

CS514: Design and Analysis of Algorithms

Algorithm design



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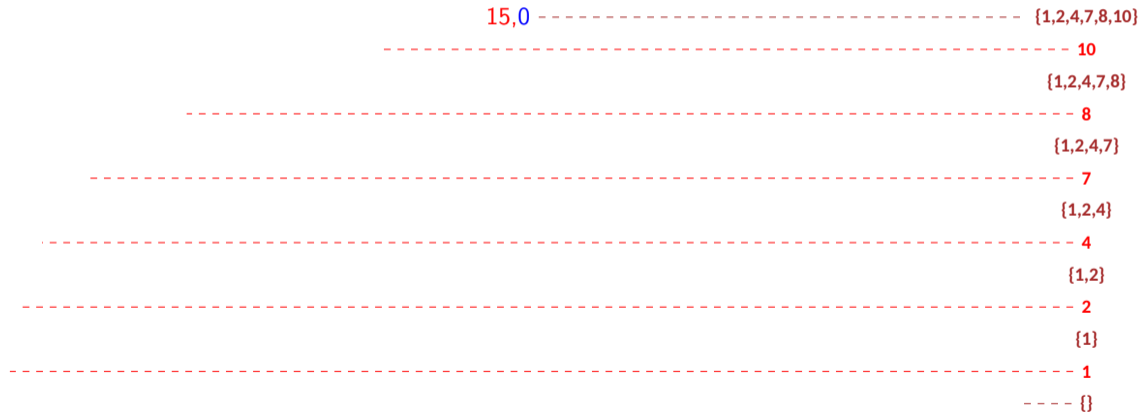
`https://www.iitp.ac.in/~arijit/`

Coin change: Recursion tree

- Given a set of coins C , what is the minimum number coins required to provide sum S ?
- Example: $C = \{1, 2, 4, 7, 8, 10\}$, $S = 15$

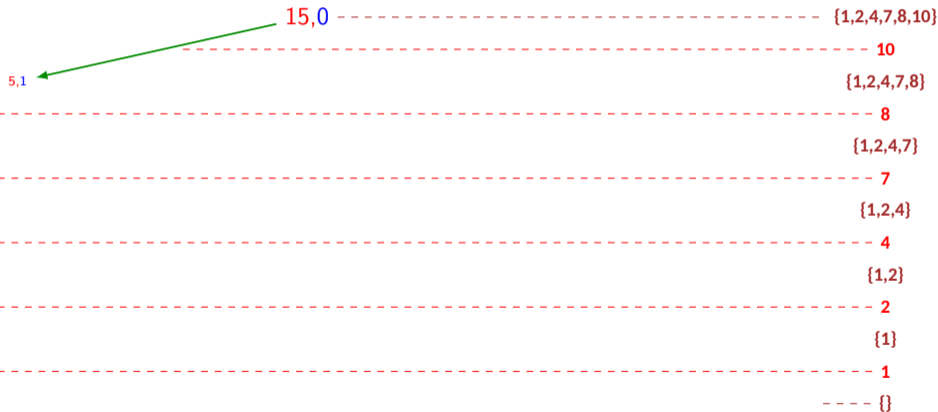
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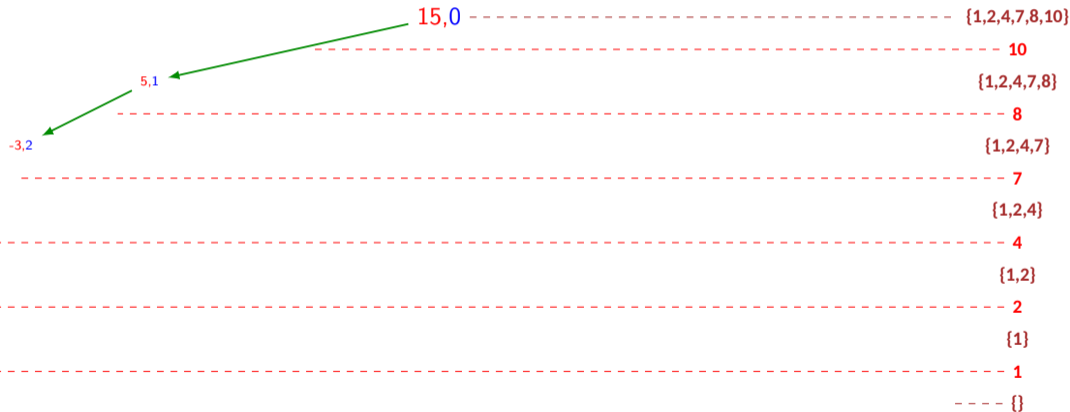
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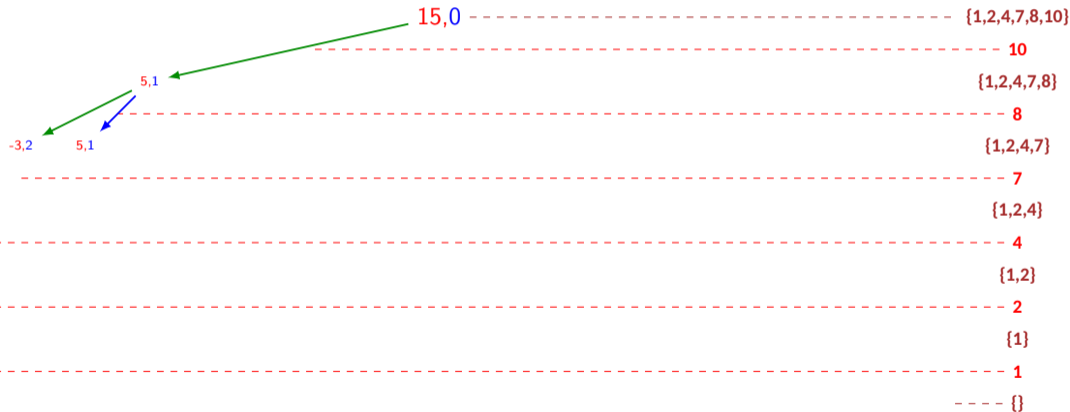
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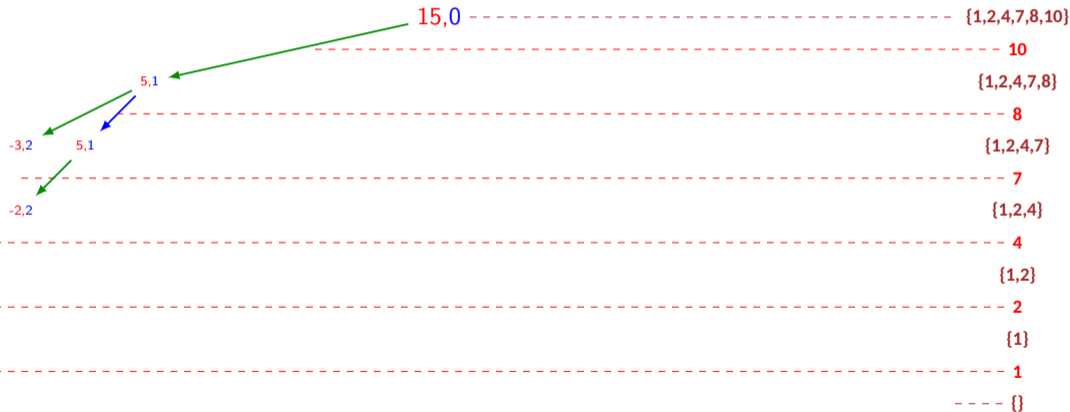
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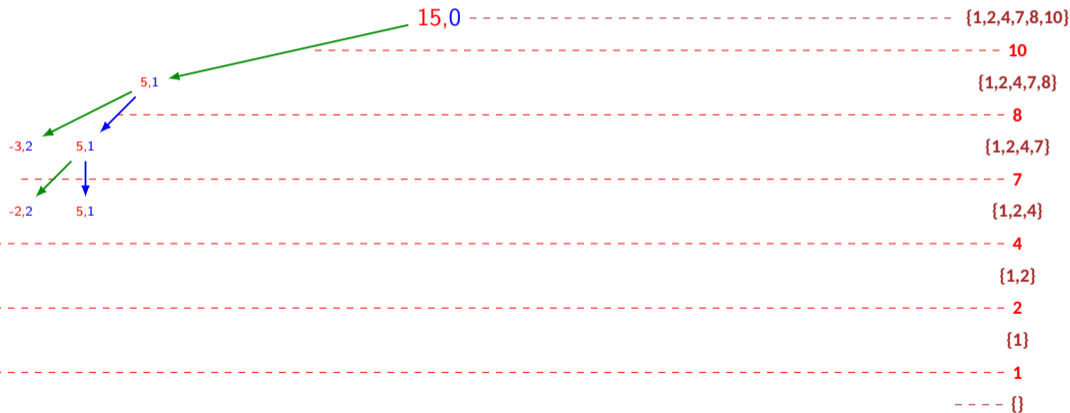
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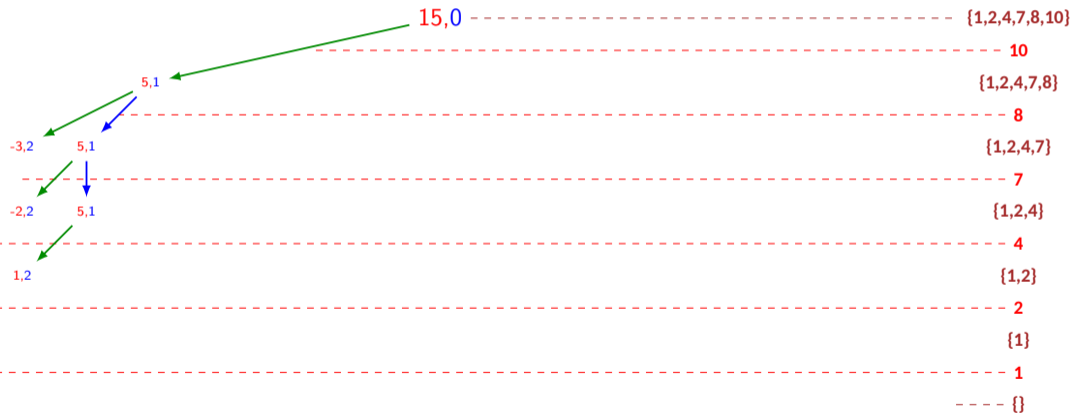
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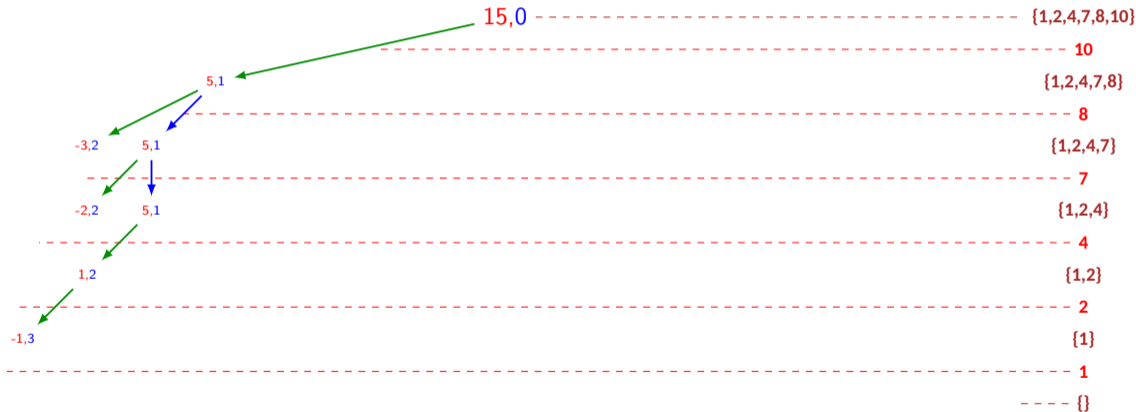
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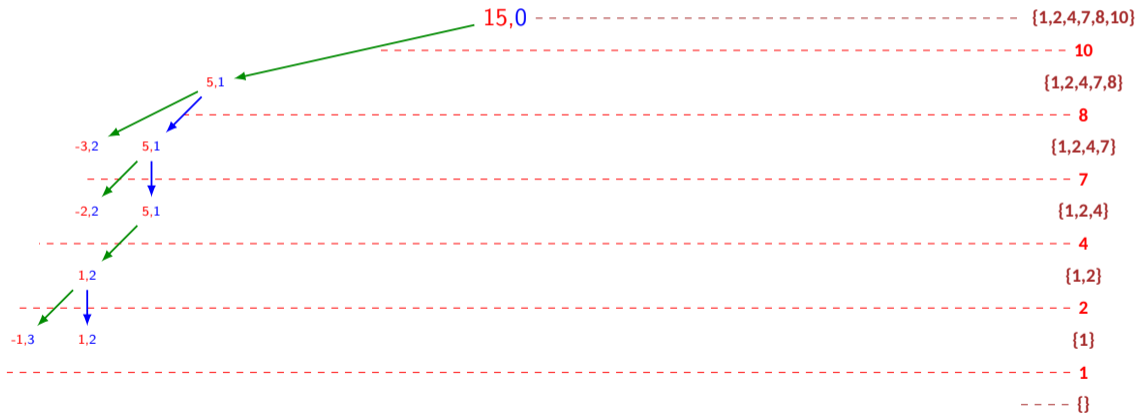
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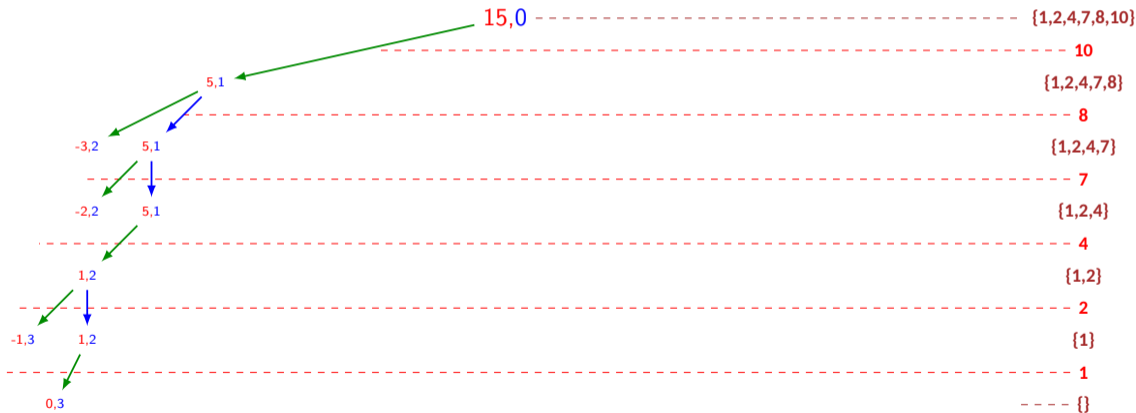
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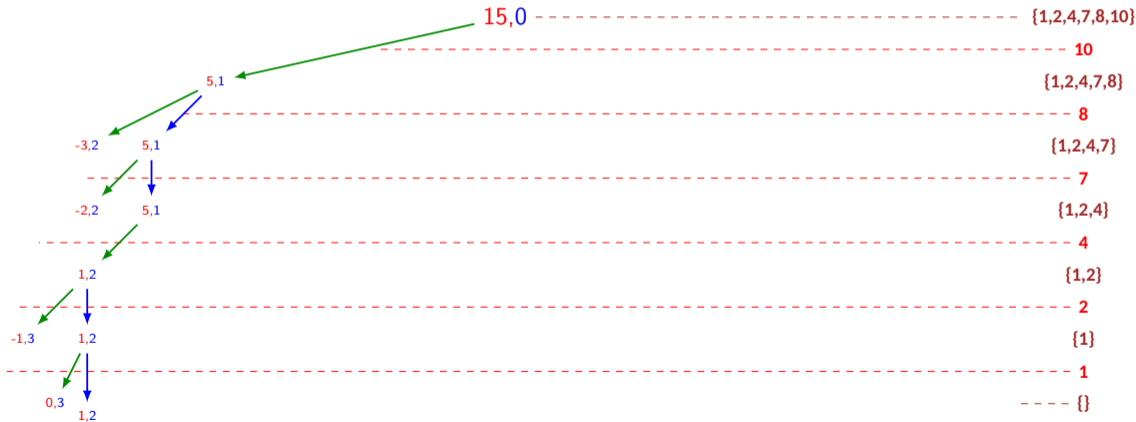
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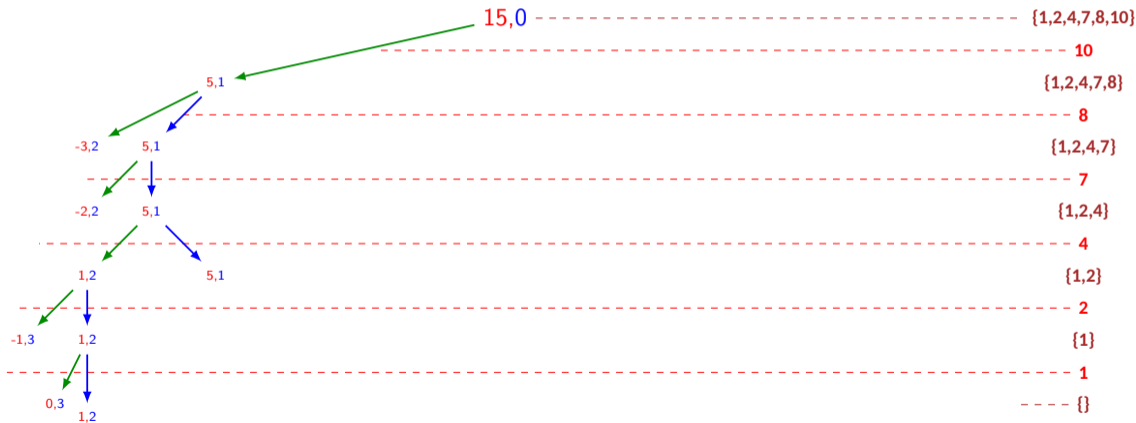
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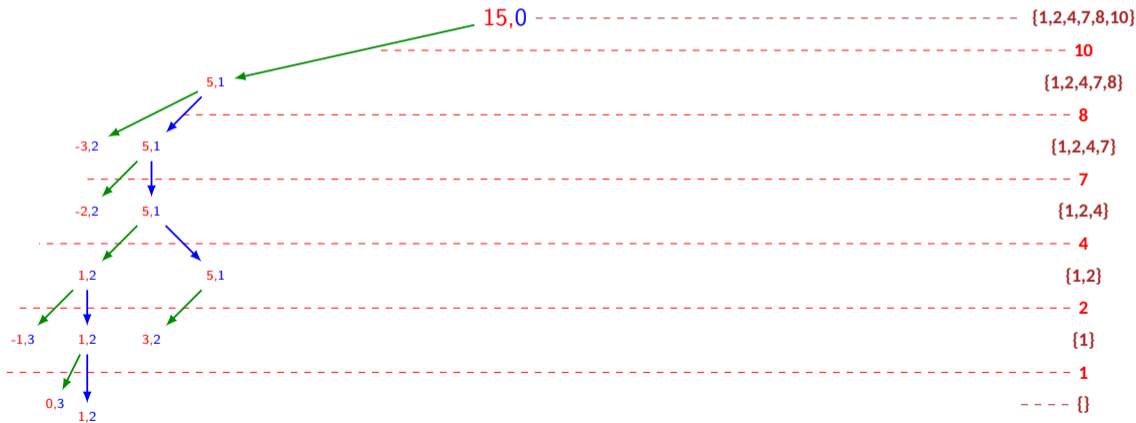
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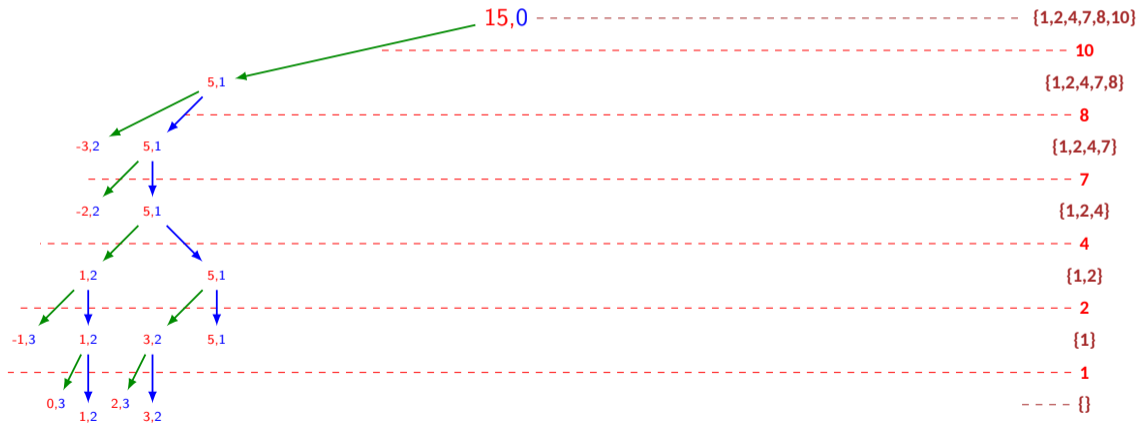
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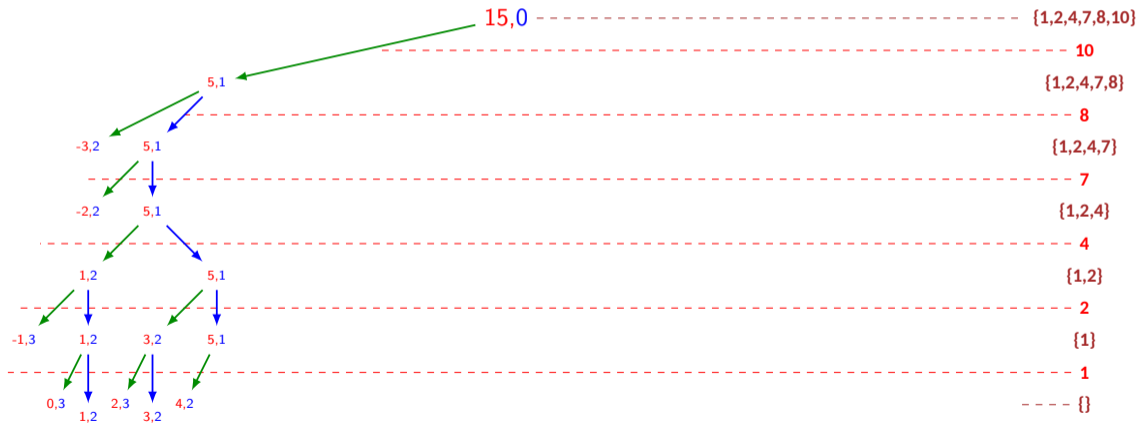
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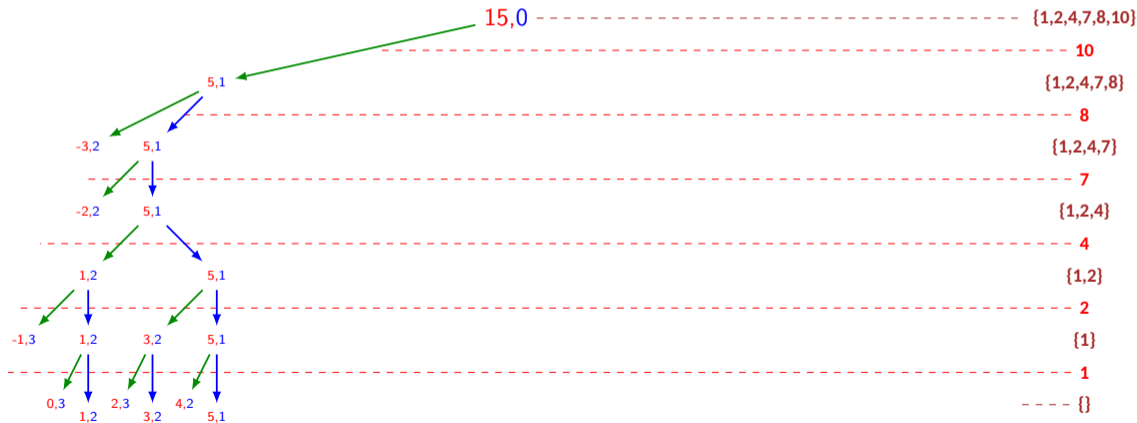
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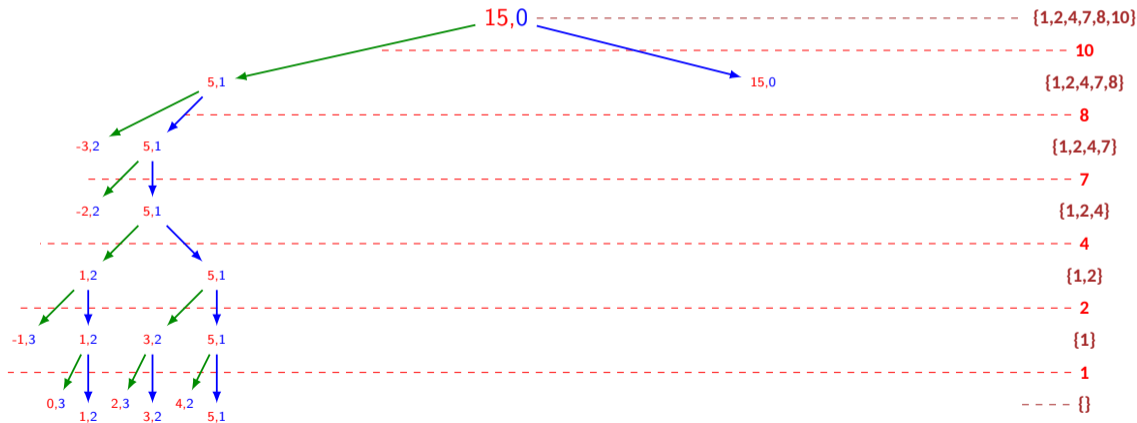
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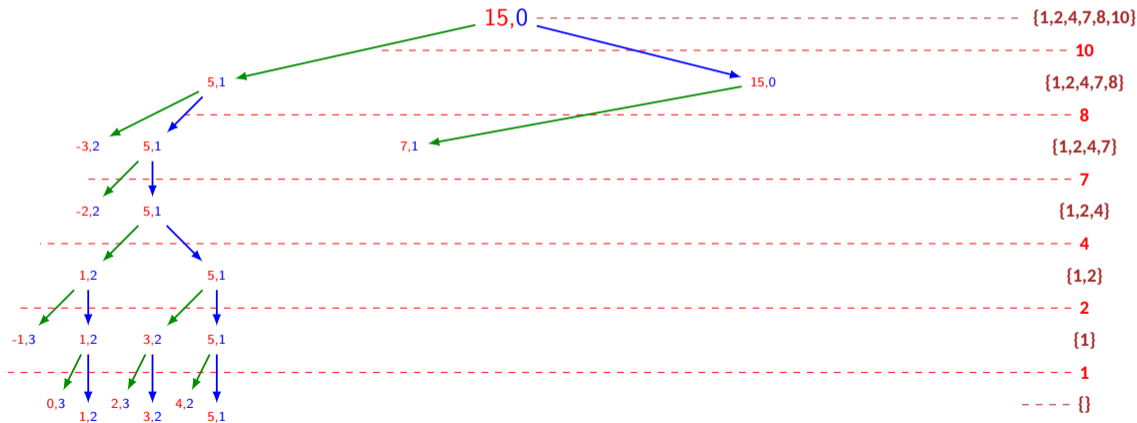
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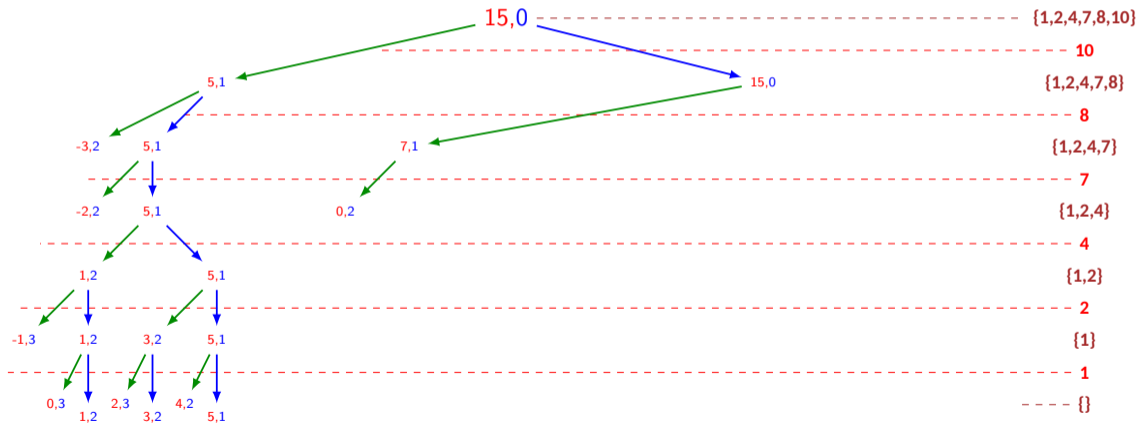
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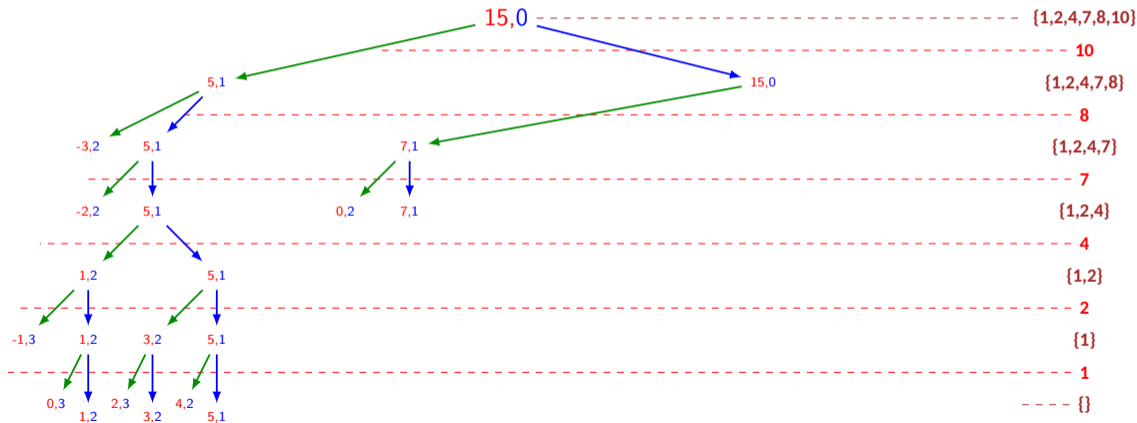
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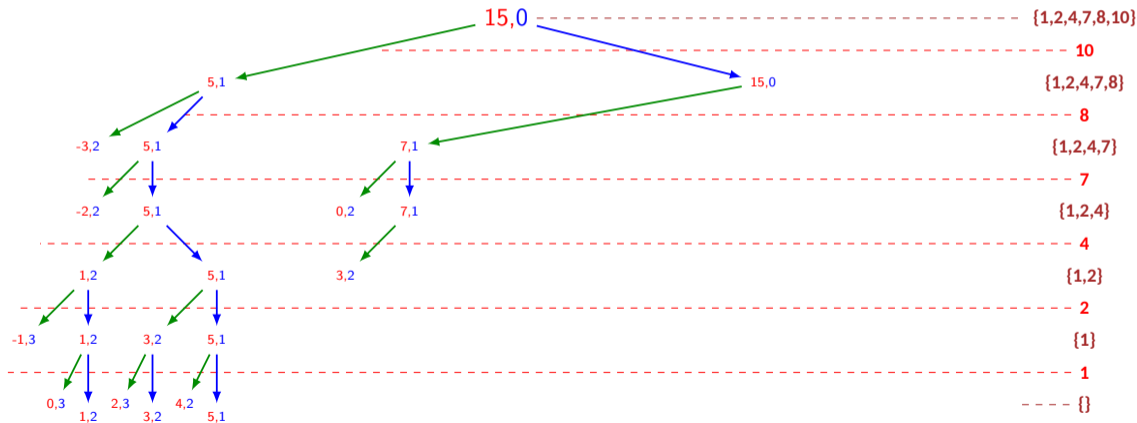
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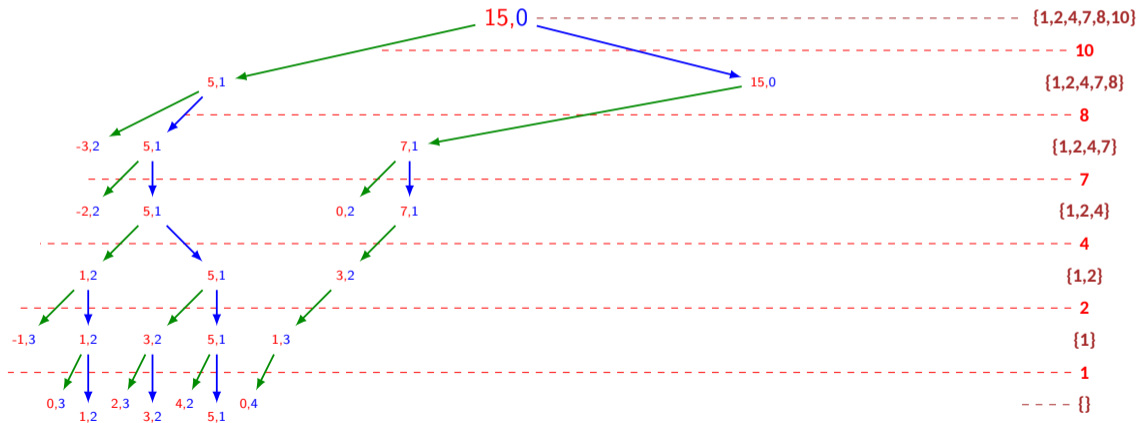
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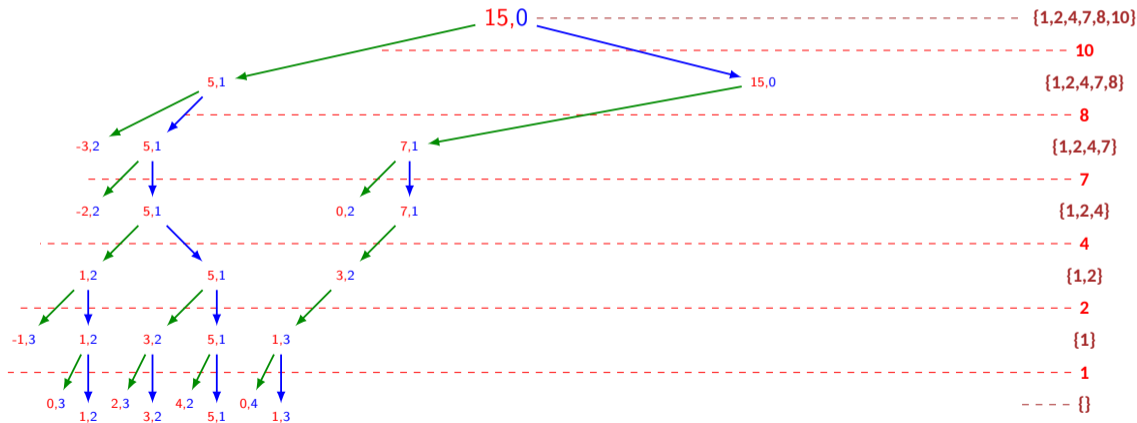
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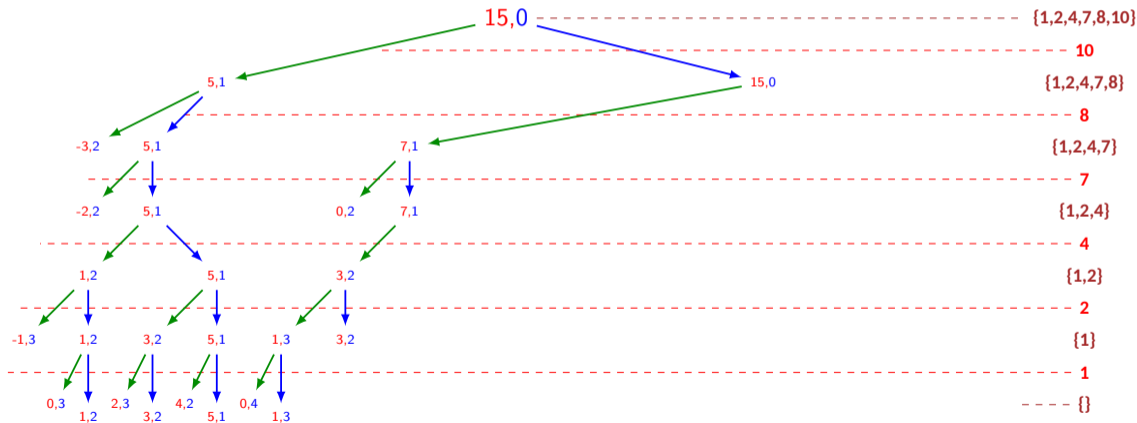
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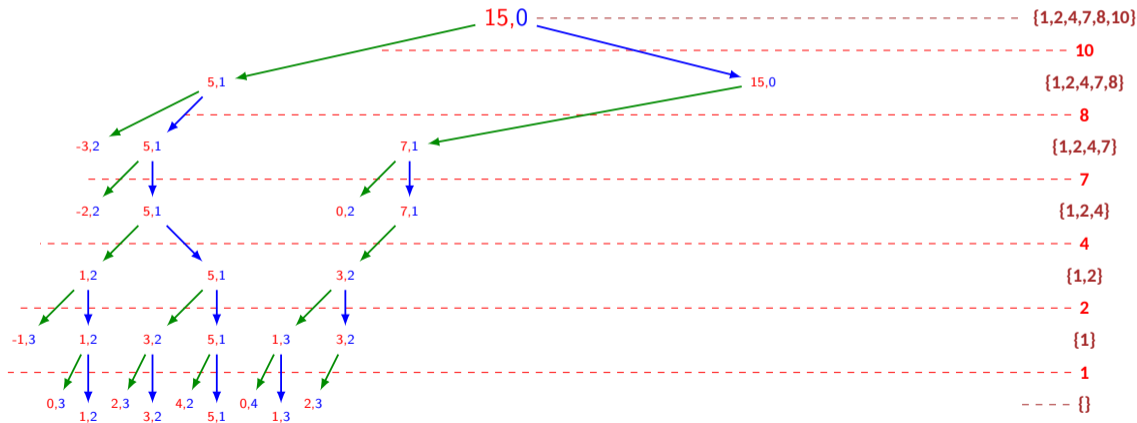
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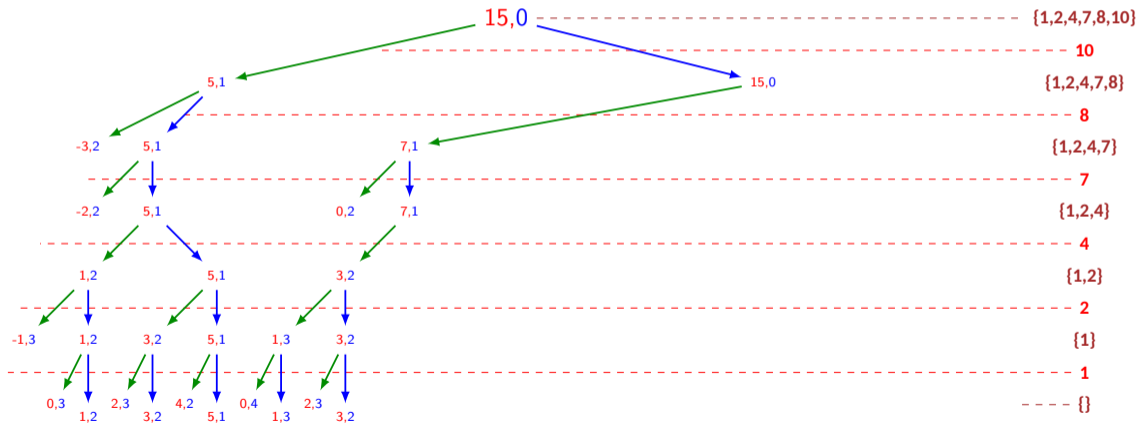
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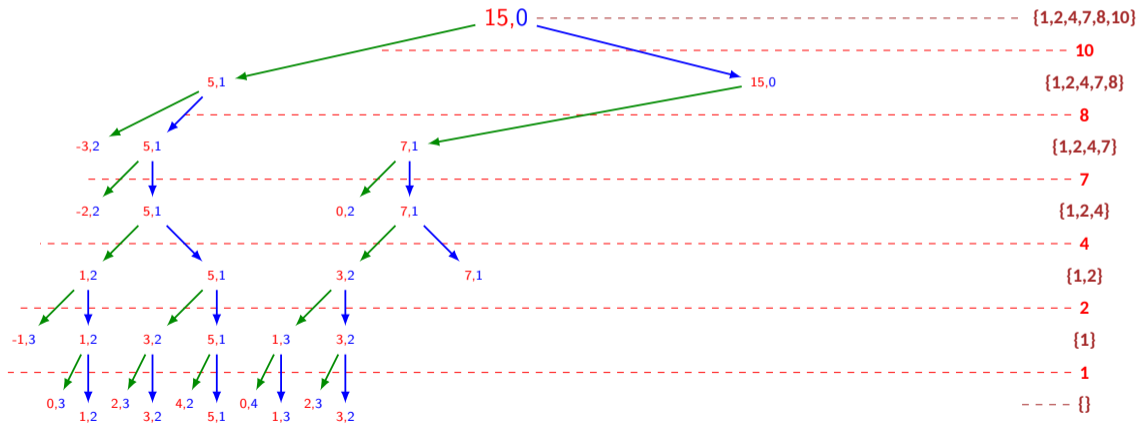
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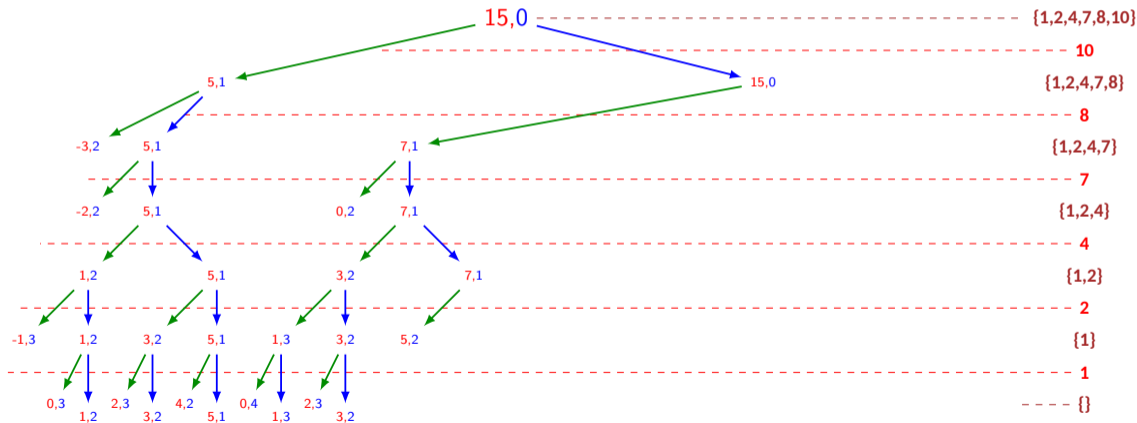
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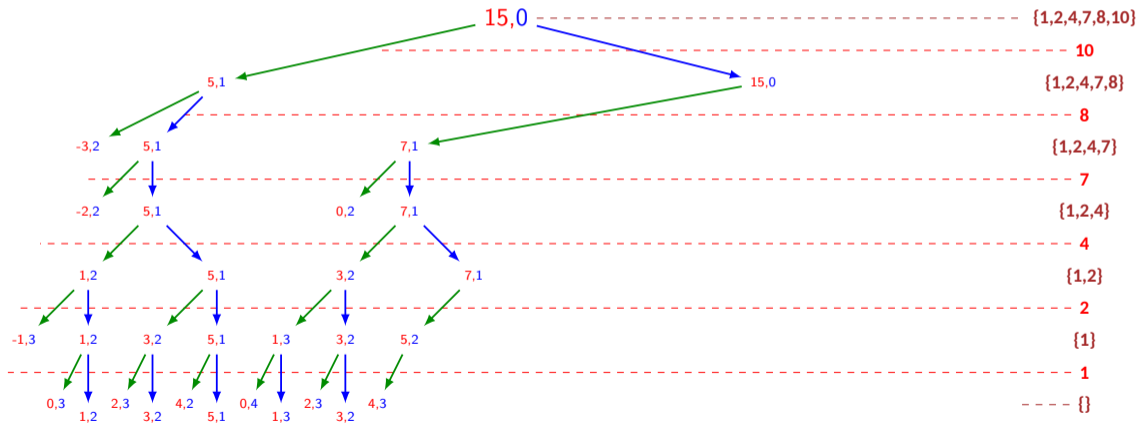
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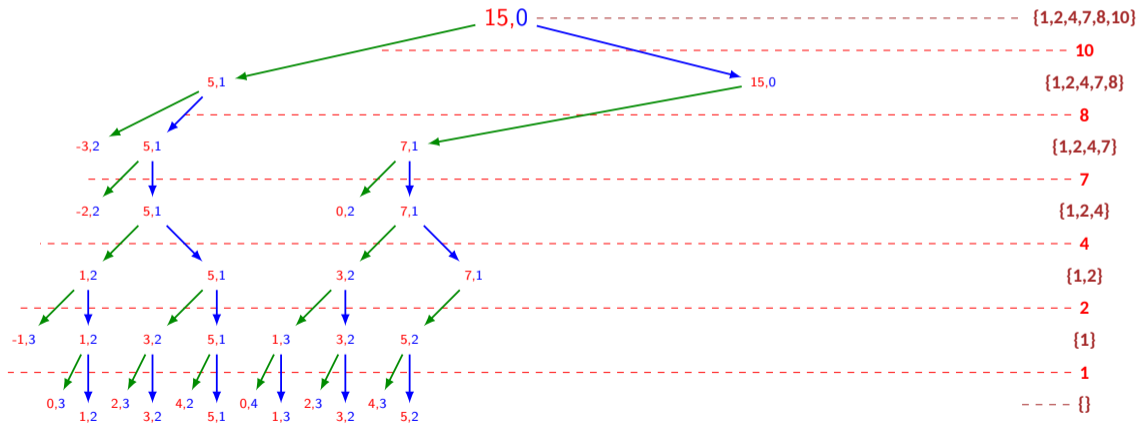
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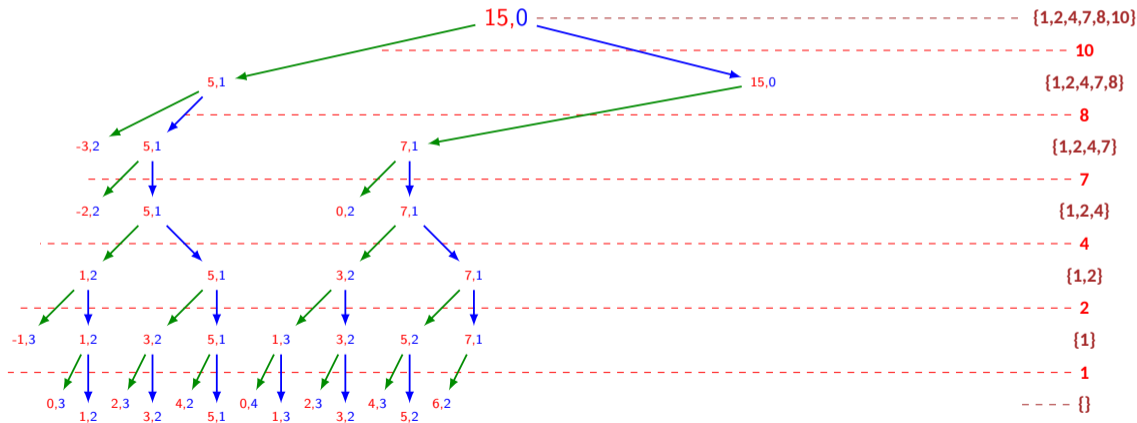
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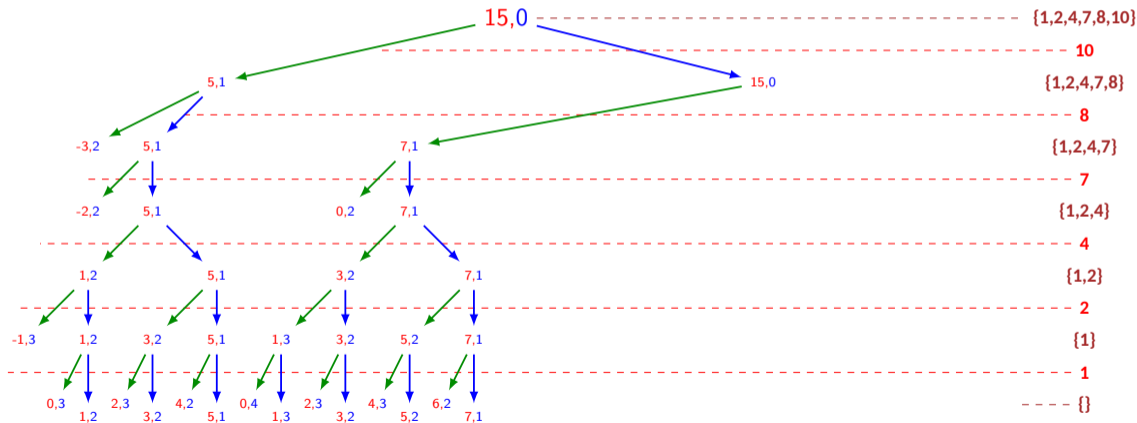
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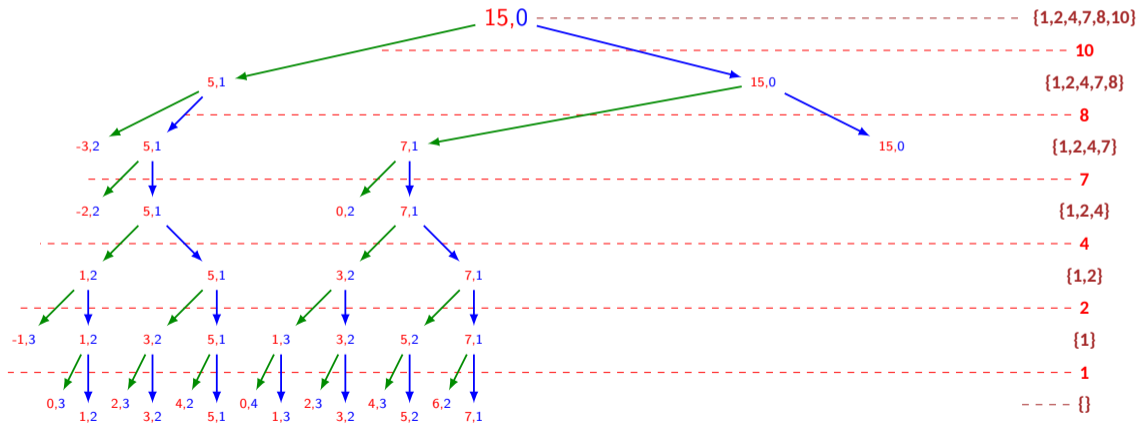
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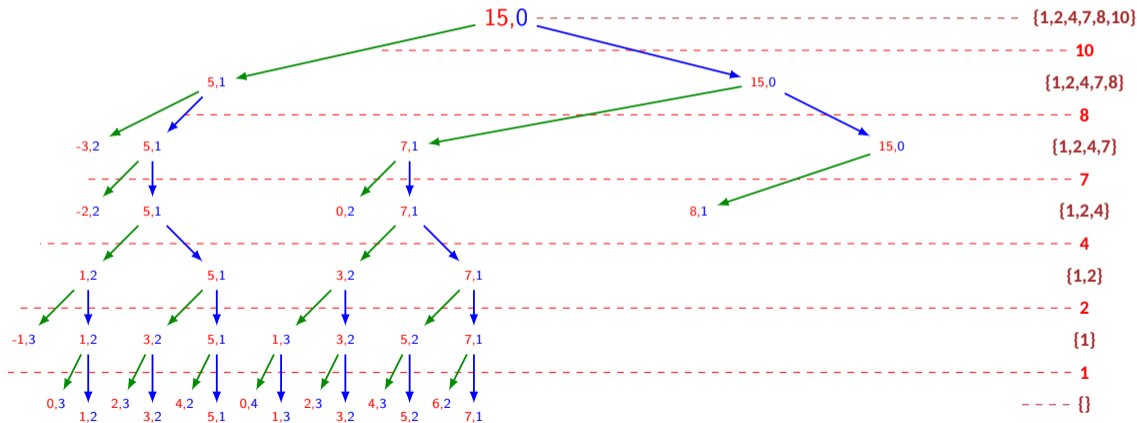
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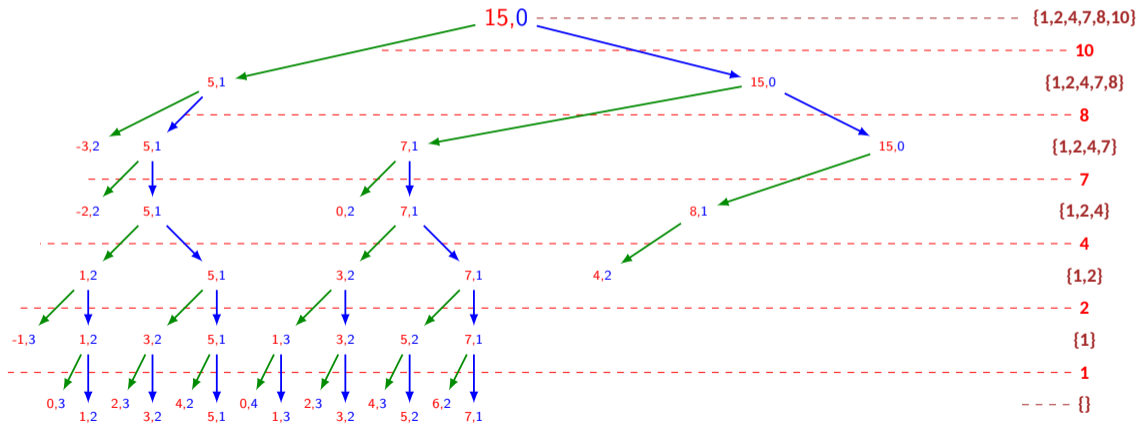
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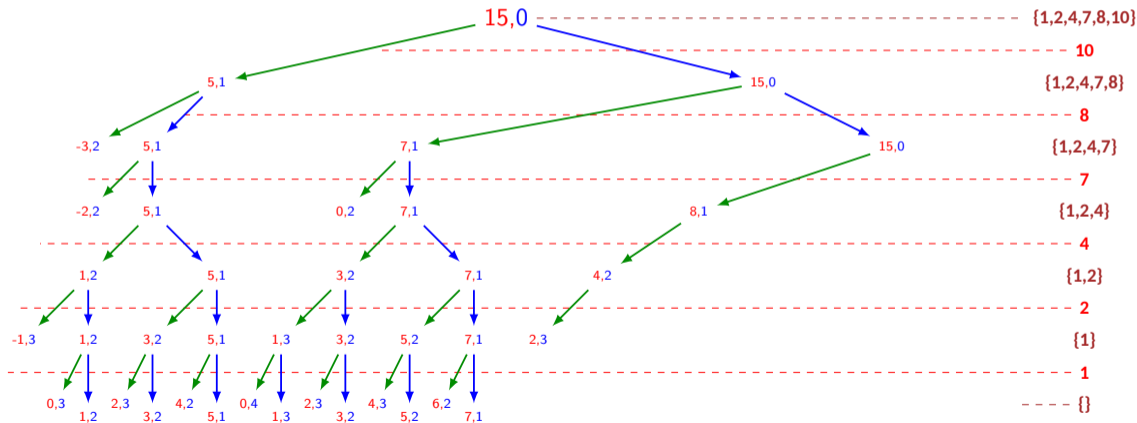
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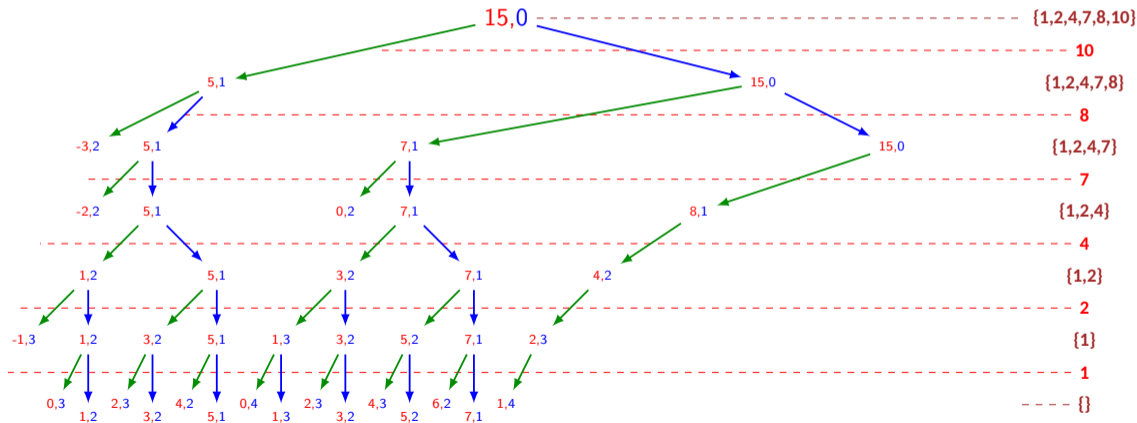
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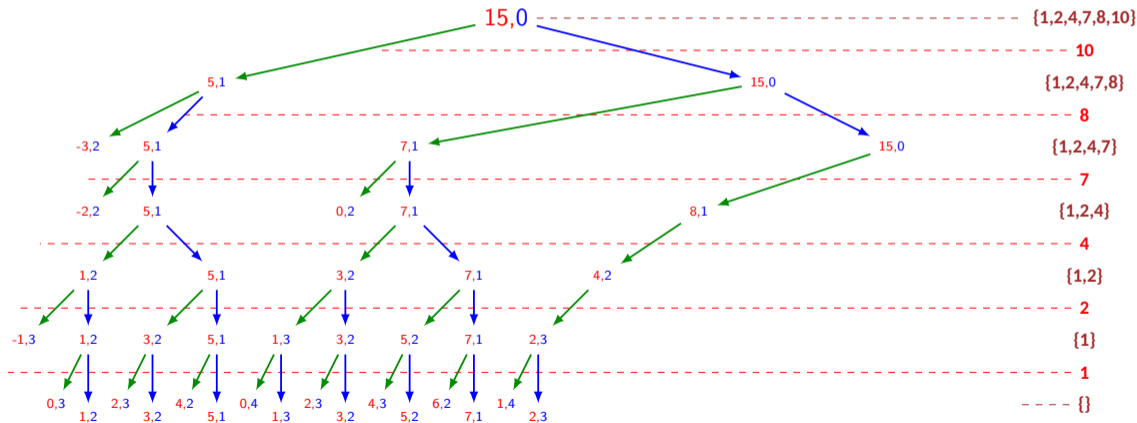
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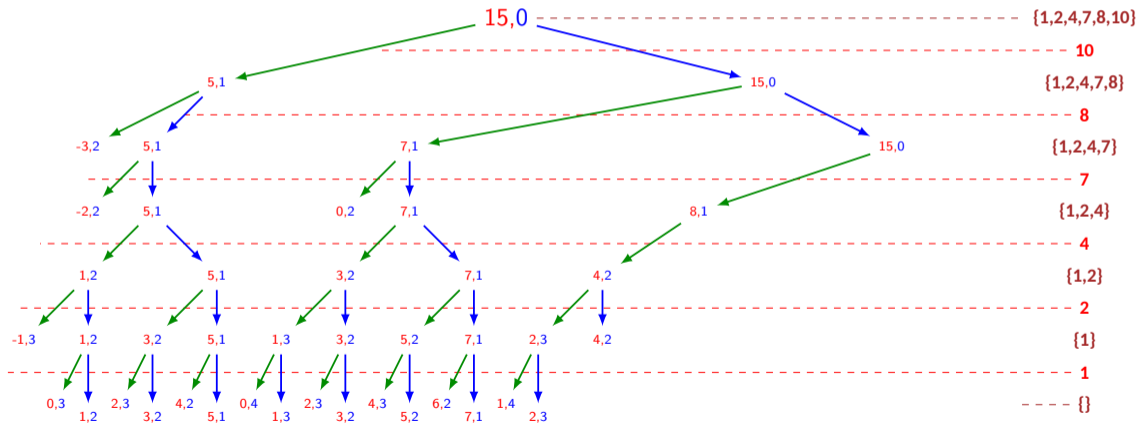
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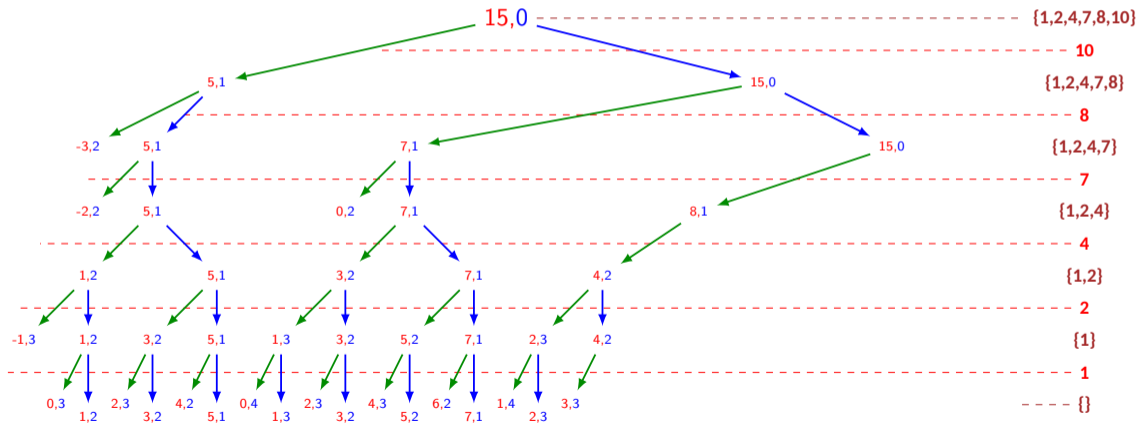
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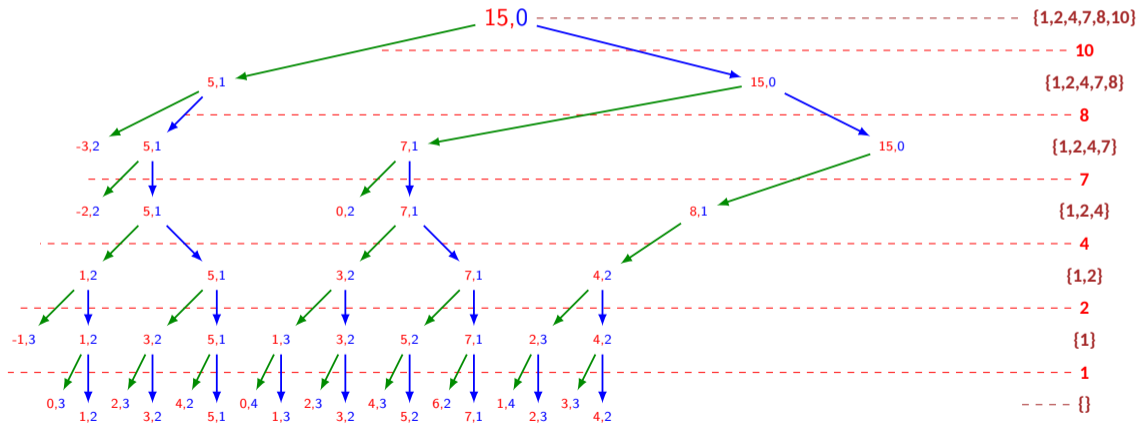
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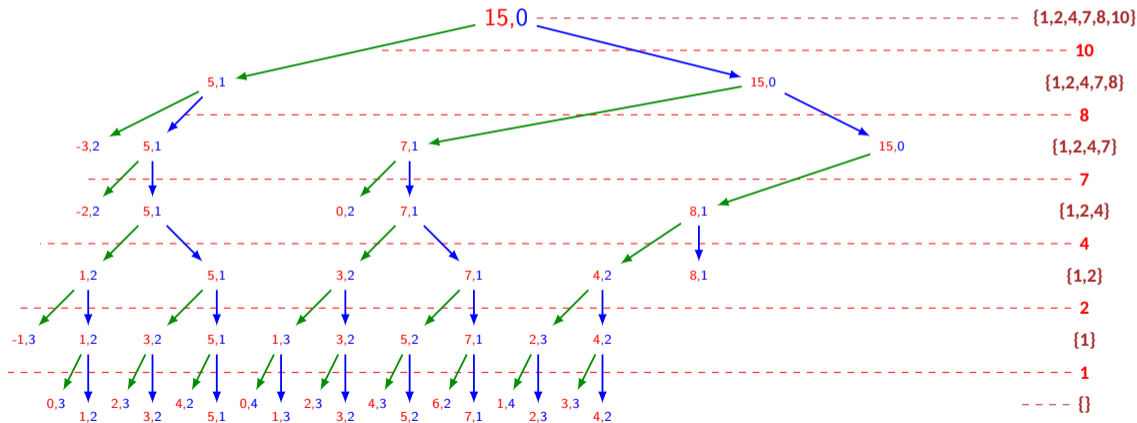
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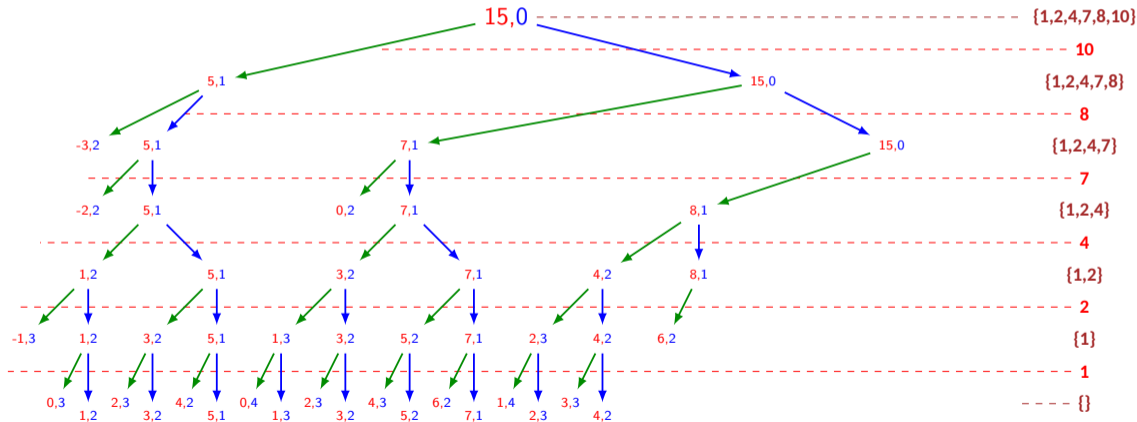
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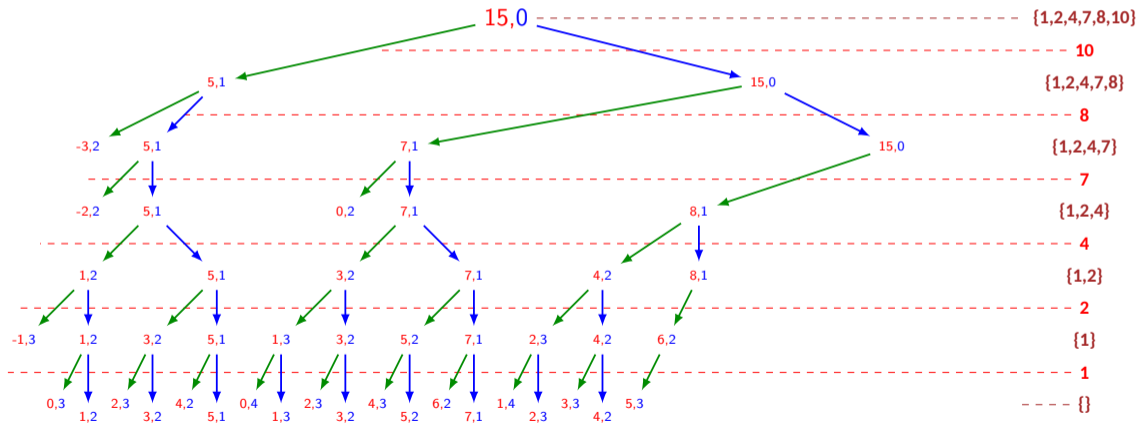
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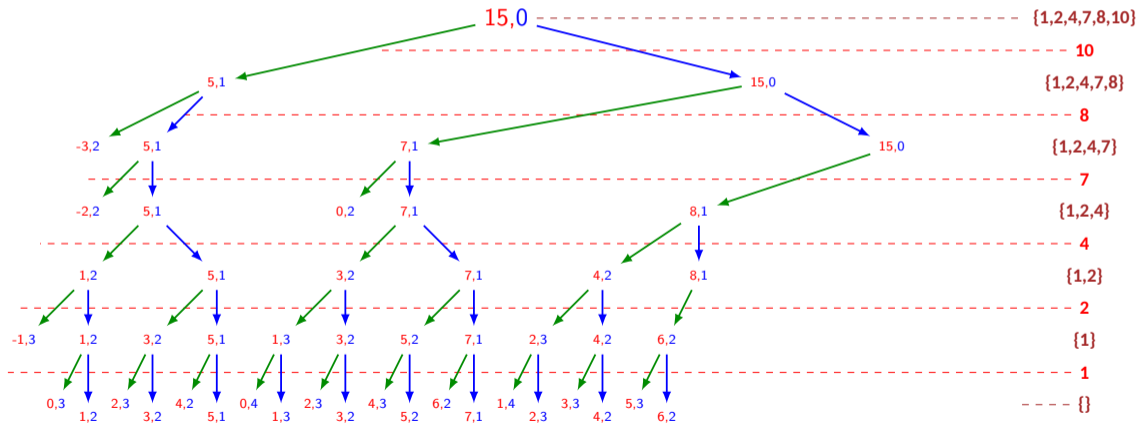
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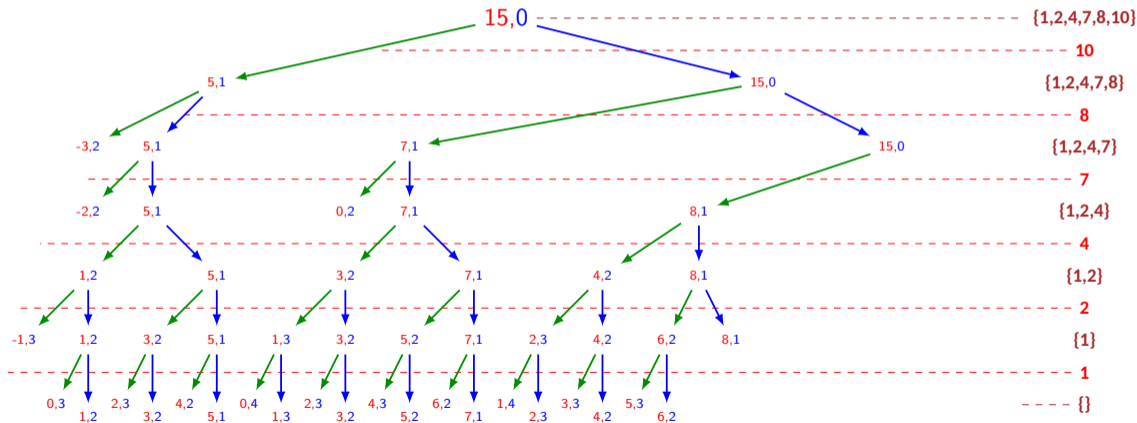
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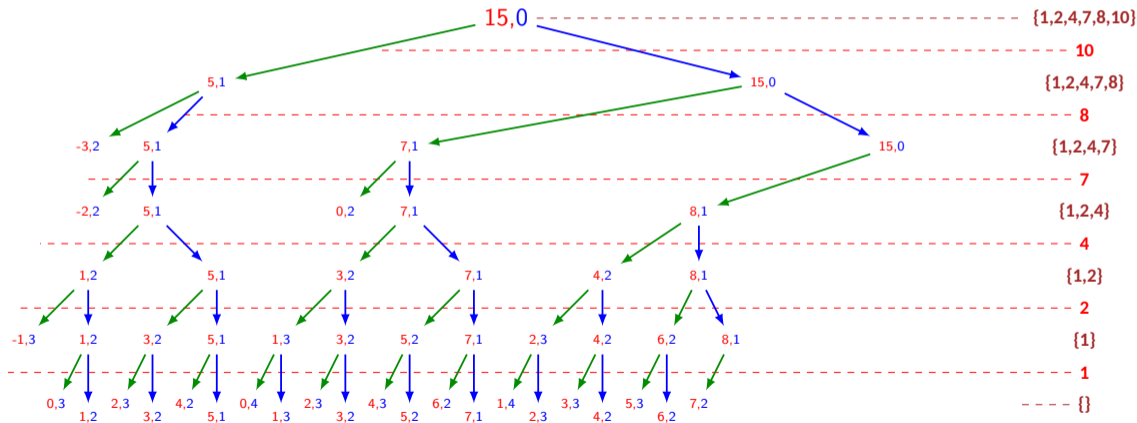
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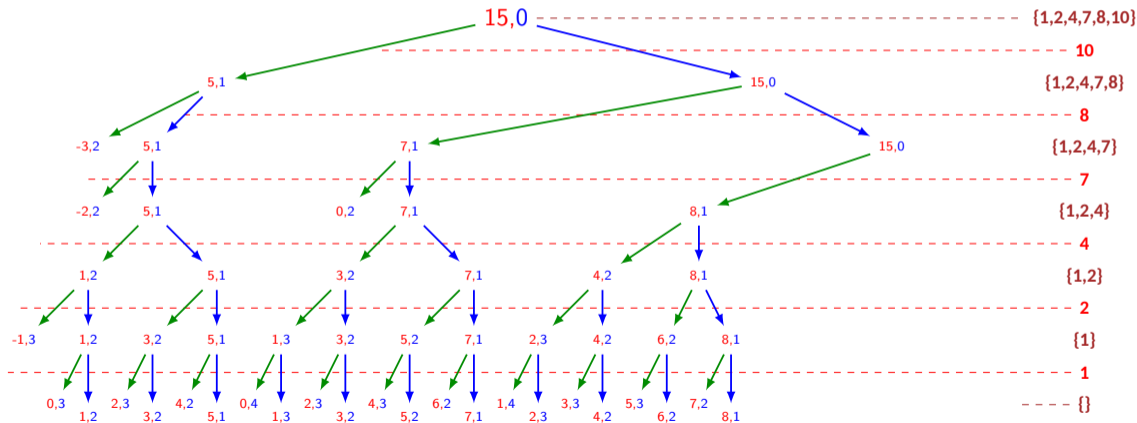
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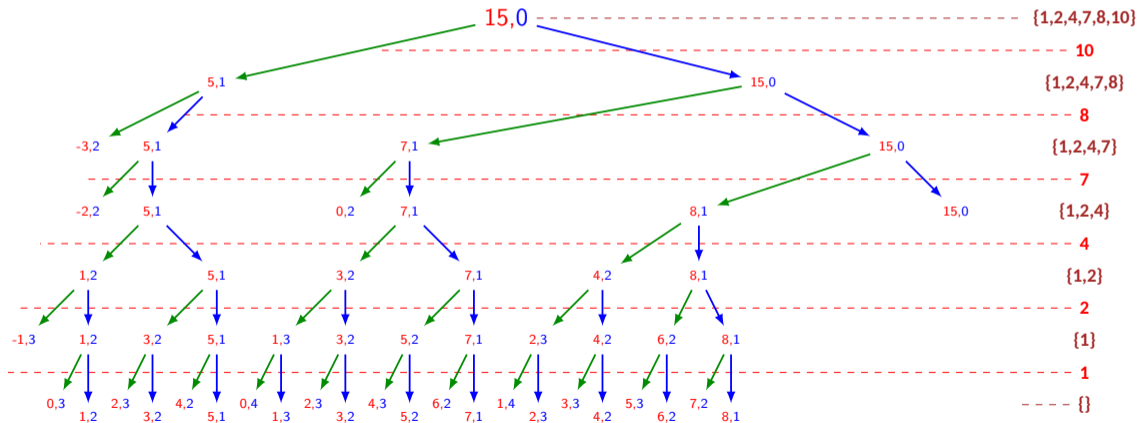
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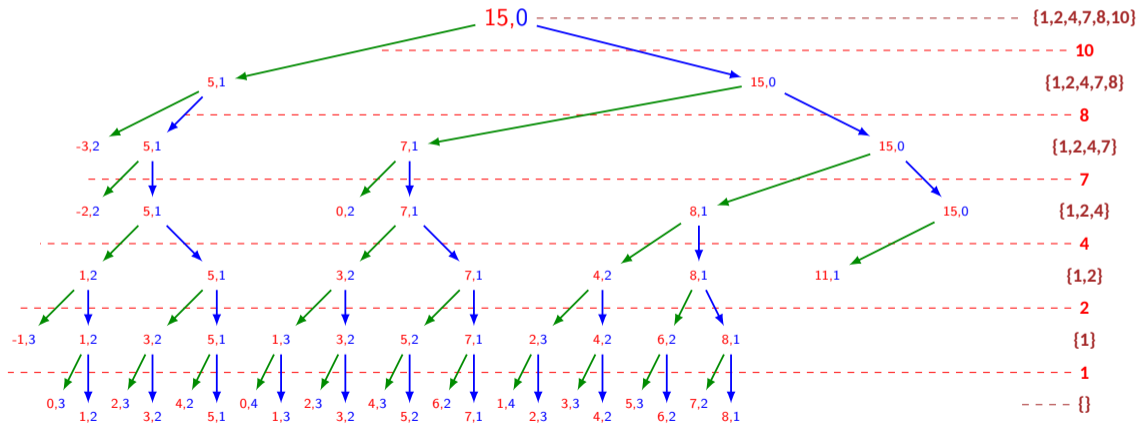
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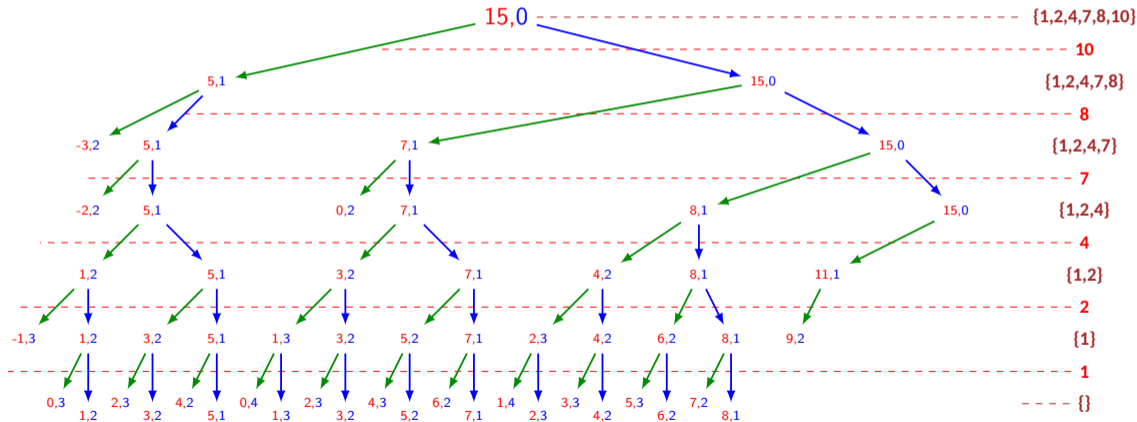
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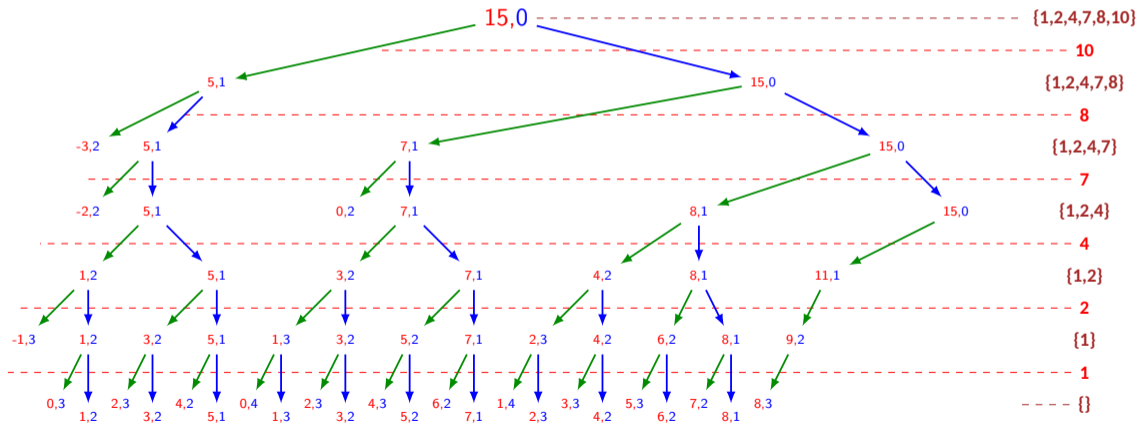
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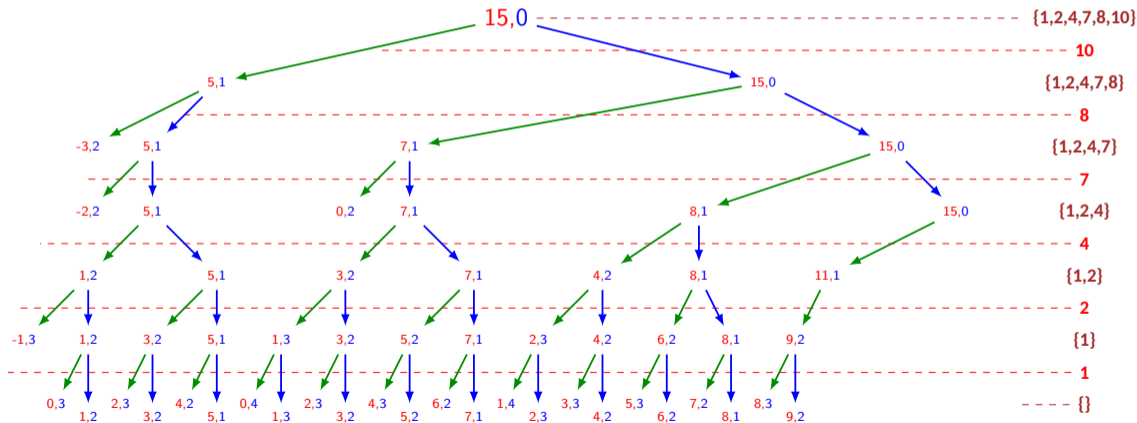
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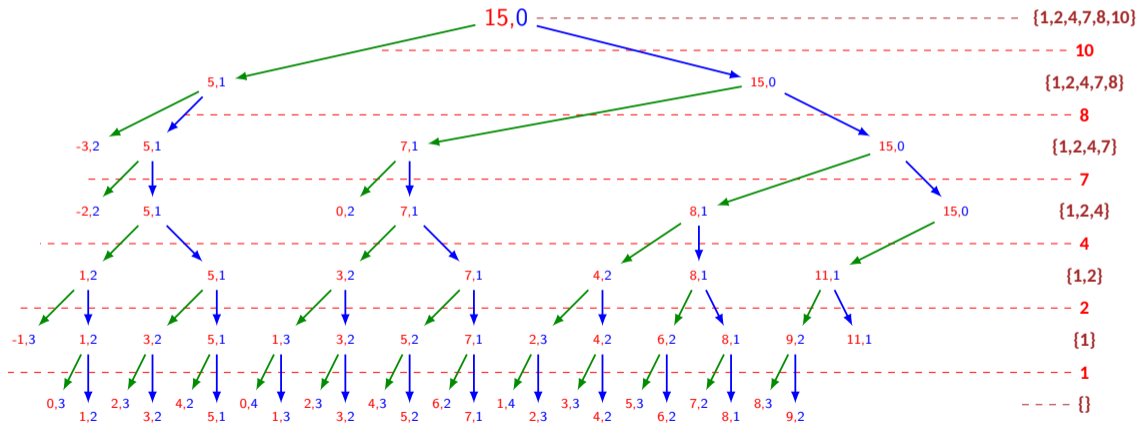
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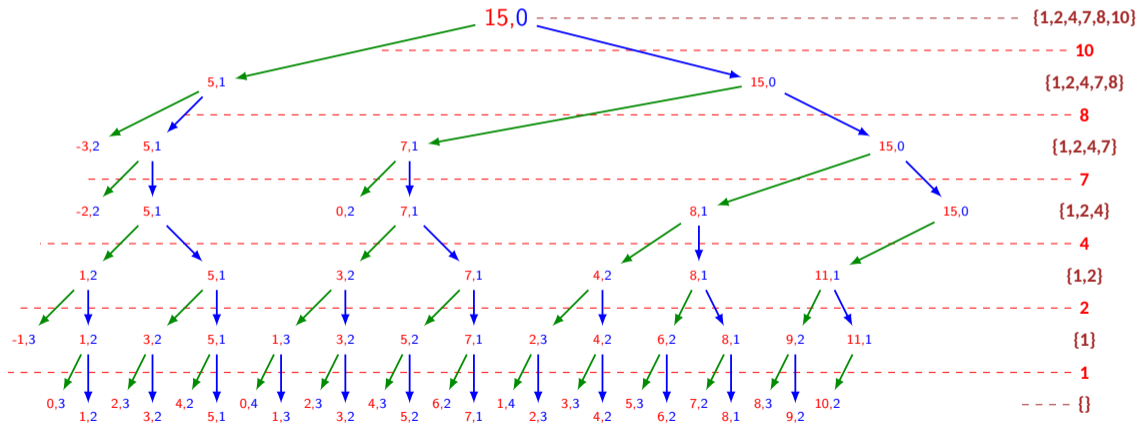
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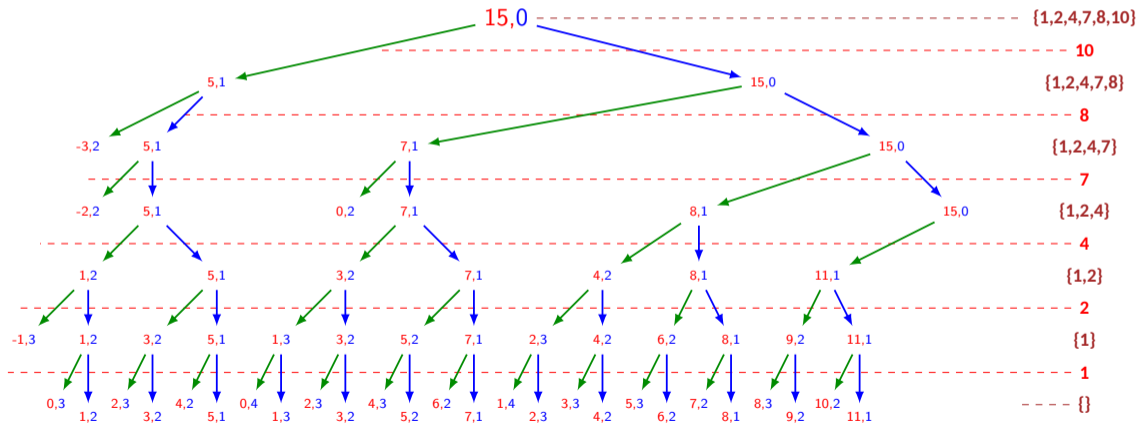
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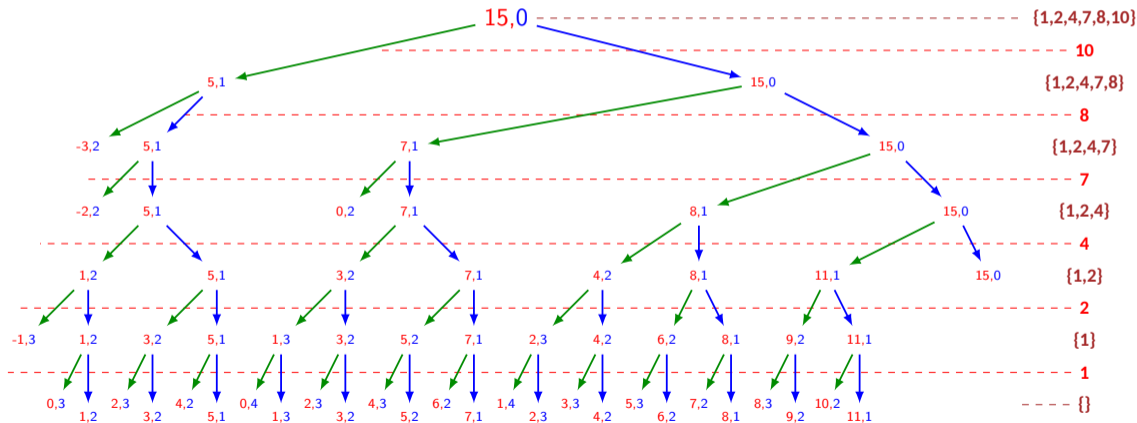
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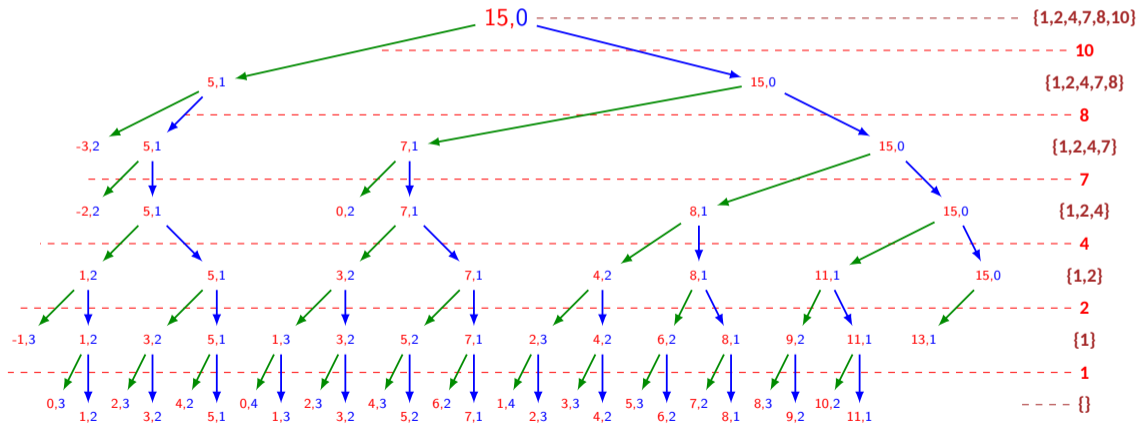
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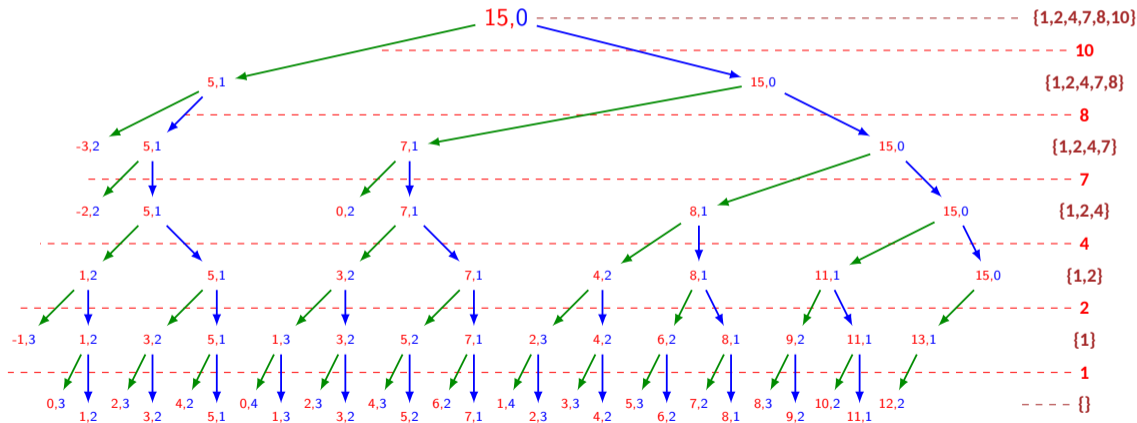
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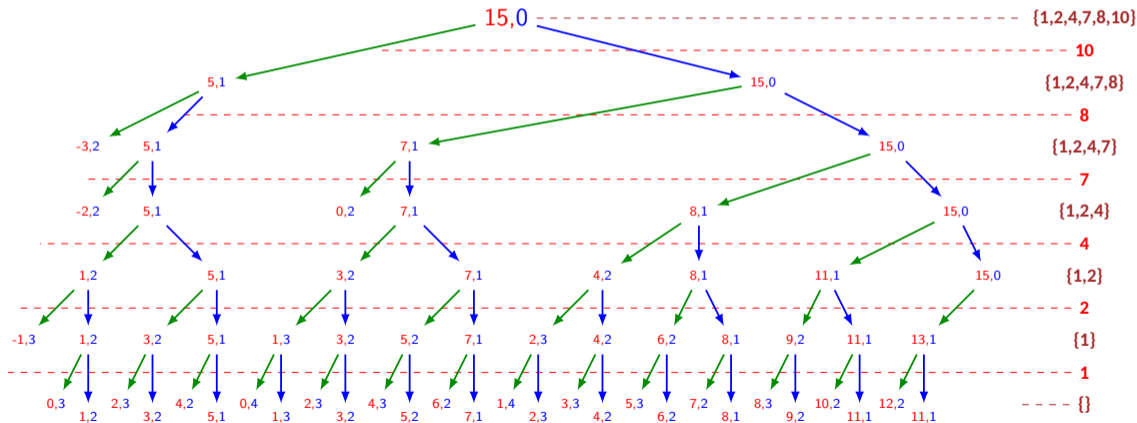
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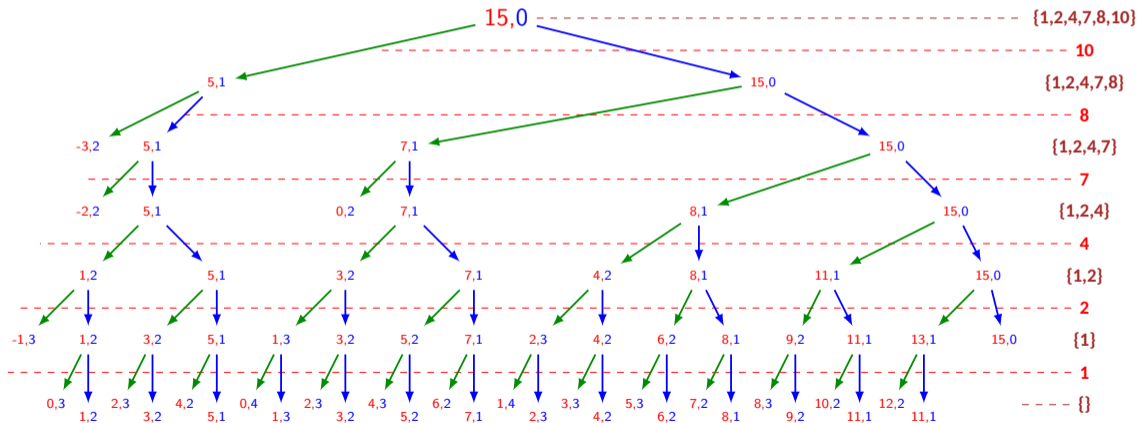
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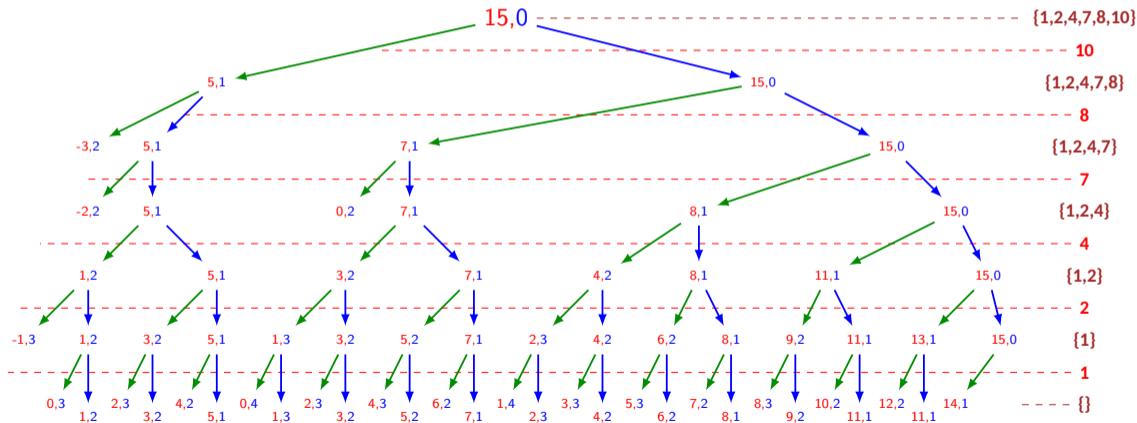
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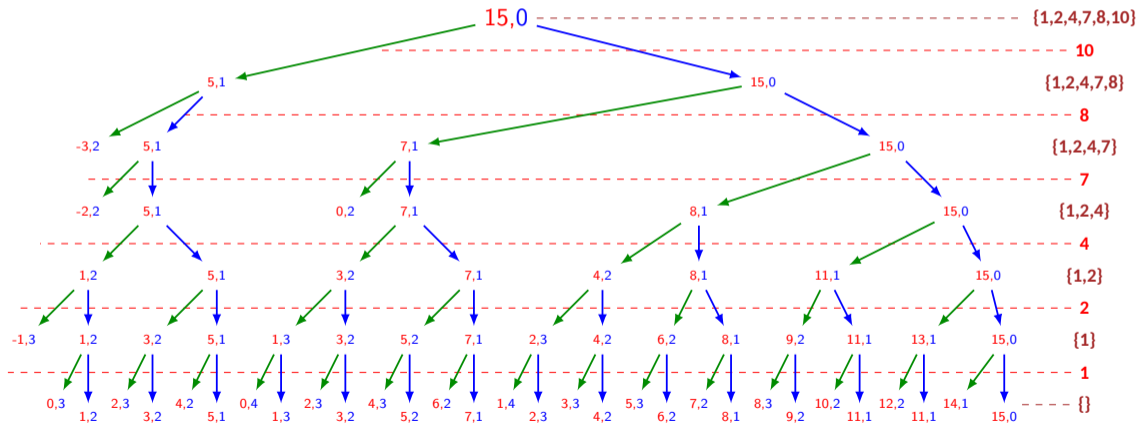
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Coin change: Bound condition

- Initialize C =set of coins, $currBest = \infty$
- $Coin(S, n, count)$:
 1. if $S < 0$ return
 2. if $(S == 0)$ { $currBest = currBest > count ? count : currBest$; return }
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 4. $Coin(S - c_1, n - 1, count + 1)$
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 - $count + \left\lceil \frac{S}{\max\{C_{remaining}\}} \right\rceil < currBest$
 - $count + j < currBest$, where $j = \arg \max_j \left[\left(\sum_{i=1}^j c_i \right) < S \right]$, assuming sorted coin set

Thank you!