# CS5201: Advanced Artificial Intelligence

## Propositional Logic

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# First few examples

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- If I am the President then I am well-known. I am not the President. So I am not well-known.

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- **If I am the President then I am well-known. I am the President. So I am well-known.**
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  - $a$: I am the President
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- Coding sentences
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  - $F_2$: $a$

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  - $a$: **I am the President**
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- **Coding sentences**
  - $F_1$: $a \rightarrow b$
  - $F_2$: $a$
  - $G$: $b$
- **The final formula for deduction** $(F_1 \wedge F_2) \rightarrow G$ **that is** $((a \rightarrow b) \wedge a) \rightarrow b$

# Deduction using Propositional Logic: Example 1

- **If I am the President then I am well-known. I am the President. So I am well-known.**
- **Coding: Variables**
  - $a$: **I am the President,** $b$: **I am well-known**
- **Coding sentences**
  - $F_1$: $a \to b$, $F_2$: $a$, $G$: $b$
- **The final formula for deduction** $(F_1 \wedge F_2) \to G$ **that is** $((a \to b) \wedge a) \to b$

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| $a$ | $b$ | $a \rightarrow b$ | $(a \rightarrow b) \wedge a$ | $((a \rightarrow b) \wedge a) \rightarrow b$ |
|---|---|---|---|---|

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- If I am the President then I am well-known. I am the President. So I am well-known.
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  - $a$: I am the President, $b$: I am well-known
- Coding sentences
  - $F_1$: $a \to b$, $F_2$: $a$, $G$: $b$
- The final formula for deduction $(F_1 \wedge F_2) \to G$ that is $((a \to b) \wedge a) \to b$

| $a$ | $b$ | $a \to b$ | $(a \to b) \wedge a$ | $((a \to b) \wedge a) \to b$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | F | T | F | T |

# Deduction using Propositional Logic: Example 2

- **If I am the President then I am well-known. I am not the President. So I am not well-known.**

# Deduction using Propositional Logic: Example 2

- If I am the President then I am well-known. I am not the President. So I am not well-known.
- Coding: Variables

# Deduction using Propositional Logic: Example 2

- If I am the President then I am well-known. I am not the President. So I am not well-known.
- Coding: Variables
  - $a$: I am the President, $b$: I am well-known

# Deduction using Propositional Logic: Example 2

- If I am the President then I am well-known. I am not the President. So I am not well-known.
- Coding: Variables
  - $a$: I am the President, $b$: I am well-known
- Coding sentences

# Deduction using Propositional Logic: Example 2

- If I am the President then I am well-known. I am not the President. So I am not well-known.
- Coding: Variables
  - $a$: I am the President, $b$: I am well-known
- Coding sentences
  - $F_1$: $a \rightarrow b$,

# Deduction using Propositional Logic: Example 2

- If I am the President then I am well-known. I am not the President. So I am not well-known.
- Coding: Variables
  - $a$: I am the President, $b$: I am well-known
- Coding sentences
  - $F_1$: $a \rightarrow b$, $F_2$: $\sim a$,

# Deduction using Propositional Logic: Example 2

- If I am the President then I am well-known. I am not the President. So I am not well-known.
- Coding: Variables
  - $a$: I am the President, $b$: I am well-known
- Coding sentences
  - $F_1$: $a \rightarrow b$, $F_2$: $\sim a$, $G$: $\sim b$

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- **If I am the President then I am well-known. I am not the President. So I am not well-known.**
- **Coding: Variables**
  - $a$: **I am the President,** $b$: **I am well-known**
- **Coding sentences**
  - $F_1$: $a \rightarrow b$, $F_2$: $\sim a$, $G$: $\sim b$
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| $a$ | $b$ | $a \rightarrow b$ | $(a \rightarrow b) \wedge \sim a$ | $((a \rightarrow b) \wedge \sim a) \rightarrow \sim b$ |
|---|---|---|---|---|

# Deduction using Propositional Logic: Example 2

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- **Coding: Variables**
  - $a$: **I am the President,** $b$: **I am well-known**
- **Coding sentences**
  - $F_1$: $a \rightarrow b$, $F_2$: $\sim a$, $G$: $\sim b$
- **The final formula for deduction** $(F_1 \wedge F_2) \rightarrow G$ **that is** $((a \rightarrow b) \wedge \sim a) \rightarrow \sim b$

| $a$ | $b$ | $a \rightarrow b$ | $(a \rightarrow b) \wedge \sim a$ | $((a \rightarrow b) \wedge \sim a) \rightarrow \sim b$ |
|---|---|---|---|---|
| T | T | T | F | T |
| T | F | F | F | T |
| F | T | T | T | F |
| F | F | T | T | T |

# Deduction using Propositional Logic: Example 3

- **If I am the President then I am well-known. I am the President. So I am well-known.**
- **Coding: Variables**
  - $a$: **I am the President,**
  - $b$: **I am well-known**
- **Coding sentences**
  - $F_1$: $a \rightarrow b$, $F_2$: $a$, $G$: $b$
- **The final formula for deduction** $(F_1 \wedge F_2) \rightarrow G$ **that is** $((a \rightarrow b) \wedge a) \rightarrow b$

| $a$ | $b$ | $a \rightarrow b$ | $(a \rightarrow b) \wedge a$ | $((a \rightarrow b) \wedge a) \rightarrow b$ |
|---|---|---|---|---|
| **T** | **T** | **T** | **T** | **T** |
| **T** | **F** | **F** | **F** | **T** |
| **F** | **T** | **T** | **F** | **T** |
| **F** | **F** | **T** | **F** | **T** |

- **If Rajat is the President then Rajat is well-known. Rajat is the President. So Rajat is well-known.**

# Deduction using Propositional Logic: Example 3

- **If I am the President then I am well-known. I am the President. So I am well-known.**
- **Coding: Variables**
  - $a$: **I am the President,**
  - $b$: **I am well-known**
- **Coding sentences**
  - $F_1$: $a \rightarrow b$, $F_2$: $a$, $G$: $b$
- **The final formula for deduction** $(F_1 \wedge F_2) \rightarrow G$ **that is** $((a \rightarrow b) \wedge a) \rightarrow b$

| $a$ | $b$ | $a \rightarrow b$ | $(a \rightarrow b) \wedge a$ | $((a \rightarrow b) \wedge a) \rightarrow b$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | F | T |
| F | T | T | F | T |
| F | F | T | F | T |

- **If Rajat is the President then Rajat is well-known. Rajat is the President. So Rajat is well-known.**
- **Coding: Variables**
  - $a$: **Rajat is the President,**
  - $b$: **Rajat is well-known**
- **Coding sentences**
  - $F_1$: $a \rightarrow b$, $F_2$: $a$, $G$: $b$
- **The final formula for deduction** $(F_1 \wedge F_2) \rightarrow G$ **that is** $((a \rightarrow b) \wedge a) \rightarrow b$

# Deduction using Propositional Logic: Example 4,5

- **If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore Asha is not elected VP.**

# Deduction using Propositional Logic: Example 4,5

- If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore Asha is not elected VP.
- If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is chosen as G-Sec. Therefore Asha is elected VP.

# More examples

- **If Asha is elected VP then Rajat is chosen as G-Sec or Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore if Asha is elected VP then Bharati is chosen as Treasurer.**

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- **If Asha is elected VP then Rajat is chosen as G-Sec or Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore if Asha is elected VP then Bharati is chosen as Treasurer.**
- **If Asha is elected VP then either Rajat is chosen as G-Sec or Bharati is chosen as Treasurer but not both. Rajat is not chosen as G-Sec. Therefore if Asha is elected VP then Bharati is chosen as Treasurer.**

# More examples

- If Asha is elected VP then Rajat is chosen as G-Sec or Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore if Asha is elected VP then Bharati is chosen as Treasurer.
- If Asha is elected VP then either Rajat is chosen as G-Sec or Bharati is chosen as Treasurer but not both. Rajat is not chosen as G-Sec. Therefore if Asha is elected VP then Bharati is chosen as Treasurer.
- If I study, then I will not fail in CS5201. If I do not play mobile games too often, then I will study. I failed in CS5201. Therefore, I played mobile games too often.

# More examples

- If Asha is elected VP then Rajat is chosen as G-Sec or Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore if Asha is elected VP then Bharati is chosen as Treasurer.
- If Asha is elected VP then either Rajat is chosen as G-Sec or Bharati is chosen as Treasurer but not both. Rajat is not chosen as G-Sec. Therefore if Asha is elected VP then Bharati is chosen as Treasurer.
- If I study, then I will not fail in CS5201. If I do not play mobile games too often, then I will study. I failed in CS5201. Therefore, I played mobile games too often.
- Atul was telling you what he ate yesterday afternoon. He tells you, ”I had either popcorn or cutlet. Also, if I had cucumber sandwiches, then I had soda. But I didn’t drink soda or tea.” Of course, you know that Atul is the world’s worst liar, and everything he says is false. What did Atul eat?

# Propositional Logic

- Interpretation — It is the truth of a formula under a given assignment of Boolean variables to true or false
- Valid — If the formula is true under all interpretation
- Non-Valid — If there is an interpretation for which the formula is false
- Satisfiable — If there is an interpretation for which the formula is true
- Unsatisfiable — If the formula is false under all interpretations
- Propositional logic deduction problem is decidable but NP-Hard
- Validity checking — Truth table method, tree method
- Data structures — Binary decision diagrams (BDD)
- Symbolic method — Natural deduction is sound and complete

# Deduction in Propositional Logic

- **Modus Ponens:** $(a \to b), a$ **:- therefore** $b$

# Deduction in Propositional Logic

- **Modus Ponens:** $(a \rightarrow b), a$ **:- therefore** $b$
- **Modus Tollens:** $(a \rightarrow b), \sim b$ **:- therefore** $\sim a$

# Deduction in Propositional Logic

- **Modus Ponens:** $(a \rightarrow b), a$ **:- therefore** $b$
- **Modus Tollens:** $(a \rightarrow b), \sim b$ **:- therefore** $\sim a$
- **Hypothetical Syllogism:** $(a \rightarrow b), (b \rightarrow c)$ **:- therefore** $(a \rightarrow c)$

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- **Modus Tollens:** $(a \rightarrow b), \sim b$ **:- therefore** $\sim a$
- **Hypothetical Syllogism:** $(a \rightarrow b), (b \rightarrow c)$ **:- therefore** $(a \rightarrow c)$
- **Disjunctive Syllogism:** $(a \vee b), \sim a$ **:- therefore** $b$

# Deduction in Propositional Logic

- **Modus Ponens:** $(a \rightarrow b), a$ **:- therefore** $b$
- **Modus Tollens:** $(a \rightarrow b), \sim b$ **:- therefore** $\sim a$
- **Hypothetical Syllogism:** $(a \rightarrow b), (b \rightarrow c)$ **:- therefore** $(a \rightarrow c)$
- **Disjunctive Syllogism:** $(a \vee b), \sim a$ **:- therefore** $b$
- **Constructive Dilemma:** $(a \rightarrow b) \wedge (c \rightarrow d), (a \vee c)$ **:- therefore** $(b \vee d)$

# Deduction in Propositional Logic

- **Modus Ponens:** $(a \to b), a$ **:- therefore** $b$
- **Modus Tollens:** $(a \to b), \sim b$ **:- therefore** $\sim a$
- **Hypothetical Syllogism:** $(a \to b), (b \to c)$ **:- therefore** $(a \to c)$
- **Disjunctive Syllogism:** $(a \vee b), \sim a$ **:- therefore** $b$
- **Constructive Dilemma:** $(a \to b) \wedge (c \to d), (a \vee c)$ **:- therefore** $(b \vee d)$
- **Destructive Dilemma:** $(a \to b) \wedge (c \to d), (\sim b \vee \sim d)$ **:- therefore** $(\sim a \vee \sim c)$

# Deduction in Propositional Logic

- **Modus Ponens:** $(a \to b), a$ **:- therefore** $b$
- **Modus Tollens:** $(a \to b), \sim b$ **:- therefore** $\sim a$
- **Hypothetical Syllogism:** $(a \to b), (b \to c)$ **:- therefore** $(a \to c)$
- **Disjunctive Syllogism:** $(a \vee b), \sim a$ **:- therefore** $b$
- **Constructive Dilemma:** $(a \to b) \wedge (c \to d), (a \vee c)$ **:- therefore** $(b \vee d)$
- **Destructive Dilemma:** $(a \to b) \wedge (c \to d), (\sim b \vee \sim d)$ **:- therefore** $(\sim a \vee \sim c)$
- **Simplification:** $(a \wedge b)$ **:- therefore** $a$

# Deduction in Propositional Logic

- **Modus Ponens:** $(a \rightarrow b), a$ **:- therefore** $b$
- **Modus Tollens:** $(a \rightarrow b), \sim b$ **:- therefore** $\sim a$
- **Hypothetical Syllogism:** $(a \rightarrow b), (b \rightarrow c)$ **:- therefore** $(a \rightarrow c)$
- **Disjunctive Syllogism:** $(a \vee b), \sim a$ **:- therefore** $b$
- **Constructive Dilemma:** $(a \rightarrow b) \wedge (c \rightarrow d), (a \vee c)$ **:- therefore** $(b \vee d)$
- **Destructive Dilemma:** $(a \rightarrow b) \wedge (c \rightarrow d), (\sim b \vee \sim d)$ **:- therefore** $(\sim a \vee \sim c)$
- **Simplification:** $(a \wedge b)$ **:- therefore** $a$
- **Conjunction:** $a, b$ **:- therefore** $(a \wedge b)$

# Deduction in Propositional Logic

- **Modus Ponens:** $(a \to b), a$ **:- therefore** $b$
- **Modus Tollens:** $(a \to b), \sim b$ **:- therefore** $\sim a$
- **Hypothetical Syllogism:** $(a \to b), (b \to c)$ **:- therefore** $(a \to c)$
- **Disjunctive Syllogism:** $(a \vee b), \sim a$ **:- therefore** $b$
- **Constructive Dilemma:** $(a \to b) \wedge (c \to d), (a \vee c)$ **:- therefore** $(b \vee d)$
- **Destructive Dilemma:** $(a \to b) \wedge (c \to d), (\sim b \vee \sim d)$ **:- therefore** $(\sim a \vee \sim c)$
- **Simplification:** $(a \wedge b)$ **:- therefore** $a$
- **Conjunction:** $a, b$ **:- therefore** $(a \wedge b)$
- **Addition:** $a$ **:- therefore** $(a \vee b)$

# Deduction in Propositional Logic

- **Modus Ponens:** $(a \to b), a$ **:- therefore** $b$
- **Modus Tollens:** $(a \to b), \sim b$ **:- therefore** $\sim a$
- **Hypothetical Syllogism:** $(a \to b), (b \to c)$ **:- therefore** $(a \to c)$
- **Disjunctive Syllogism:** $(a \vee b), \sim a$ **:- therefore** $b$
- **Constructive Dilemma:** $(a \to b) \wedge (c \to d), (a \vee c)$ **:- therefore** $(b \vee d)$
- **Destructive Dilemma:** $(a \to b) \wedge (c \to d), (\sim b \vee \sim d)$ **:- therefore** $(\sim a \vee \sim c)$
- **Simplification:** $(a \wedge b)$ **:- therefore** $a$
- **Conjunction:** $a, b$ **:- therefore** $(a \wedge b)$
- **Addition:** $a$ **:- therefore** $(a \vee b)$
- **Natural deduction is Sound and Complete**

# Propositional logic deduction - 1

- **Knowledge base: If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned**
- **Target: Can we prove that the unicorn is mythical? Magical? Horned?**

# Propositional logic deduction - 1

- **Knowledge base: If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned**
- **Target: Can we prove that the unicorn is mythical? Magical? Horned?**
- **Propositions:** $a$**-mythical,** $b$**-mortal,** $c$**-mammal,** $d$**-horned,** $e$**-magical**

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- **Goal** $(\mathcal{G})$**:** $a, e, d$

# Propositional logic deduction - 2

- **Knowledge base: Holmes owns two suits: one black and one tweed. He always wears either a tweed suit or sandals. Whenever he wears his tweed suit and a purple shirt, he chooses to not wear a tie. He never wears the tweed suit unless he is also wearing either a purple shirt or sandals. Whenever he wears sandals, he also wears a purple shirt. Yesterday, Holmes wore a bow tie.**
- **Target: What else did he wear?**
- **Propositions:** *bs*-**black suit,** *ts*-**tweed suit,** *s*-**sandals,** *ps*-**purple shirt,** *t*-**tie**

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  - $s \rightarrow ps$
  - $t$
- **Goal** $(\mathcal{G})$: **All satisfying solutions**

# Problem: Glasses

- You are about to leave for college in the morning and discover that you don't have your glasses. You know the following statements are true:
  - If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.
  - If my glasses are on the kitchen table, then I saw them at breakfast.
  - I did not see my glasses at breakfast.
  - I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.
  - If I was reading the newspaper in the living room then my glasses are on the coffee table.
- Where are the glasses?

# Puzzle-1

- As a reward for saving his daughter from pirates, the King has given you the opportunity to win a treasure hidden inside one of three trunks. The two trunks that do not hold the treasure are empty. To win, you must select the correct trunk. Trunks 1 and 2 are each inscribed with the message "This trunk is empty," and Trunk 3 is inscribed with the message "The treasure is in Trunk 2." The Queen, who never lies, tells you that only one of these inscriptions is true, while the other two are wrong. Which trunk should you select to win?

# Puzzle-2

- **There is an island that has two kinds of inhabitants, knights, who always tell the truth, and their opposites, knaves, who always lie. You encounter two people A and B. What are A and B if A says "B is a knight" and B says "The two of us are opposite types"?**

# Insufficiency of Propositional Logic

- **Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school.**

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- No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.

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- No contractors are dependable. Some engineers are contractors. Therefore some engineers are not dependable.
- All dancers are graceful. Ayesha is a student. Ayesha is a dancer. Therefore some student is graceful.
- Every passenger is either in first class or second class. Each passenger is in second class if and only if he or she is not wealthy. Some passengers are wealthy. Not all passengers are wealthy. Therefore some passengers are in second class.

*Thank you!*