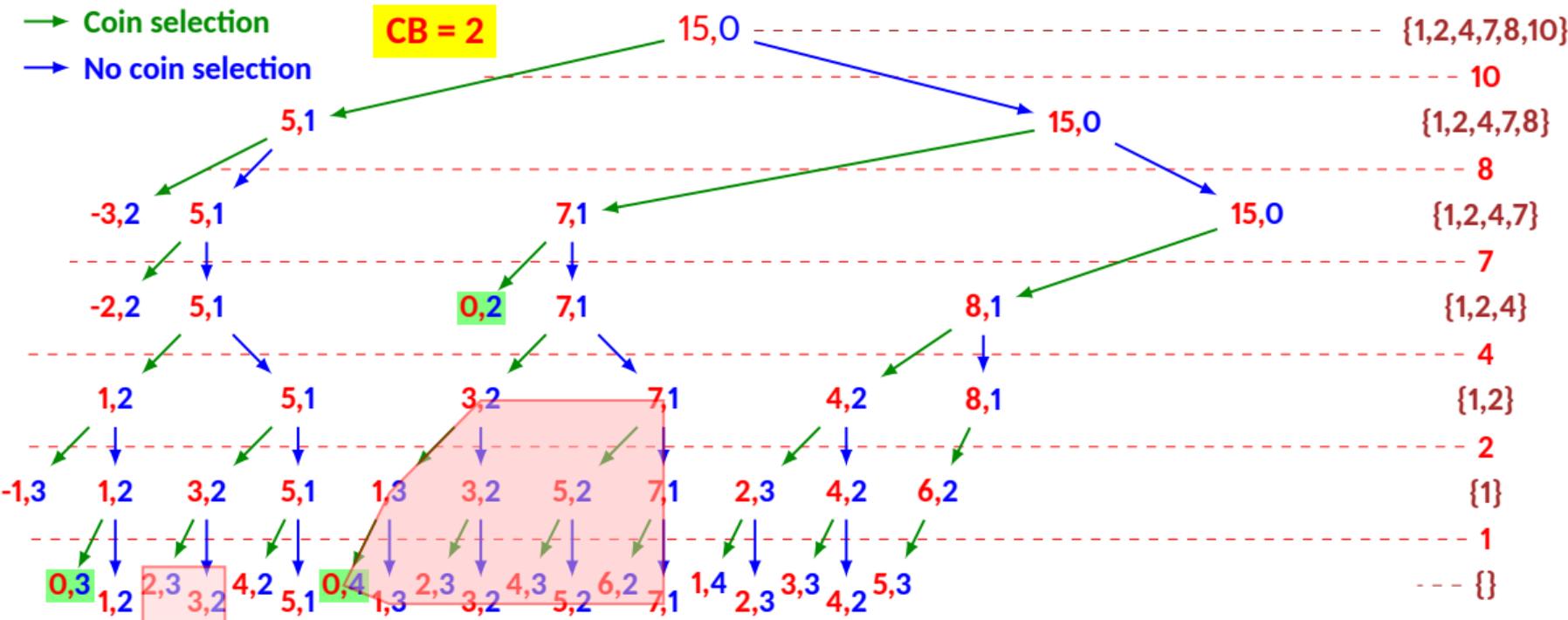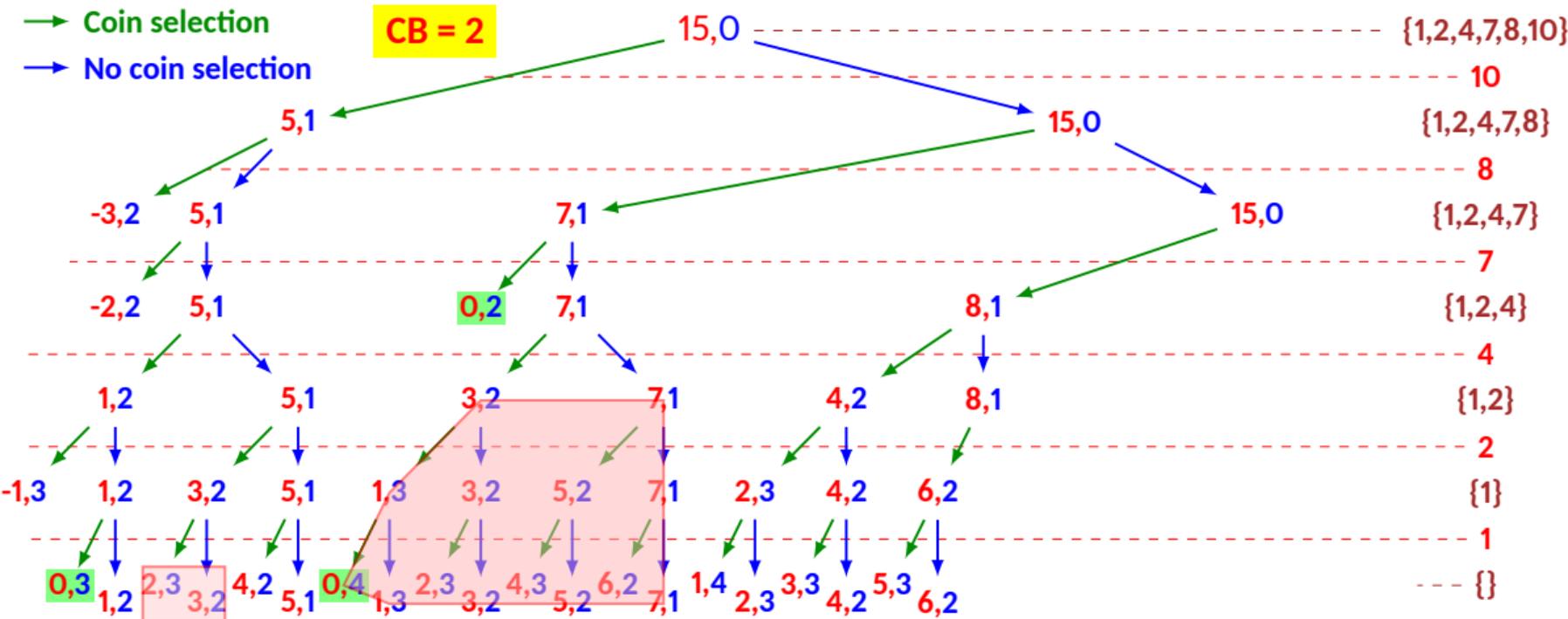0,3 1,2 2,3 3,2 4,2 5,1 0,4 1,3 2,3 3,2 4,3 5,2 6,2 7,1 1,4 2,3 3,3   — — {}

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CB = 2

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15,0 {1,2,4,7,8}

5,1 — — — — — — 8

-3,2   5,1   7,1   15,0 {1,2,4,7}

— — — — — — 7

-2,2   5,1   0,2   7,1   8,1 {1,2,4}

— — — — 4

1,2   5,1   3,2   7,1   4,2 {1,2}

— — 2

-1,3 1,2 3,2 5,1   1,3 3,2 5,2 7,1   2,3 4,2 {1}

— — 1

0,3 1,2 2,3 3,2 4,2 5,1   0,4 1,3 2,3 3,2 4,3 5,2 6,2 7,1   1,4 2,3 3,3 4,2 — — {}

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No coin selection

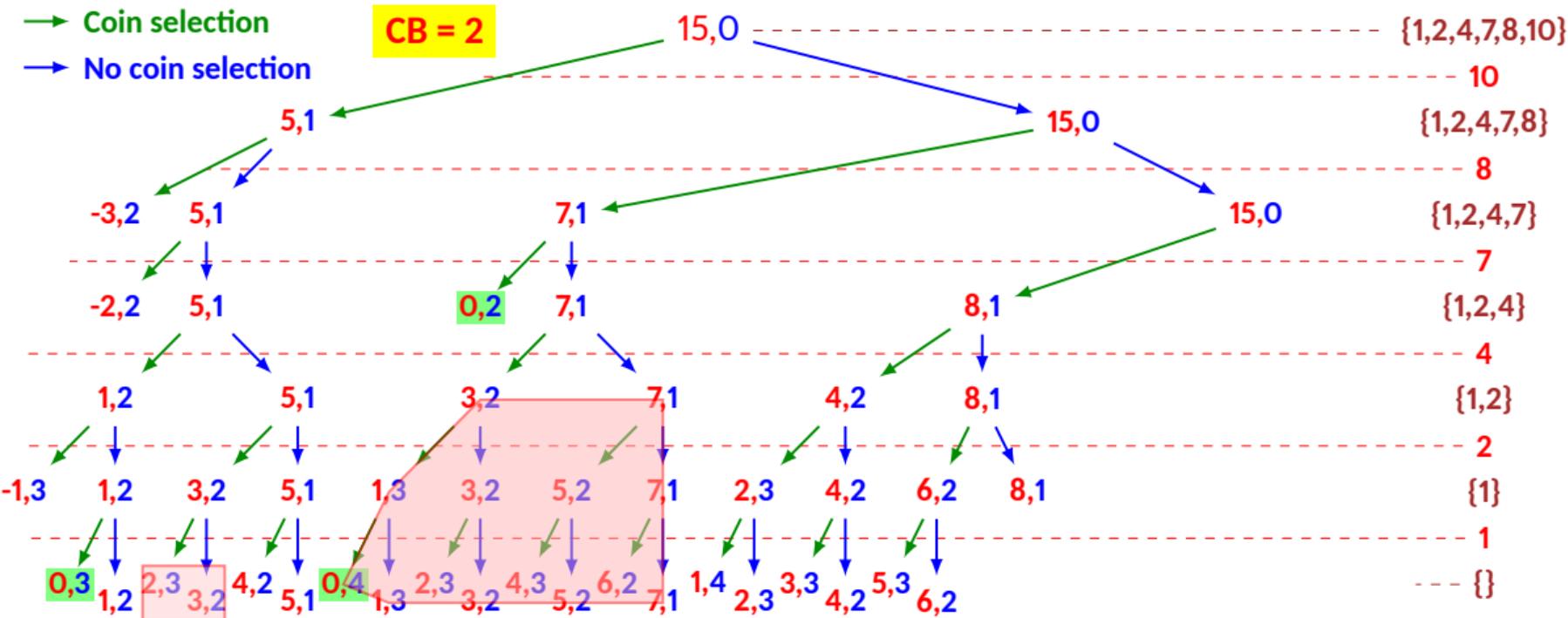CB = 2

{1,2,4,7,8,10}
10
{1,2,4,7,8}
8
{1,2,4,7}
7
{1,2,4}
4
{1,2}
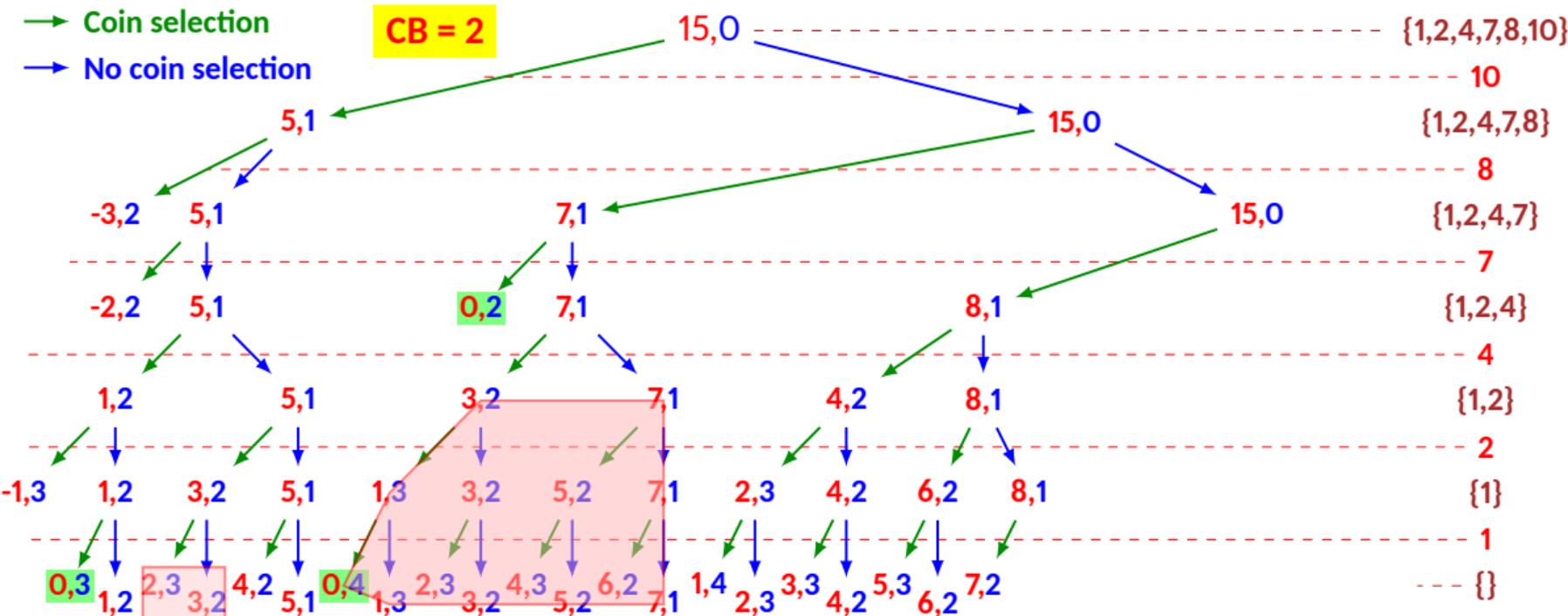2
{1}
1
{}

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CB = 2

Coin selection
No coin selection

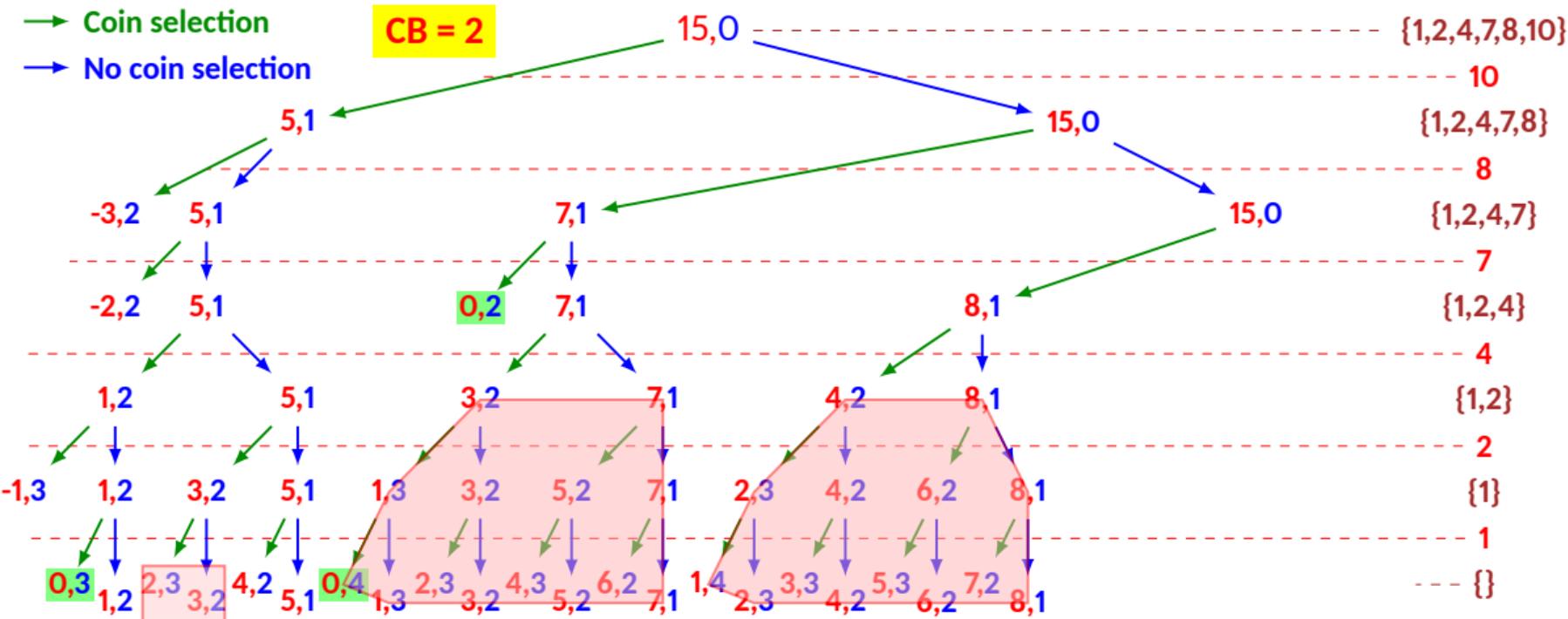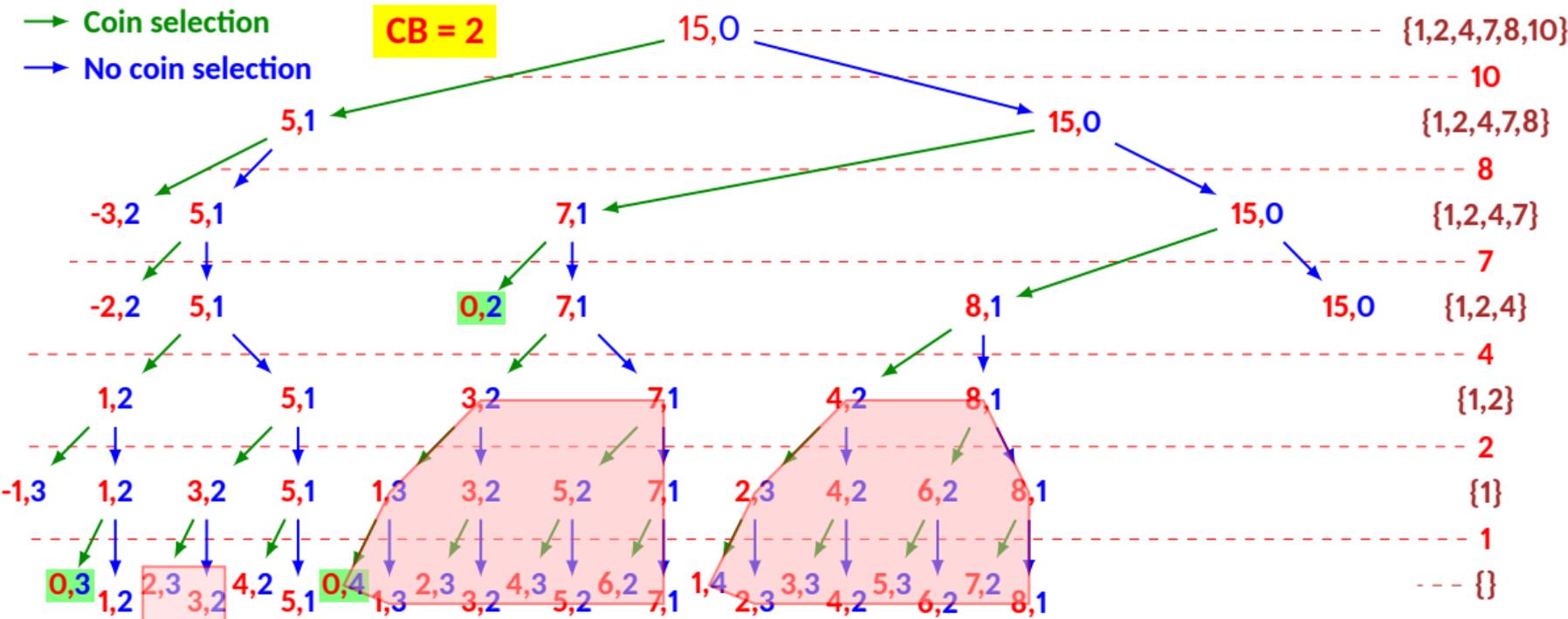{1,2,4,7,8,10}

10

{1,2,4,7,8}

8

{1,2,4,7}

7

{1,2,4}

4

{1,2}

2

{1}

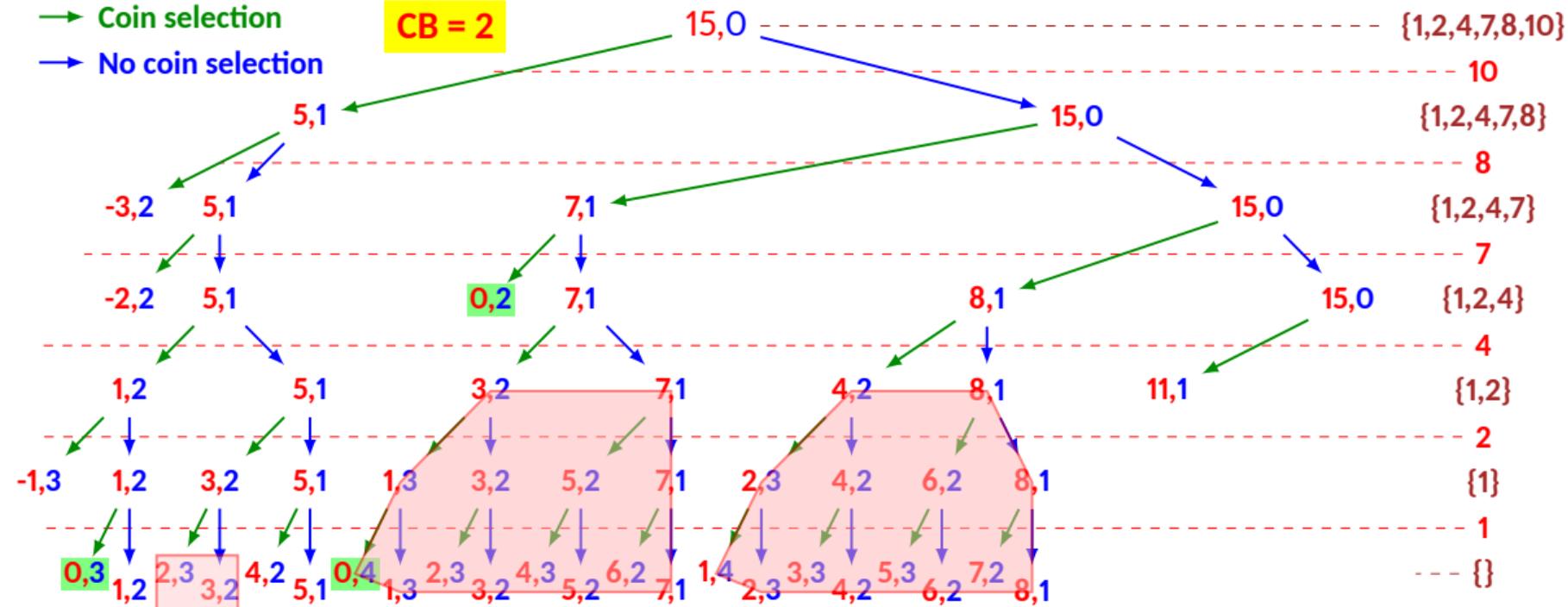1

{}

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No coin selection
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15,0 — — — — — — — — — — — — — — — {1,2,4,7,8,10}
— — — — — — — — — — — — — — — — — — 10
5,1   15,0 — — — {1,2,4,7,8}
— — — — — — — — — — — — — — — — — — 8
-3,2 5,1   7,1   15,0 — {1,2,4,7}
— — — — — — — — — — — — — — — — — — 7
-2,2 5,1   0,2 7,1   8,1   15,0 {1,2,4}
— — — — — — — — — — — — — — — — — — 4
1,2 5,1   3,2 7,1   4,2 8,1   11,1 {1,2}
— — — — — — — — — — — — — — — — — — 2
-1,3 1,2 3,2 5,1 1,3 3,2 5,2 7,1 2,3 4,2 6,2 8,1 9,2 11,1 {1}
— — — — — — — — — — — — — — — — — — 1
0,3 1,2 2,3 3,2 4,2 5,1 0,4 1,3 2,3 3,2 4,3 5,2 6,2 7,1 1,4 2,3 3,3 4,2 5,3 6,2 7,2 8,1 8,3 9,2 {}
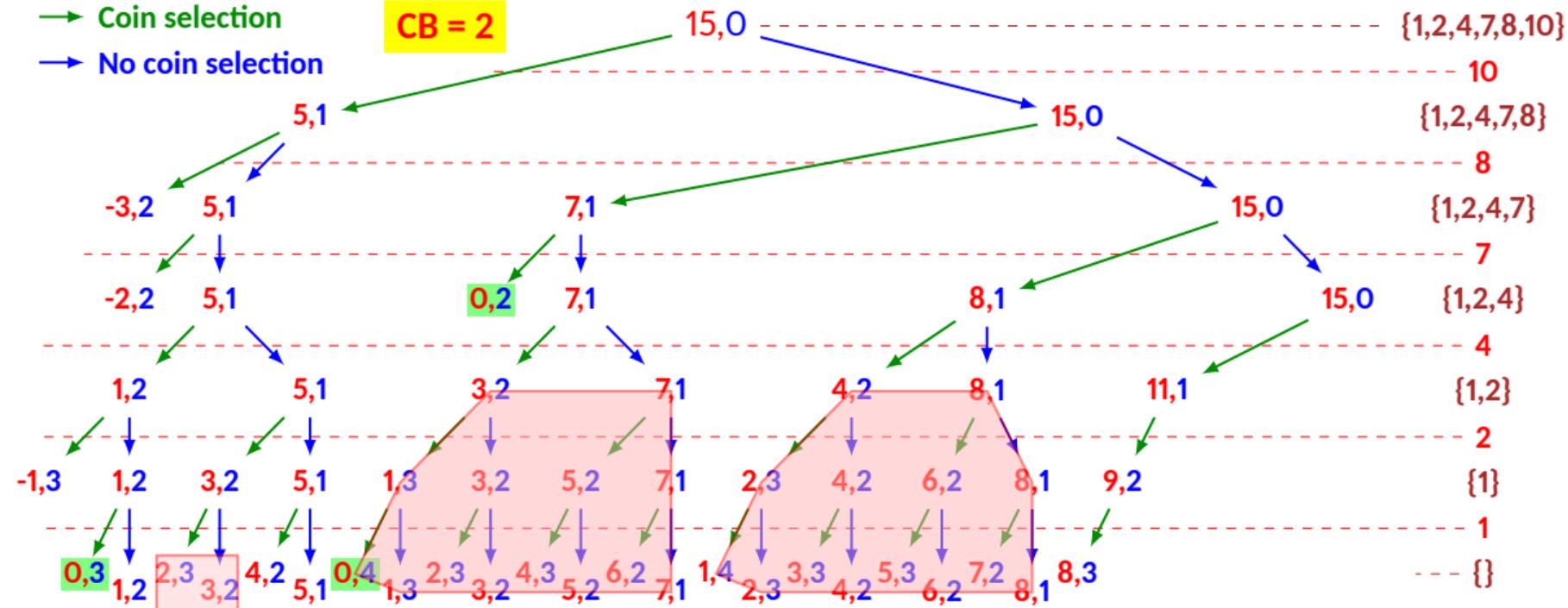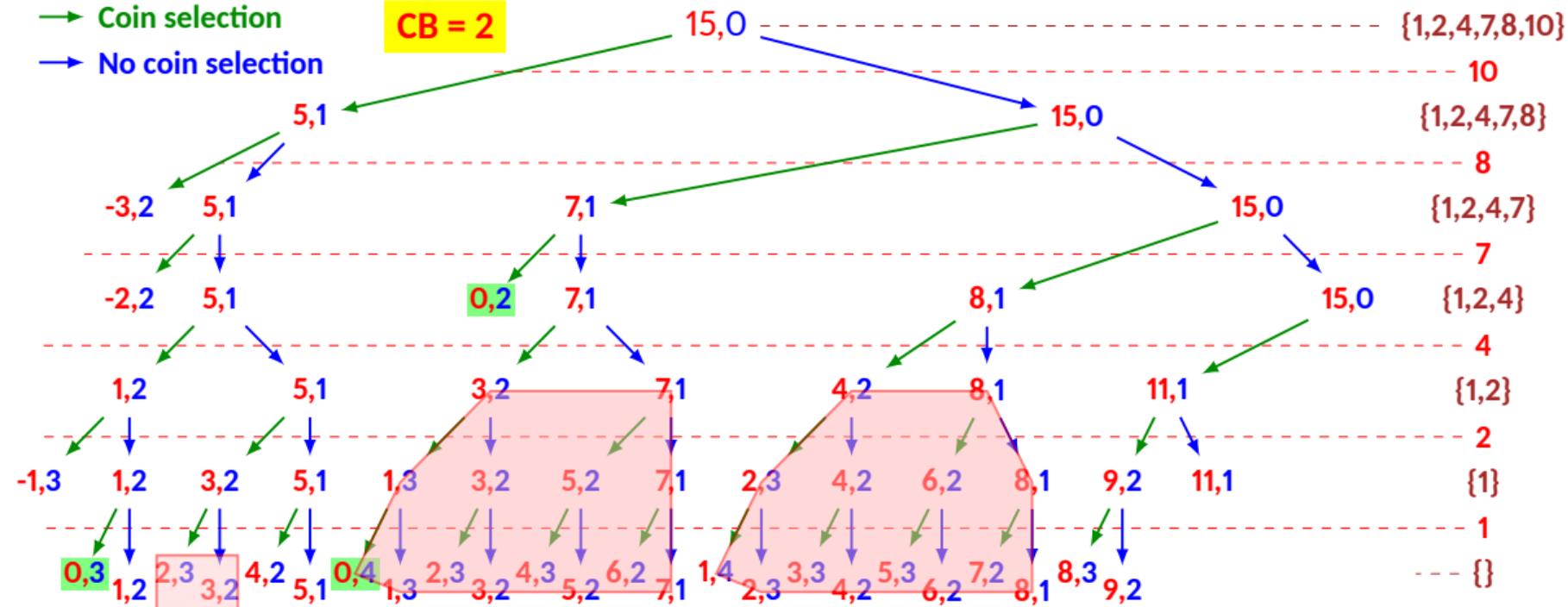
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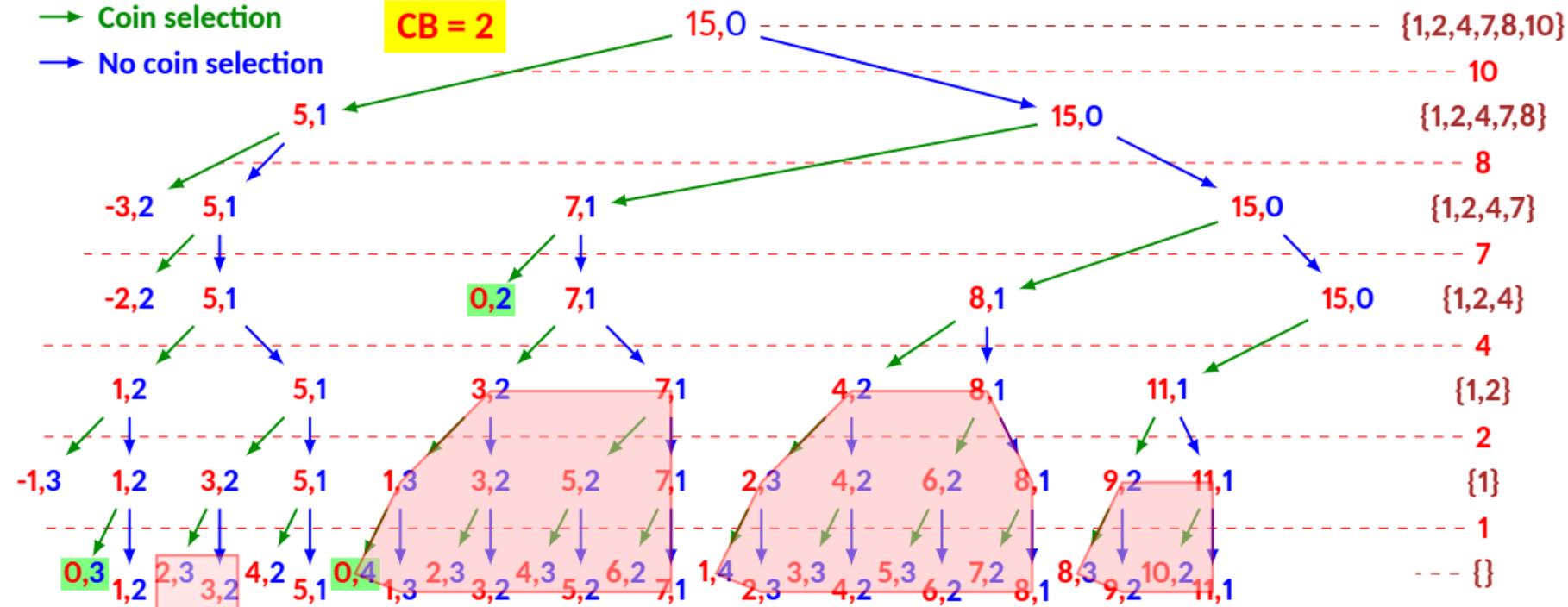
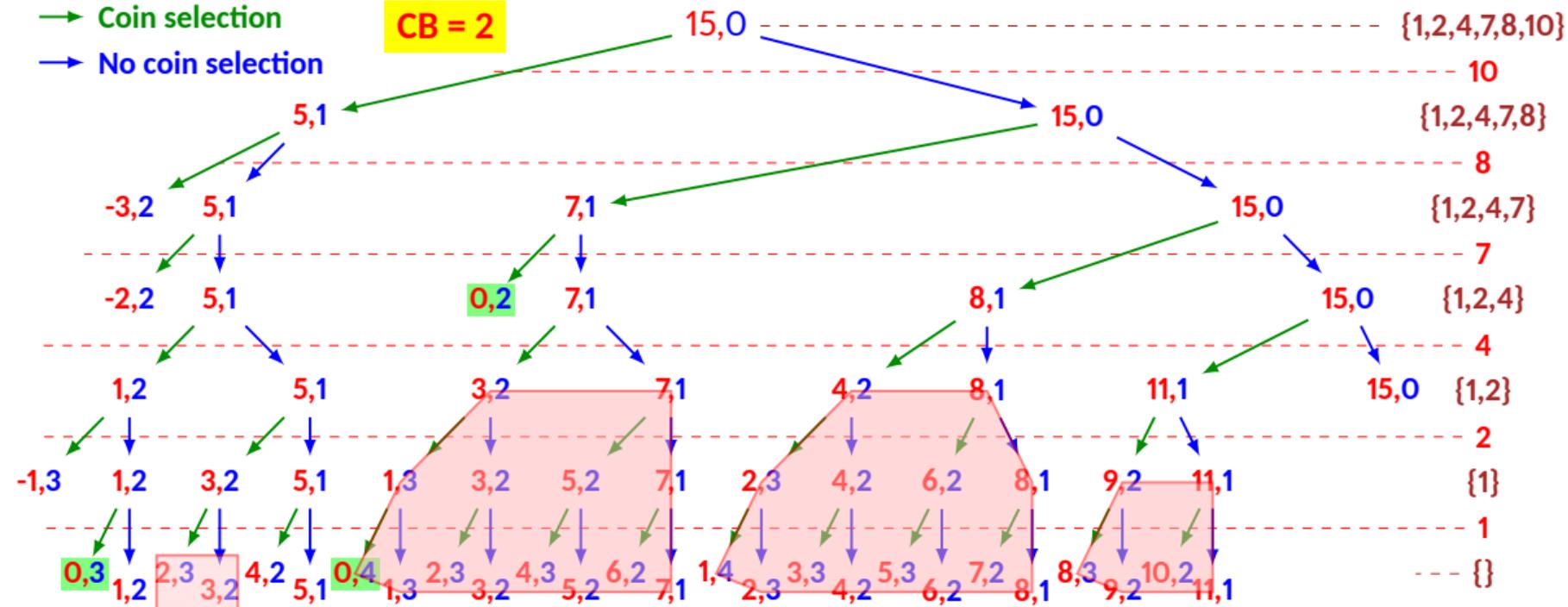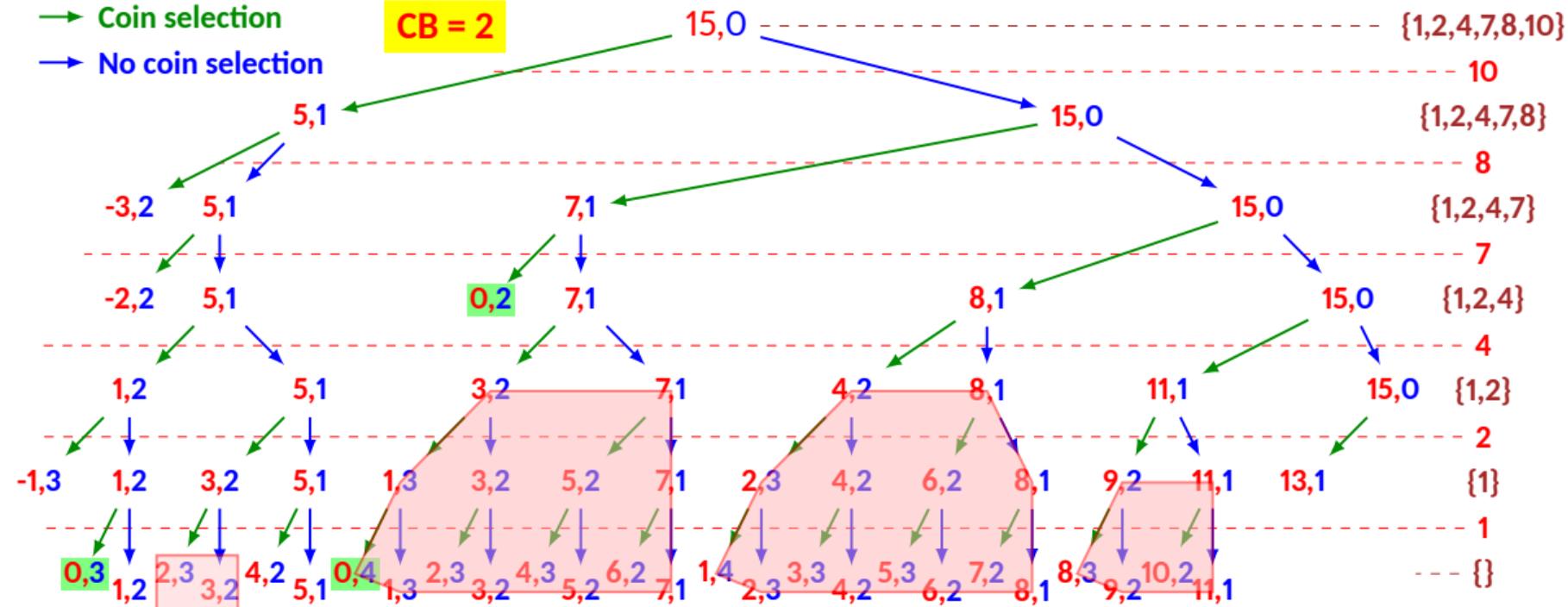# Coin change: DFBB

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— 10
5,1   15,0   {1,2,4,7,8}
— 8
-3,2 5,1   7,1   15,0   {1,2,4,7}
— 7
-2,2 5,1   0,2 7,1   8,1   15,0 {1,2,4}
— 4
1,2 5,1   3,2 7,1   4,2 8,1   11,1 15,0 {1,2}
— 2
-1,3 1,2 3,2 5,1 1,3 3,2 5,2 7,1 2,3 4,2 6,2 8,1 9,2 11,1 13,1 {1}
— 1
0,3 1,2 2,3 3,2 4,2 5,1 0,4 1,3 2,3 3,2 4,3 5,2 6,2 7,1 1,4 2,3 3,3 4,2 5,3 6,2 7,2 8,1 8,3 9,2 10,2 11,1 — — {}

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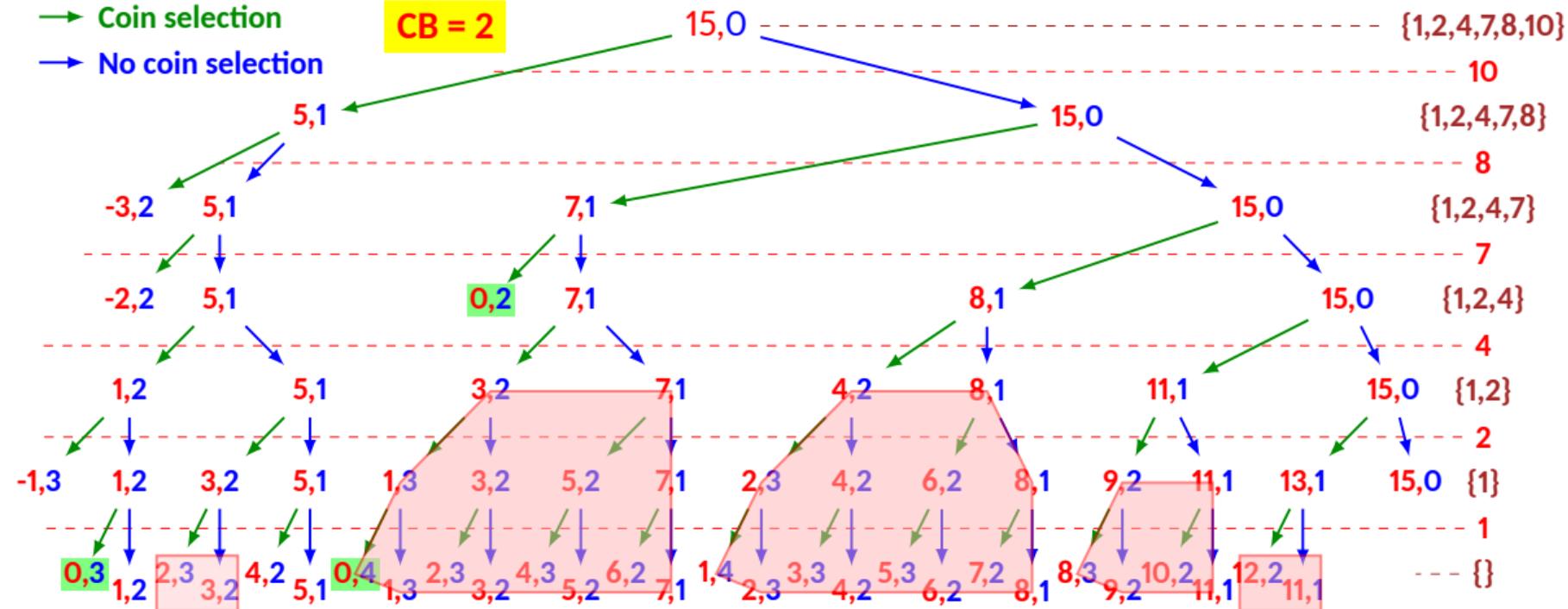

Coin selection

No coin selection

CB = 2

15,0 — — — {1,2,4,7,8,10}

10

5,1 — — 15,0 {1,2,4,7,8}

8

-3,2 5,1   7,1   15,0 {1,2,4,7}

7

-2,2 5,1   0,2 7,1   8,1   15,0 {1,2,4}

4

1,2 5,1   3,2 7,1 4,2 8,1 11,1 15,0 {1,2}

2

-1,3 1,2 3,2 5,1 1,3 3,2 5,2 7,1 2,3 4,2 6,2 8,1 9,2 11,1 13,1 {1}

1

0,3 1,2 2,3 3,2 4,2 5,1 0,4 1,3 2,3 3,2 4,3 5,2 6,2 7,1 1,4 2,3 3,3 4,2 5,3 6,2 7,2 8,1 8,3 9,2 10,2 11,1 12,2 11,1 — — {}

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— 10
5,1   15,0   {1,2,4,7,8}
— 8
-3,2 5,1   7,1   15,0   {1,2,4,7}
— 7
-2,2 5,1   0,2 7,1   8,1   15,0   {1,2,4}
— 4
1,2 5,1   3,2   7,1   4,2   8,1   11,1   15,0   {1,2}
— 2
-1,3 1,2 3,2 5,1 1,3 3,2 5,2 7,1 2,3 4,2 6,2 8,1 9,2 11,1 13,1 15,0 {1}
— 1
0,3 1,2 2,3 3,2 4,2 5,1 0,4 1,3 2,3 3,2 4,3 5,2 6,2 7,1 1,4 2,3 3,3 4,2 5,3 6,2 7,2 8,1 8,3 9,2 10,2 11,1 12,2 11,1 — — {}

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→ Coin selection
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$\{1,2,4,7,8,10\}$
10
$\{1,2,4,7,8\}$
8
$\{1,2,4,7\}$
7
$\{1,2,4\}$
4
$\{1,2\}$
2
$\{1\}$
1
$\{\}$

Iterative deepening A* (IDA*)

1. **Set Cut-off Bound to** $f(s)$
2. **Perform DFBB with Cut-off Bound. Backtrack from any node whose** $f(n) >$ **Cut-off Bound.**
3. **If Solution is Found, at the end of one Iteration, Terminate with Solution**
4. **If Solution is not found in any iteration, then update Cut-off Bound to the lowest** $f(n)$ **among all nodes from which the algorithm Backtracked.**
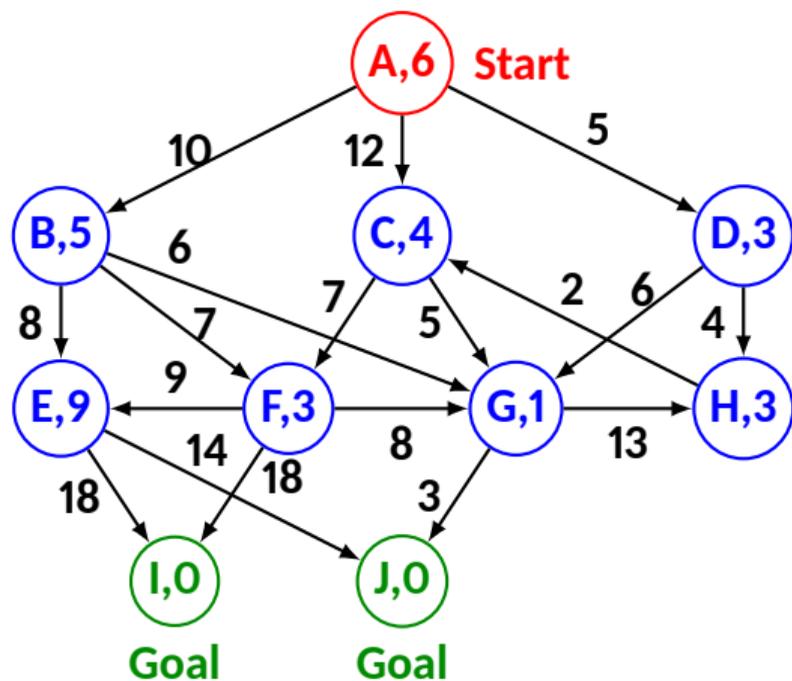5. **Go to Step 2**

Execution of IDA*
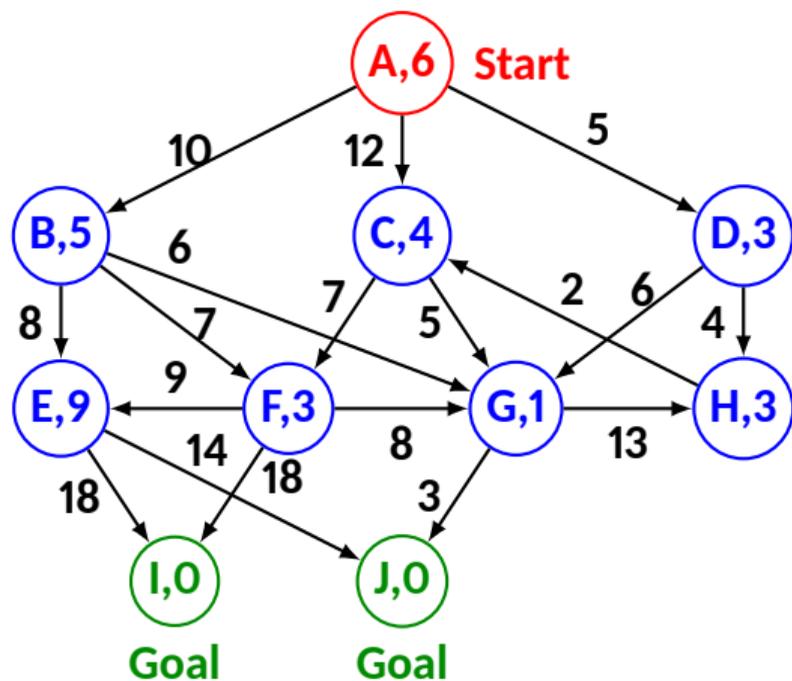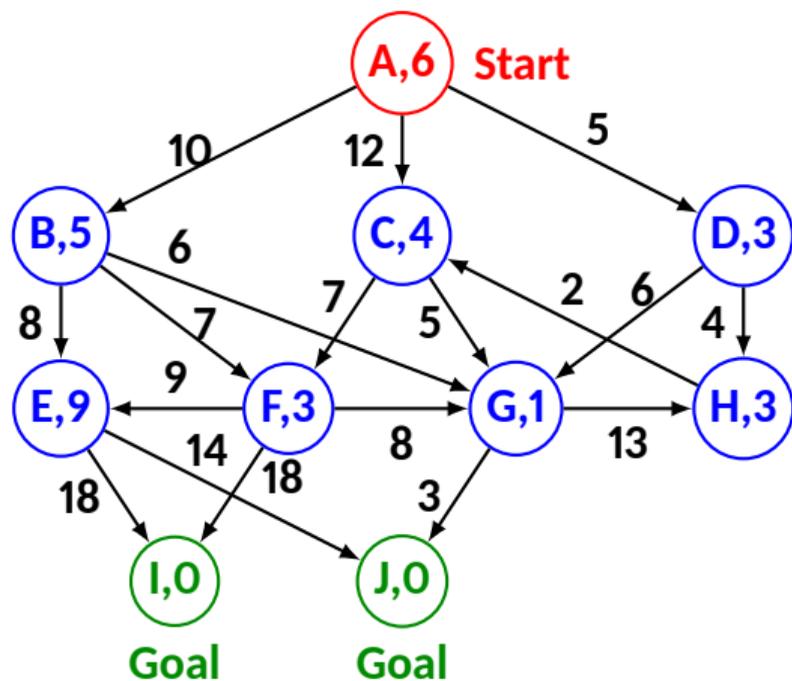
Execution of IDA*

Cutoff	OPEN
6	(B,15), (C,16), (D,8)

Execution of IDA*
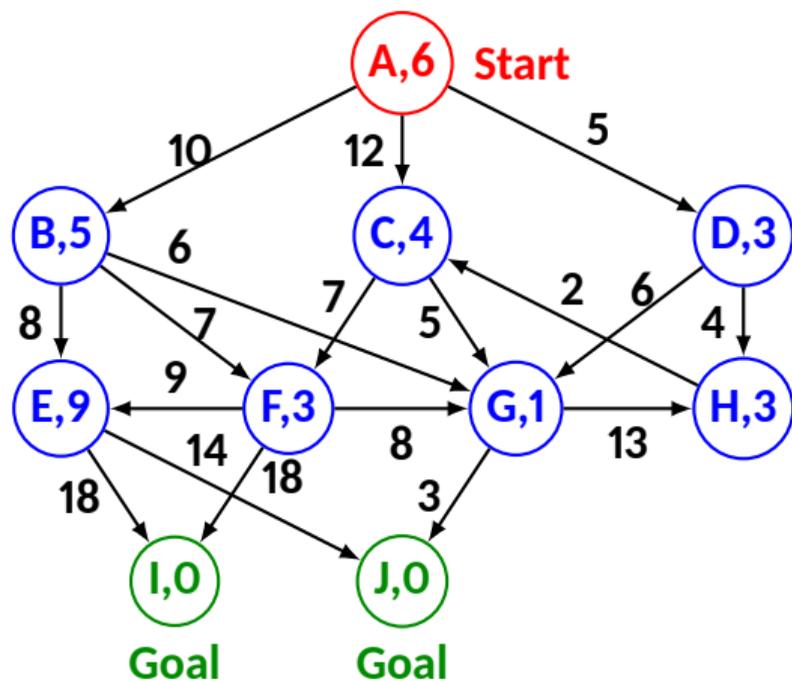
Cutoff	OPEN
6	(B,15), (C,16), (D,8)
8	(B,15), (C,16), (G,12), (H,12)

Execution of IDA*

Cutoff	OPEN
6	(B,15), (C,16), (D,8)
8	(B,15), (C,16), (G,12), (H,12)
12	(B,15), (C,15), (J,14)

Execution of IDA*
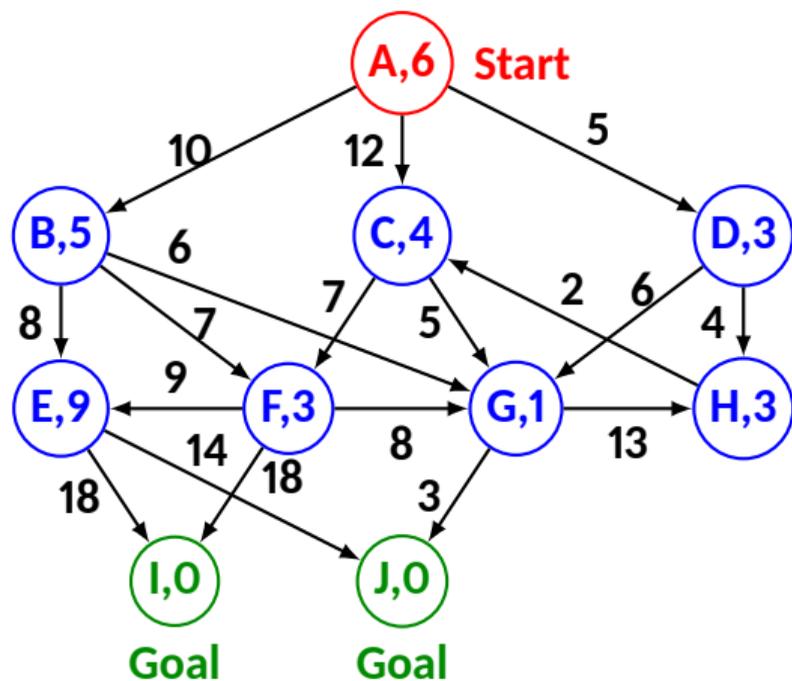
Cutoff	OPEN
6	(B,15), (C,16), (D,8)
8	(B,15), (C,16), (G,12), (H,12)
12	(B,15), (C,15), (J,14)
14	Terminate

Maximization problem
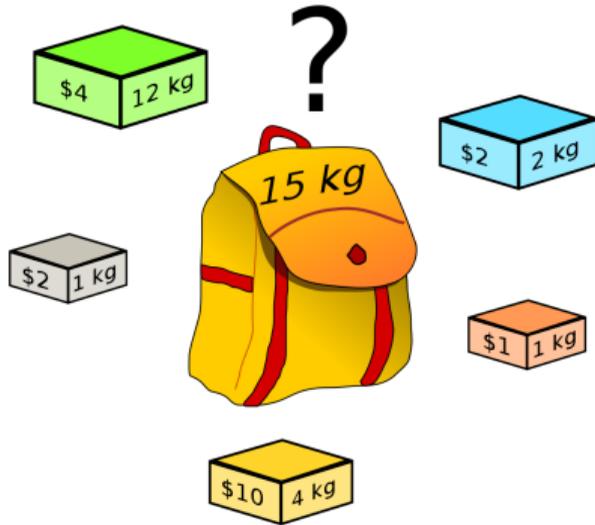
- $h(n)$ should be an overestimate of the actual cost
- In the A* algorithm, choose the node that has the largest $f(n)$ value for expansion
- What will be the state-space?
- Develop some heuristic functions and apply A*, IDA*

image source:Wikipedia

Thank you!