CS514: Design and Analysis of Algorithms

Arijit Mondal

Dept of CSE

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https://www.iitp.ac.in/~arijit/

General Information

- Class timings
  - Monday — 1800-1900
  - Tuesday — 1800-1900
  - Thursday — 1800-1900
- Room - 409 (Block 9)

Books

- Thomas H Cormen, Charles E Lieserson, Ronald L Rivest and Clifford Stein, *Introduction to Algorithms*, Third Edition, MIT Press/McGraw-Hill
- Sanjoy Dasgupta, Christos H. Papadimitriou and Umesh V. Vazirani, *Algorithms*, Tata McGraw-Hill, 2008.
- Steven Skiena, *The Algorithm Design Manual*, Springer
- Jon Kleinberg and Éva Tardos, *Algorithm Design*, Pearson, 2005.
- Robert Sedgewick and Kevin Wayne, *Algorithms*, fourth edition, Addison Wesley, 2011.
- Udi Manber, *Algorithms – A Creative Approach*, Addison-Wesley, 1989
- Jeff Erickson, *Algorithms*
- Tim Roughgarden, *Algorithms Illuminated*

Evaluation

- Two quizzes - 20%
- Midsem - 30%
- Endsem - 50%

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Introduction

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- He wrote the celebrated Arabic text Kitab al-jabr wa'l-muqabala ("Rules of restoring and equating")
- Gradually the form and meaning of algorism became corrupted and resulted into algorithm

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  - *If* 4 *is the largest integer* n *for which there is a solution to the equation* $w^n + x^n + y^n = z^n$ *in positive integers* $w, x, y,$ *and* z, *then go to step 6*

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  - *If 4 is the largest integer n for which there is a solution to the equation $w^n + x^n + y^n = z^n$ in positive integers $w, x, y,$ and z, then go to step 6*
- *An algorithm is an explicit, precise, unambiguous, mechanically-executable sequence of elementary instructions, usually intended to accomplish a specific purpose.* – Jeff Erickson

Overview

- Algorithm and program
- Pseudo-code
- Algorithm + Data-Structures = Program
- Initial solution + Analysis + Solution Refinement + Data-Structures = Final Program
- Use of recursive definition for initial solution
- Use recurrence equation for proofs and analysis
- Solution refinement through recursion transformation and traversal
- Data structures for saving past results for future use

- Sample problems
  - Finding MAX
  - Finding MAX and MIN
  - Finding MAX and 2nd-MAX
  - Fibonacci numbers
  - Searching in ordered / unordered list
  - Sorting
  - Pattern matching
  - Permutation and combination
  - Shortest path

Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

1. max(L)
2. if $|L| = 1$ return x_1
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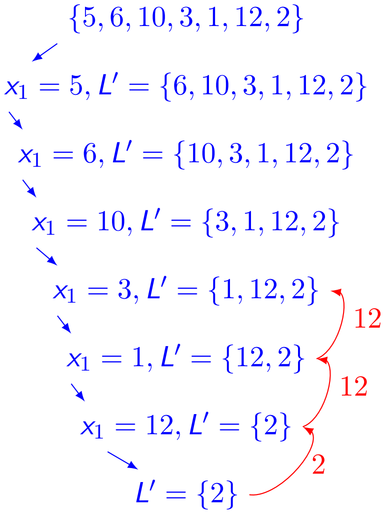

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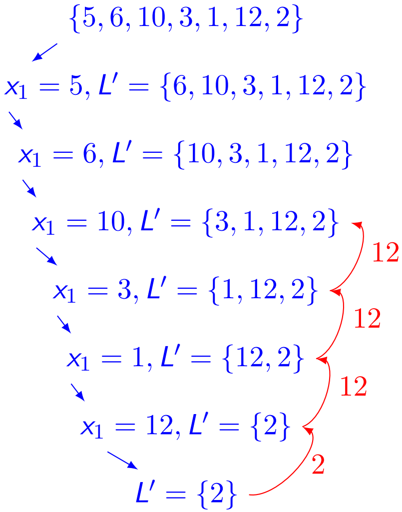

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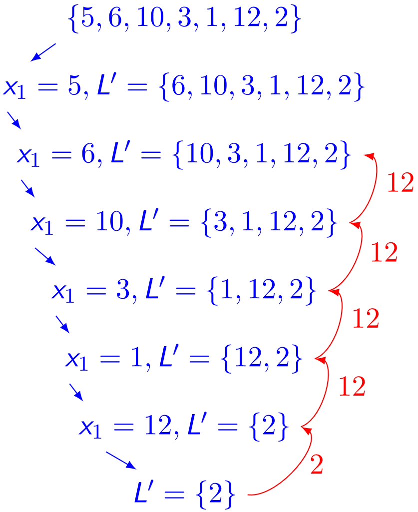

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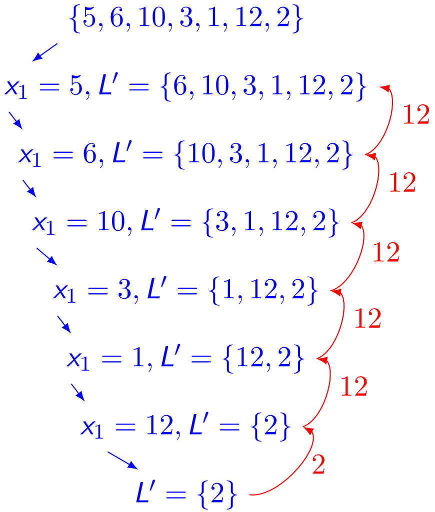

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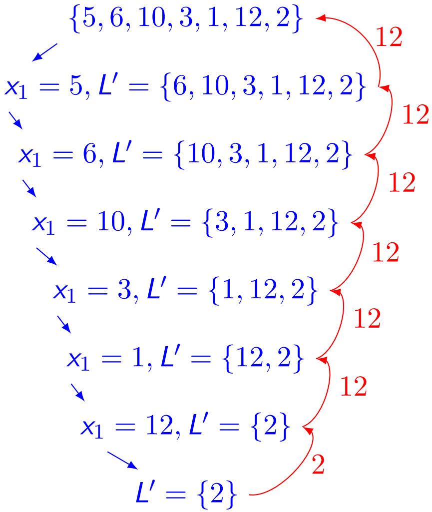

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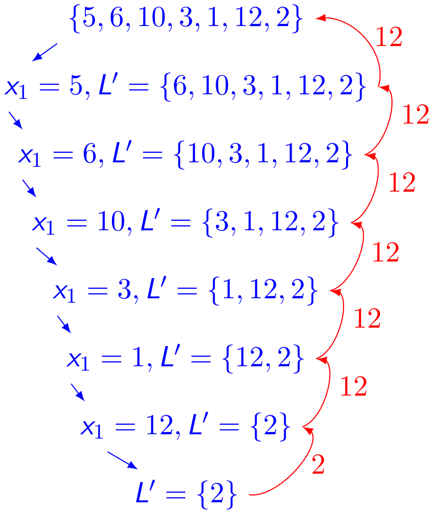

Complexity analysis:

$$T(n) = \begin{cases} T(n-1) + 1, & n > 1 \\ 0, & n = 1 \end{cases}$$

Finding MAX of n elements (2)

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- Recursive formulation:

1. max2(L)
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3. Split L into 2 non-empty sets L_1, L_2
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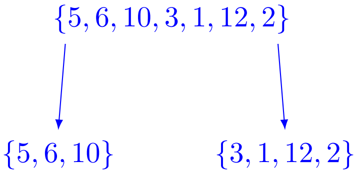

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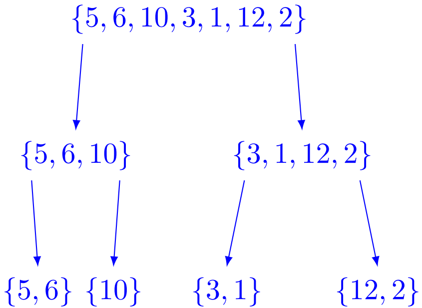

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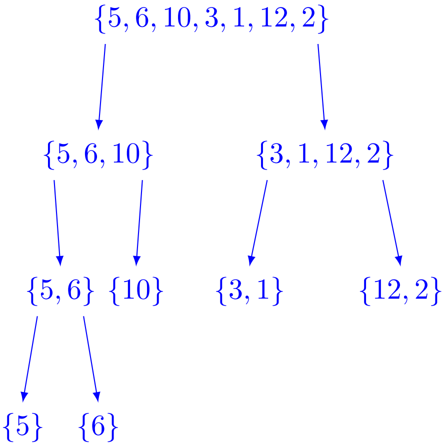

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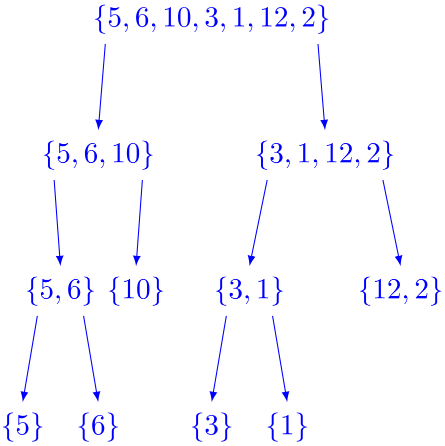

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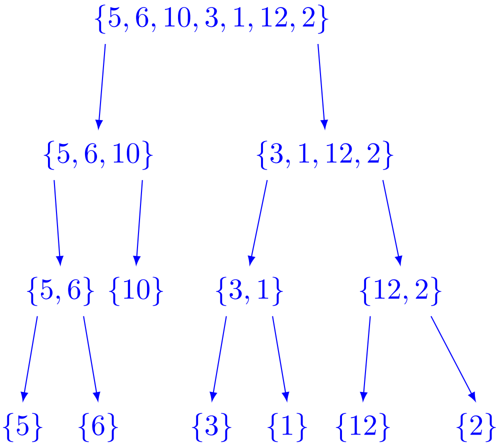

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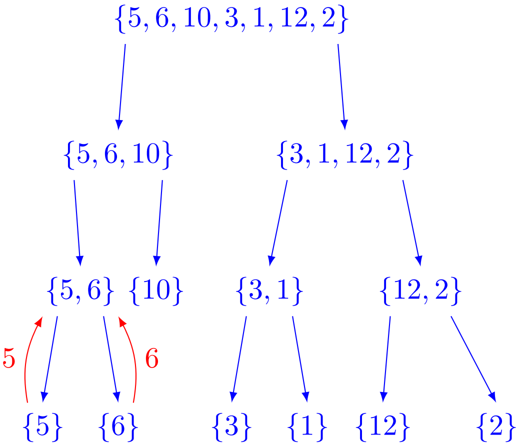

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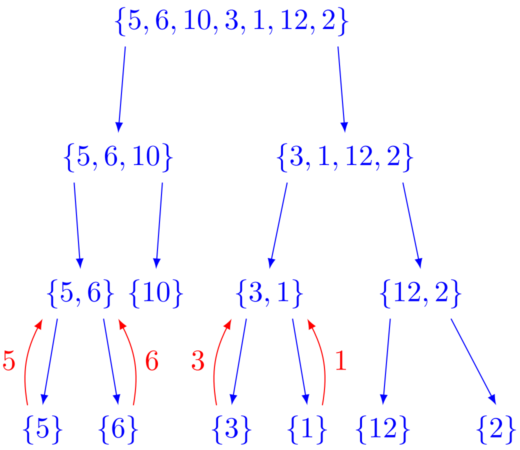

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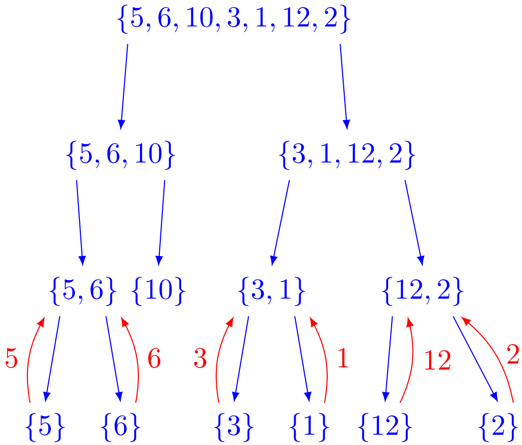

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1. max2(L)
2. if $|L| = 1$ return x_1
3. Split L into 2 non-empty sets L_1, L_2
4. $x =$max2(L_1)
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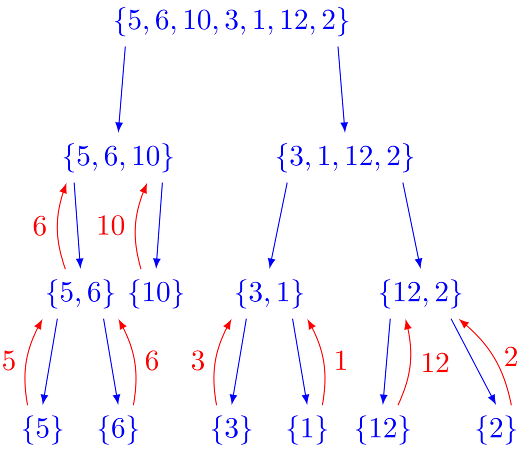

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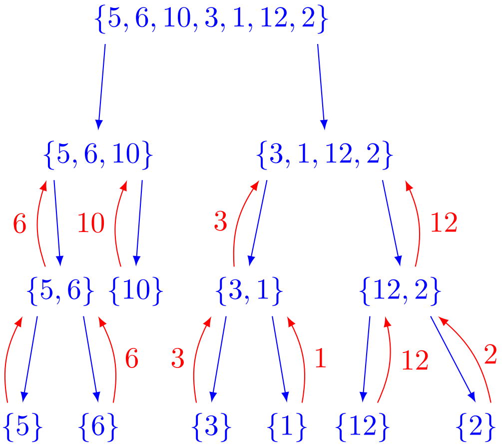

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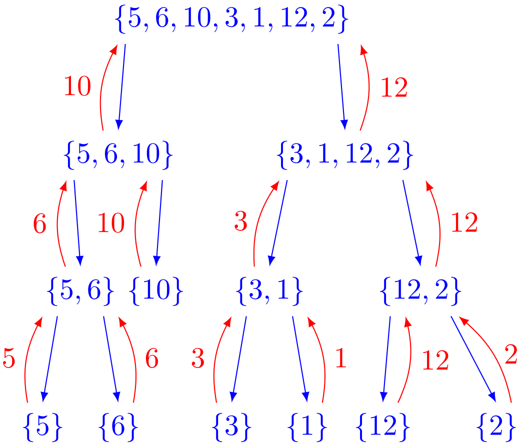

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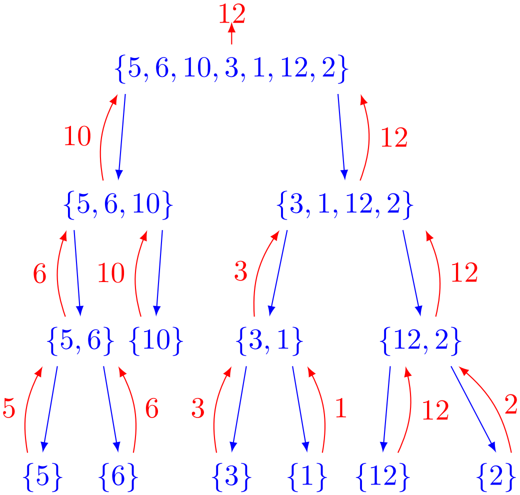

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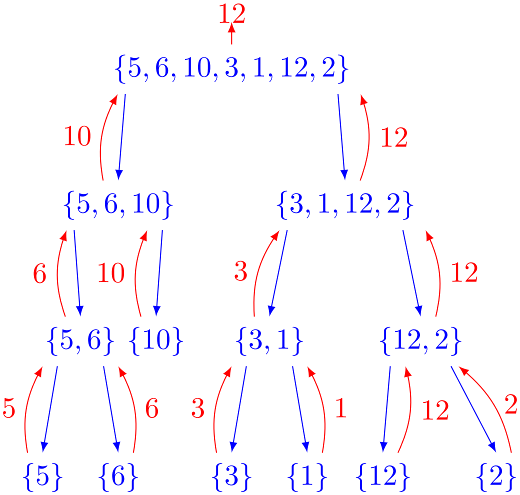
Complexity analysis:

$$
\begin{aligned}
T(n) &= T(n-k) + T(k) + 1, & n > 1 \\
&= 0, & n = 1
\end{aligned}
$$



Recursion Flow: Finding MAX of n elements (2)

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$$\{5, 6, 10, 3, 1, 12, 2\}$$

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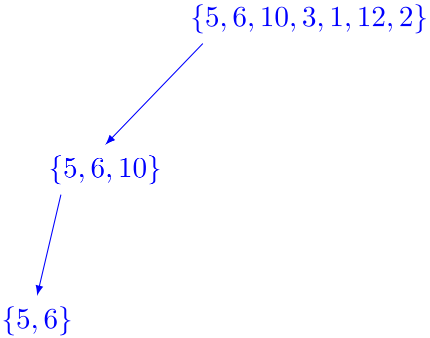

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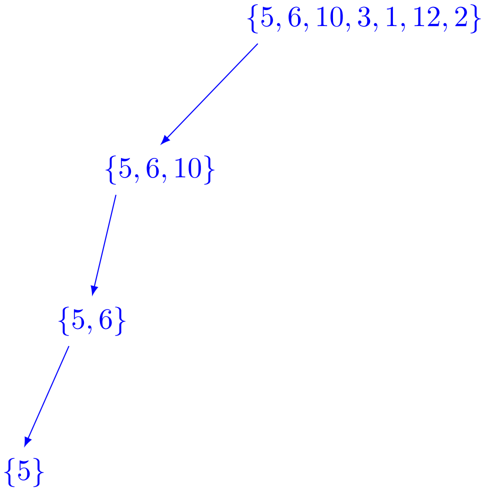

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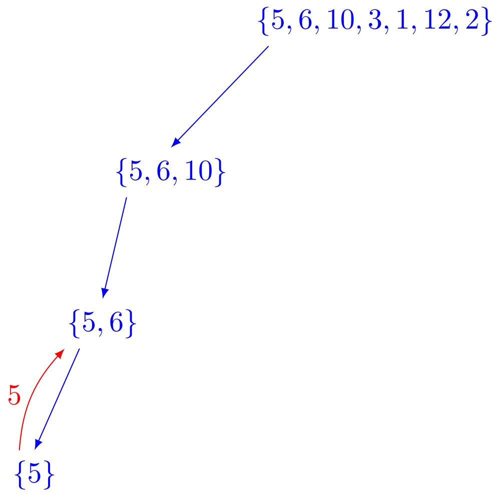

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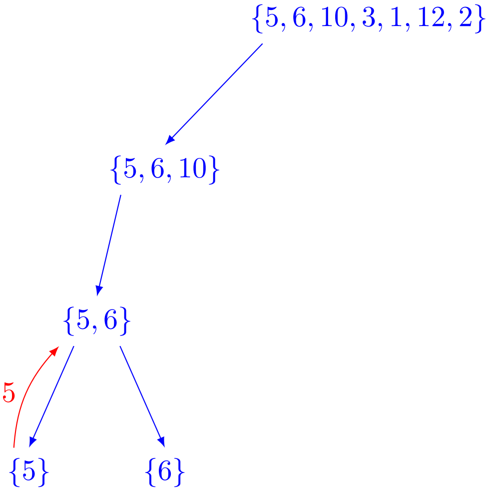

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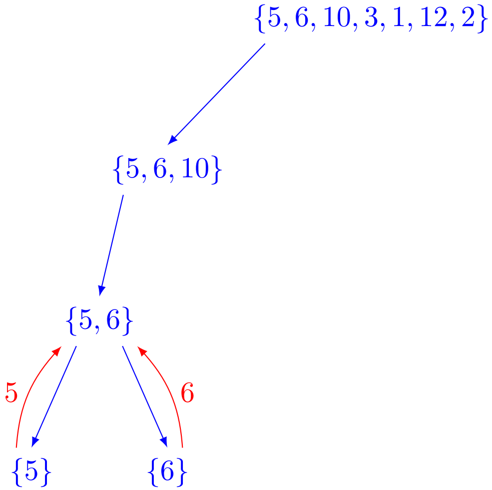

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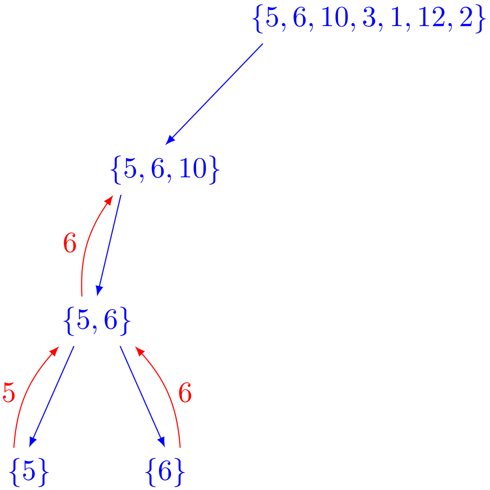

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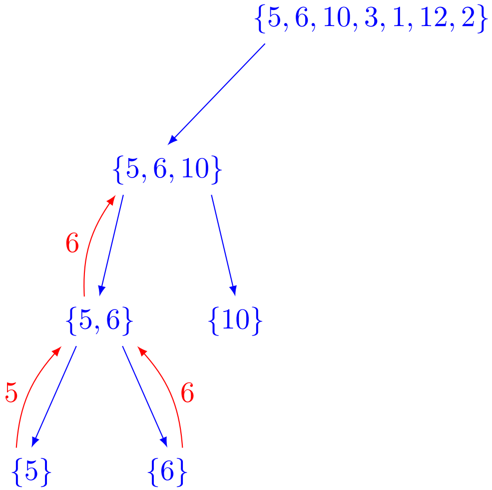

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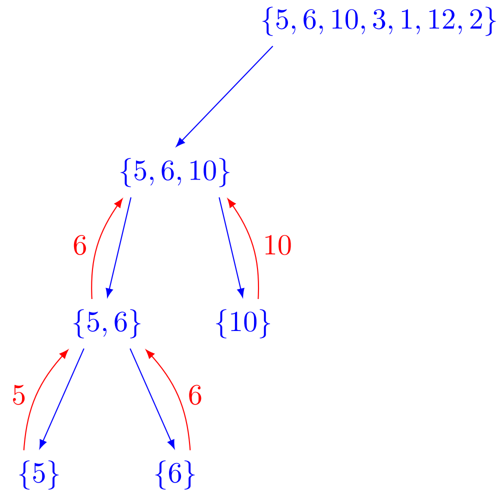

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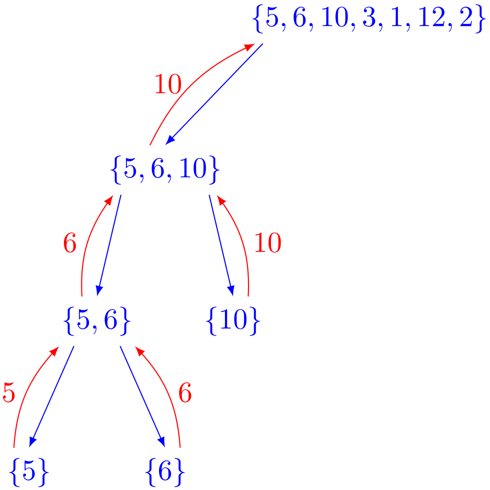

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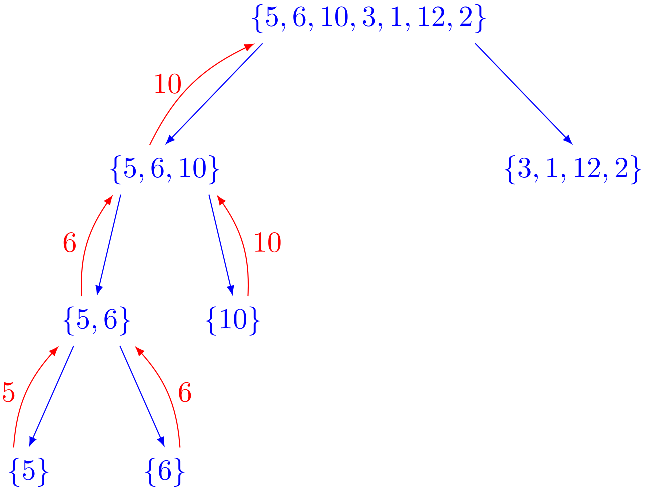

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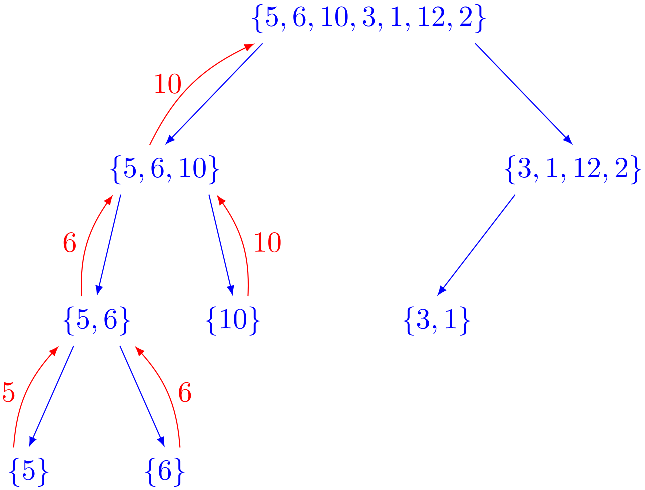

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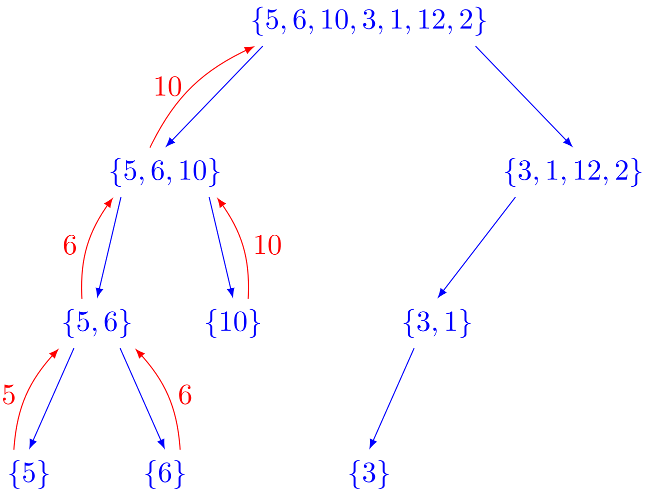

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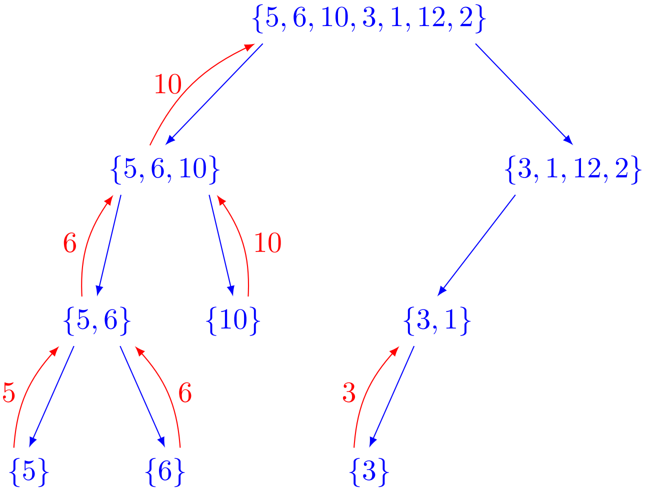

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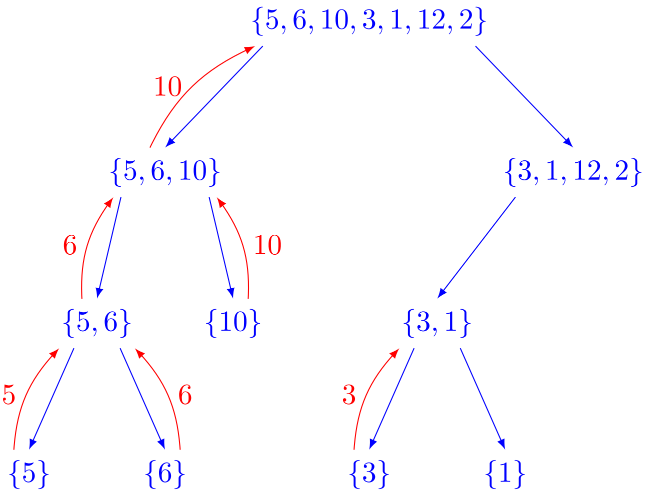

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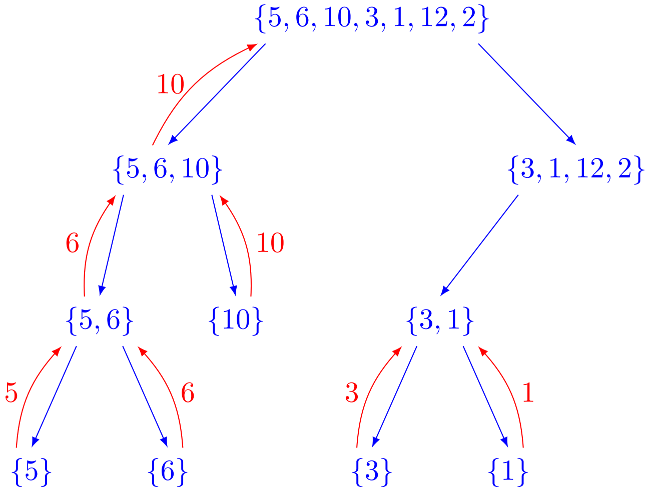

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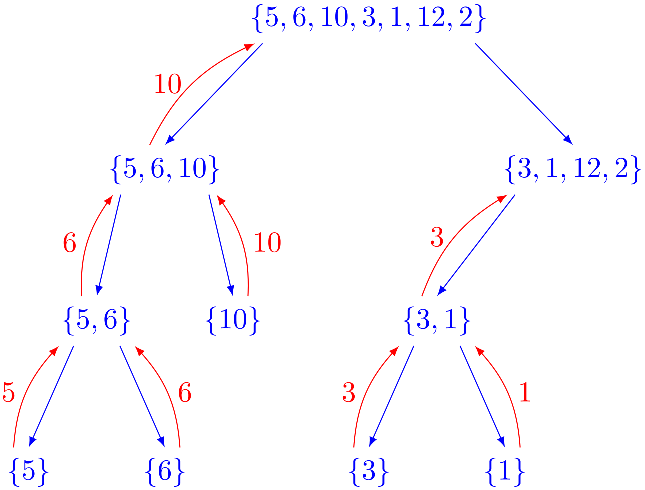

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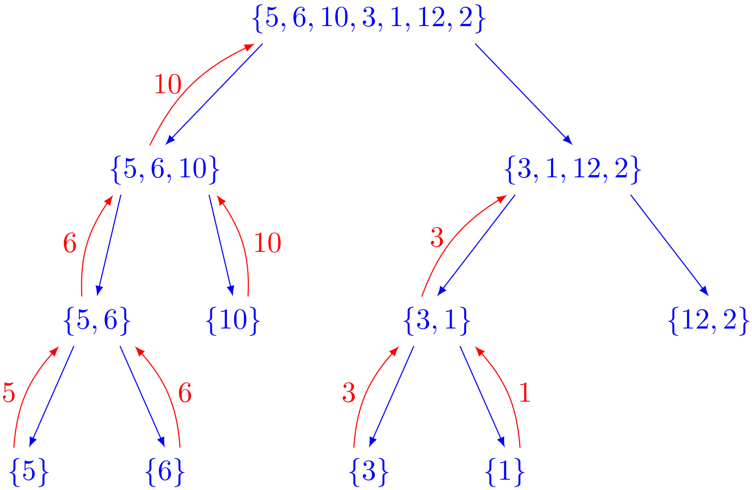

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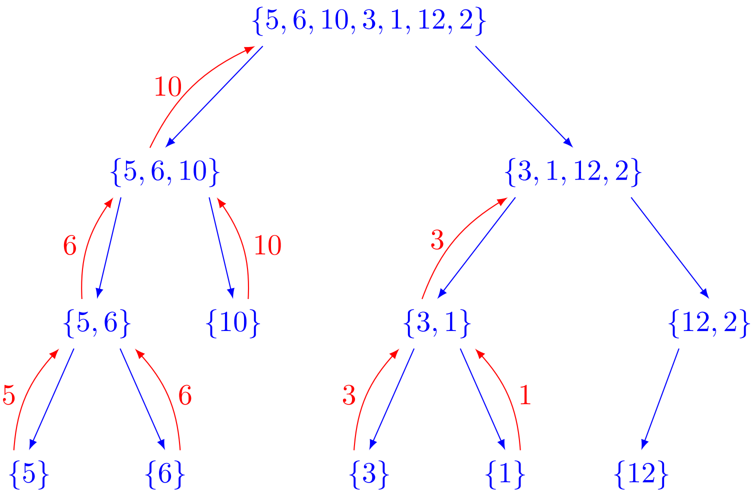

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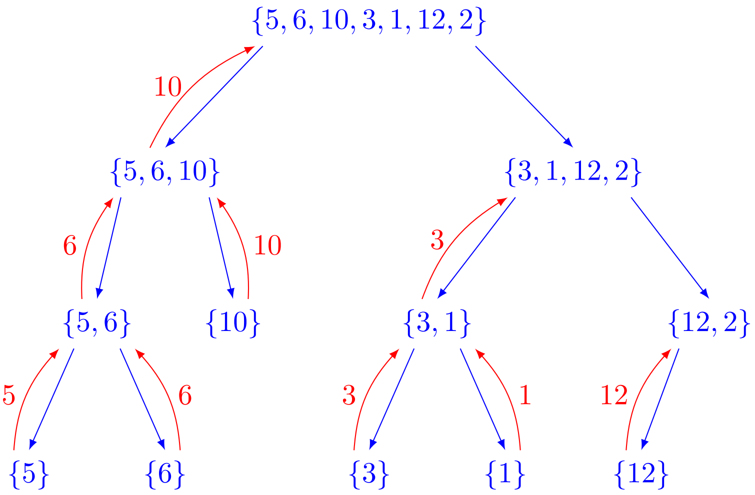

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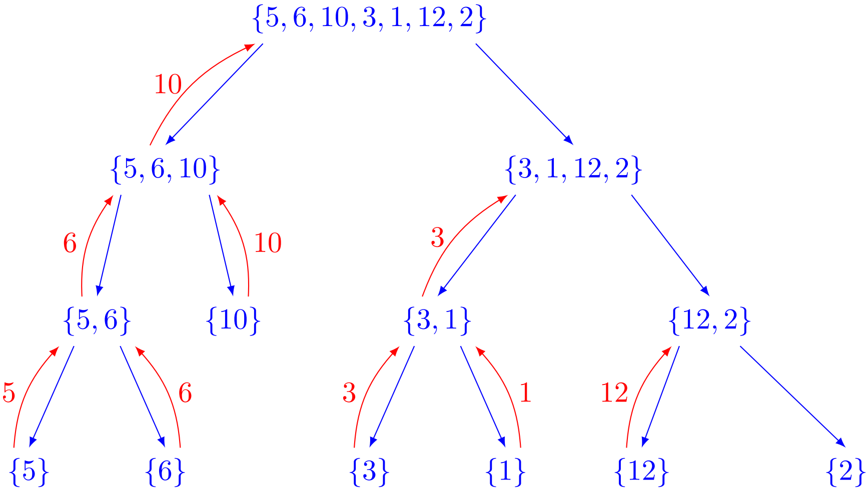

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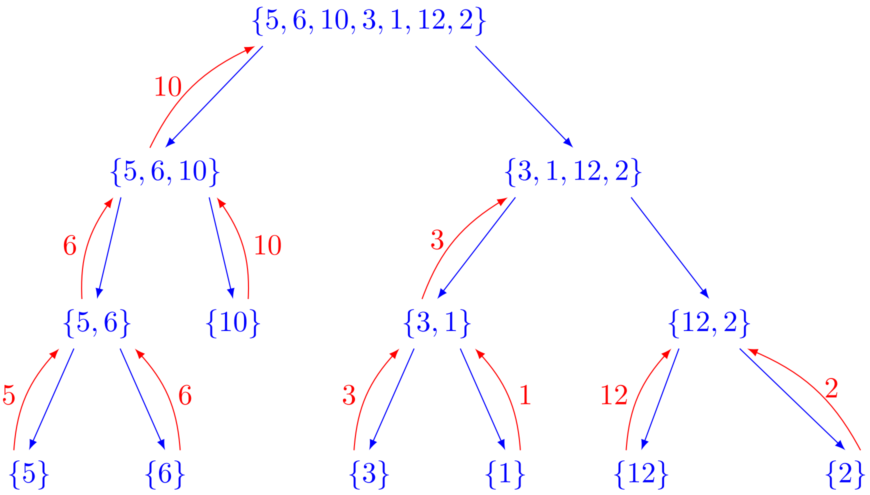

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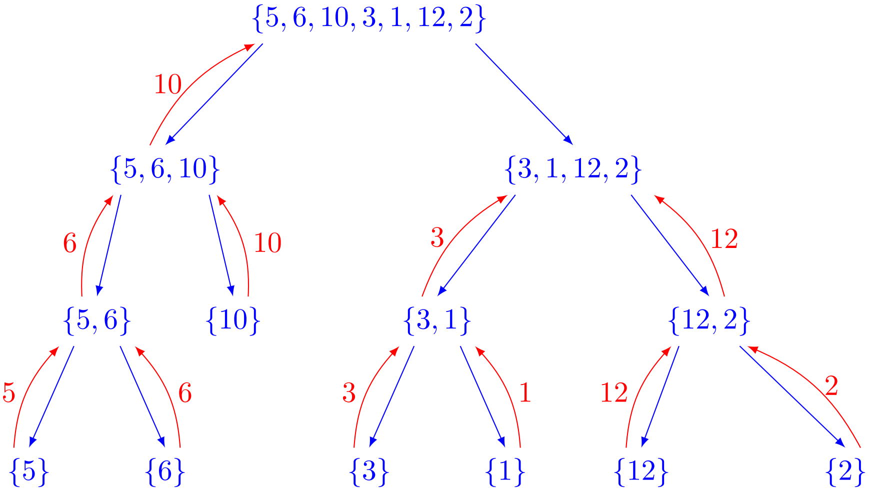

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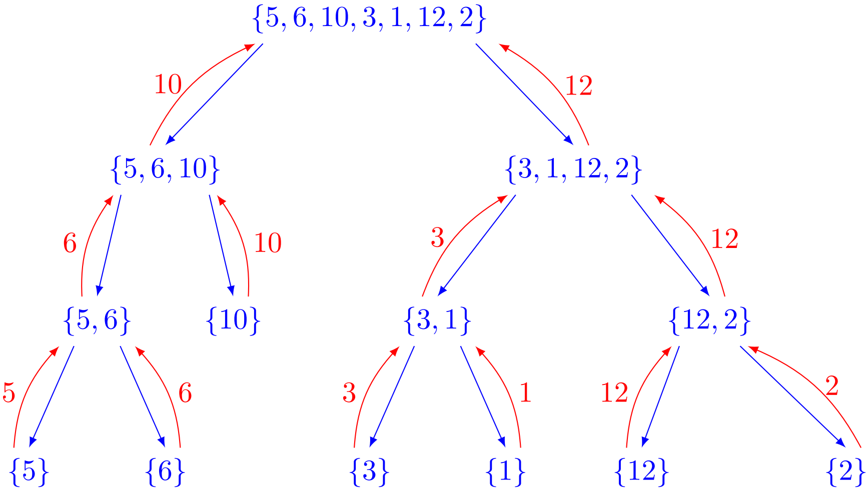

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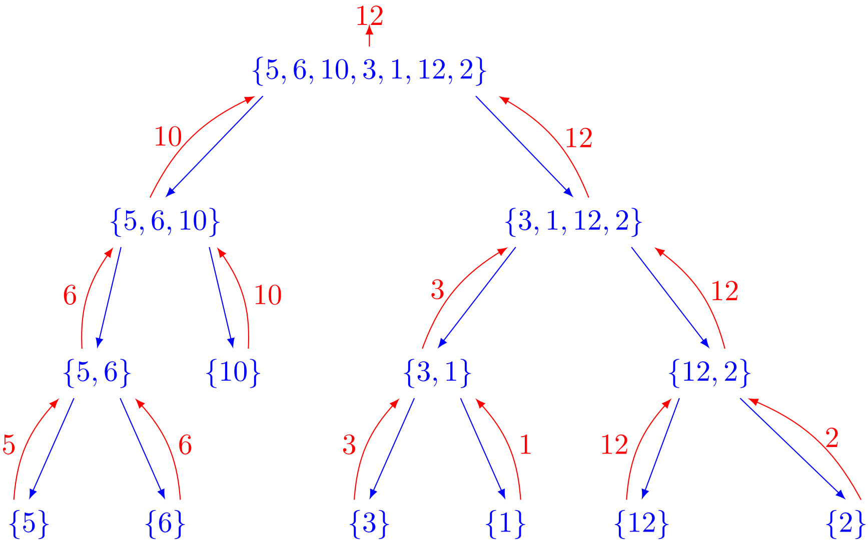


Comparison Tournament

- Finding of maximum can be viewed as a tournament of players taken two at a time

x_0

x_1

x_2

x_3

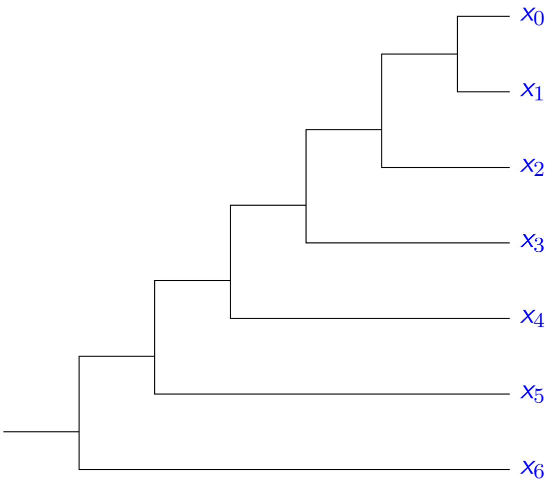
x_4

x_5

x_6

Comparison Tournament

- Finding of maximum can be viewed as a tournament of players taken two at a time



Comparison Tournament

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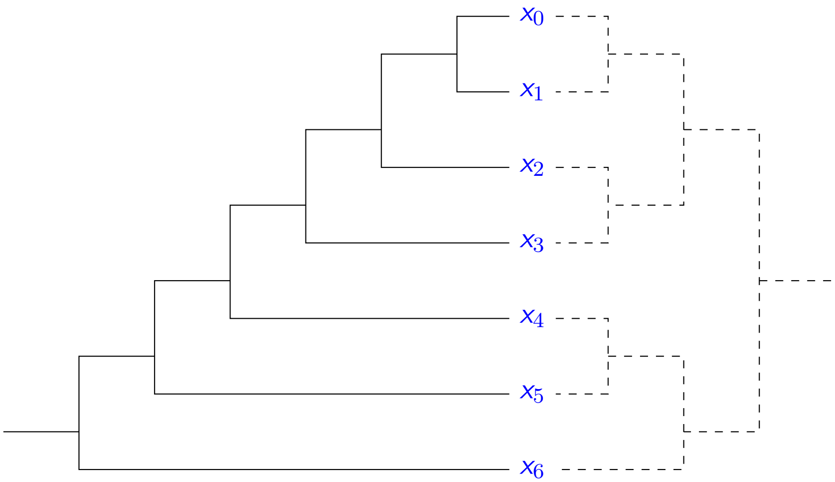


MAX & MIN (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$ and $\min\{L\}$

MAX & MIN (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$ and $\min\{L\}$
- Sequential comparison

1. maxmin(L)
2. if $|L|=1$ return $\langle x_1, x_1 \rangle$
3. $L' = L - \{x_1\}$
4. $\langle y_1, y_2 \rangle =$ maxmin(L')
5. if $x_1 > y_1$ then $m_1 = x_1$ else $m_1 = y_1$
6. if $x_1 < y_2$ then $m_2 = x_1$ else $m_2 = y_2$
7. return $\langle m_1, m_2 \rangle$

MAX & MIN (2)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$ and $\min\{L\}$
- Recursive definition

maxmin2(L)
  if $|L|=1$ return $\langle x_1, x_1 \rangle$
  if $|L|=2$ if $x_1 > x_2$ return $\langle x_1, x_2 \rangle$ else return $\langle x_2, x_1 \rangle$
  Split L into 2 non-empty sets L_1, L_2
  $\langle y_1, y_2 \rangle =$ maxmin2(L_1)
  $\langle z_1, z_2 \rangle =$ maxmin2(L_2)
  if $y_1 > z_1$ then $m_1 = y_1$ else $m_1 = z_1$
  if $y_2 < z_2$ then $m_2 = y_2$ else $m_2 = z_2$
  return $\langle m_1, m_2 \rangle$

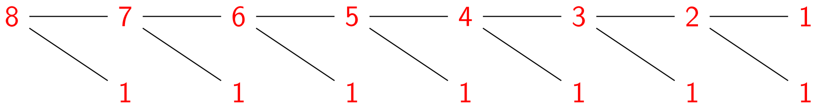
MAX & MIN (3)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$ and $\min\{L\}$
- Recursive definition - Choice of split
  - Recurrence relation:

$$
\begin{aligned}
T(n) &= 0, && n = 1 \\
&= 1, && n = 2 \\
&= T(k) + T(n-k) + 2, && n = 2
\end{aligned}
$$

Options for splitting

Options for splitting



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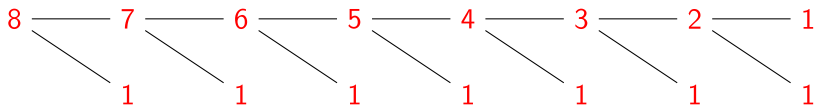
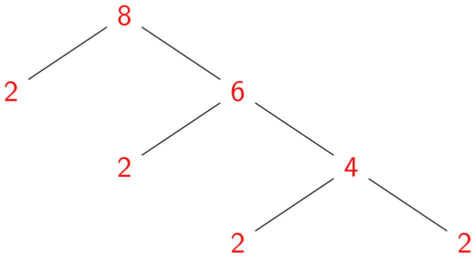
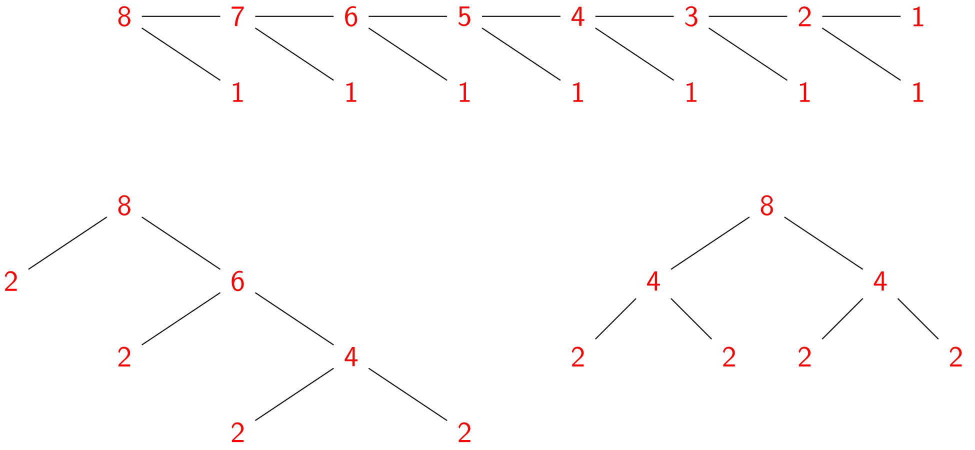




Options for splitting



MAX & 2nd-MAX (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find max$\{L\}$ and 2nd-max$\{L\}$

MAX & 2nd-MAX (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find max$\{L\}$ and 2nd-max$\{L\}$
- Recursive definition

max2ndmax(L)
    if $|L|=1$ return $\langle x_1, x_1 \rangle$
    if $|L|=2$ if $x_1 > x_2$ return $\langle x_1, x_2 \rangle$ else return $\langle x_2, x_1 \rangle$
    Split L into 2 non-empty sets L_1, L_2
    $\langle y_1, y_2 \rangle =$ max2ndmax(L_1)
    $\langle z_1, z_2 \rangle =$ max2ndmax(L_2)
    if $(y_1 > z_1)$ $\{\ m_1 = y_1;\ m_2 = z_1 > y_1 ? z_1 : y_1;\ \}$
    else $\{\ m_1 = z_1;\ m_2 = y_1 > z_2 ? y_1 : z_2;\ \}$
    return $\langle m_1, m_2 \rangle$

- Explore different splitting options

MAX & 2nd-MAX (2)

MAX & 2nd-MAX (2)

$\{5, 6, 10, 3, 1, 12, 2\}$

MAX & 2nd-MAX (2)

$$\{5, 6, 10, 3, 1, 12, 2\}$$

1	2	3	4	5	6	7	8	9	10	11	12	13

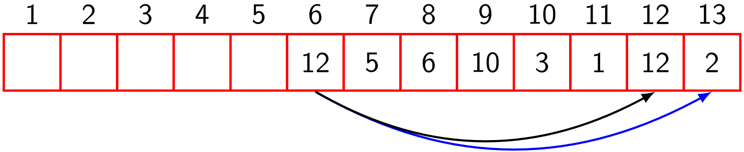
MAX & 2nd-MAX (2)

$\{5, 6, 10, 3, 1, 12, 2\}$

1	2	3	4	5	6	7	8	9	10	11	12	13
						5	6	10	3	1	12	2

MAX & 2nd-MAX (2)

$\{5, 6, 10, 3, 1, 12, 2\}$



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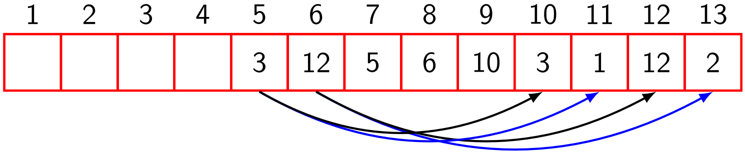

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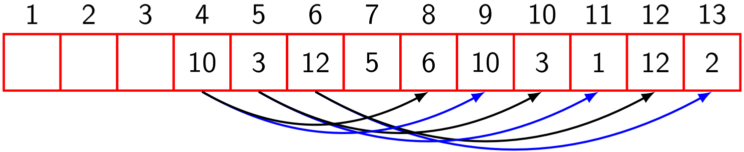

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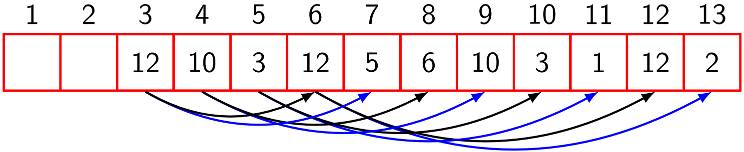
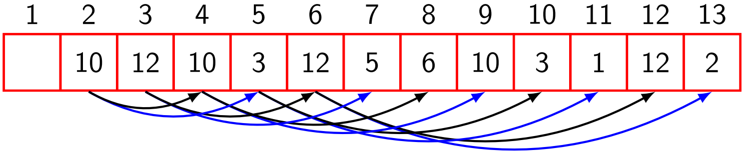


MAX & 2nd-MAX (2)

$$\{5, 6, 10, 3, 1, 12, 2\}$$



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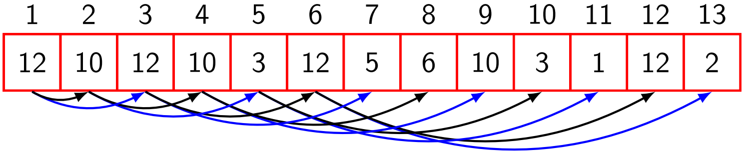

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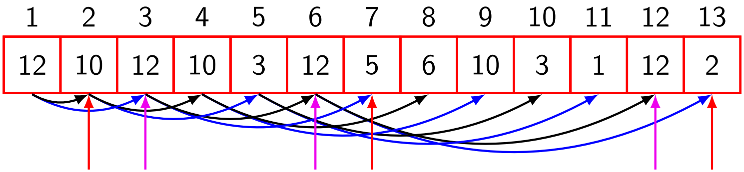

Thank you!