# CS514: Design and Analysis of Algorithms 

## Recursion: Backtracking



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## Permutation Generation

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- Given the string dog, generate all possible permutations

| 'dog' |
| :---: |
| $' ‘$ |


| 'pending' |
| :--- |
| ' used ' |

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## Permutation Generation

- permutation(pending, used)

1. if length(pending) $==0$ \{ print used; return \}
2. for $\mathrm{i}=1$ to length(pending)
3. $\quad \mathrm{c}=$ pending $[\mathrm{i}]$
4. rest $=$ pending $[1 . .(i-1)(i+1) \ldots]$
5. permutation $\left(\right.$ rest,$\left.\{\text { used }\}^{*}\{c\}\right)$

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## Combination Generation

- combination(pending, used)

1. if length(pending) $==0$ \{ print used; return \}
2. $\mathrm{c}=$ pending [1]
3. rest $=$ pending $[2 \ldots$.
4. combination(rest, $\{\text { used }\}^{*}\{c\}$ )
5. combination(rest, $\{$ used $\}$ )

## Coin change

- Given a set of coins $C$, is it possible to provide a sum $S$ using $C$ ?
- Example: $C=\{1,2,4,7,8,10\}, S=15$


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## N-Queens



Need to place N -queens on this board
Rules:

- No queens are attacking each other


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Other variants:

- At least a queen on the main diagonal
- Two queens on the two main diagonals
- Enumeration of all solutions


## 4 Queens

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|  |  |  |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
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## 4 Queens

|  |  |  |  |
| :--- | :--- | :--- | :--- |
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|  |  |  |  |



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|  |  |  |  |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |



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- There is a pile of $n=13$ coins. There are two players. At each step, a player can take either one or two coins from the pile. The player who takes the last coin looses the game. If you are the first one to make a move, what will be your strategy?


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## Exercise-1

- During a robbery, a burglar finds much more loot than he had expected and has to decide what to take. His bag will hold a total weight of at most $W$ kgs. There are $n$ items to pick from, of weight $w_{1}, \ldots, w_{n}$ and INR value $v_{1}, \ldots, v_{n}$. What's the most valuable combination of items he can fit into his bag? Develop state-space exploration based approach.


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- What will happen if repetition is allowed?


## Exercise-2

- Suppose you are given a string of letters representing text in some foreign language, but without any spaces or punctuation, and you want to break this string into its individual constituent words. For example
- ilikeicecreamandmango:
- i like ice cream and mango
- i like icecream and man go
- bothearthandsarturnspin:
- both earth and saturn spin
- bot heart hands at urns pin
- Can you develop a state-space exploration based approach to find all possible break-ups?


