

MASTER THEOREM

General Form of the Master Theorem

$$T(n) = aT(n/b) + f(n)$$

where: where $a \geq 1$ and $b > 1$ are constants and $f(n)$ is an asymptotically positive function

A more detailed formulation writes the non-recursive term as:

$$f(n) = \Theta(n^k \log^p n)$$

Then calculate:

$$d = \log_b a$$

and compare d with k .

Case 1: $d > k$

$$T(n) = \Theta(n^d)$$

The recursive polynomial contribution dominates.

Case 2: $d = k$

Now the polynomial powers are equal, so the logarithmic exponent p matters:

$$(a) p > -1 \rightarrow T(n) = \Theta(n^k \log^{p+1} n)$$

$$(b) p = -1 \rightarrow T(n) = \Theta(n^k \log \log n)$$

$$(c) p < -1 \rightarrow T(n) = \Theta(n^k)$$

Case 3: $d < k$

$$(a) p \geq 0 \rightarrow T(n) = \Theta(n^k \log^p n)$$

$$(b) p < 0 \rightarrow T(n) = \Theta(n^k)$$

Use the Master Theorem to solve the following recurrences.

(a) $T(n) = 8T(n/2) + n^2$

(b) $T(n) = 4T(n/2) + n^2$

(c) $T(n) = 4T(n/2) + n^2/\log n$

(d) $T(n) = 4T(n/2) + n^2/\log^2 n$

(e) $T(n) = 4T(n/2) + n^2/\log n$

(f) $T(n) = 2T(n/2) + n^2 \log n$

(g) $T(n) = 2T(n/2) + n^2/\log n$

(h) $T(n) = 2T(n/2) + 1$

(i) $T(n) = 2T(n/2) + n$

(j) $T(n) = 2T(n/2) + n^2$

(k) $T(n) = 4T(n/2) + n$

(l) $T(n) = 4T(n/2) + n^2$

(m) $T(n) = 8T(n/2) + n^2$

(n) $T(n) = 2T(n/2) + n \log n$

(o) $T(n) = 2T(n/2) + n/\log n$

(p) $T(n) = 2T(n/2) + n/\log^2 n$

For Case 2 ($d=k$), prove equations (a), (b) and (c).