CE 213 - Fluid Mechanics

Basic Equation of Fluid Statics

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Learning Objectives

- Basic equation of fluid statics
  - Compressible fluids
  - Isothermal fluids
  - No-isothermal fluids
- Scales of pressure measurement

Fluid Statics

A fluid element, in isolation from its surroundings, will experience two types of forces:

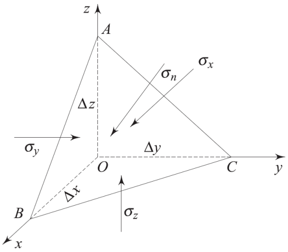
- Body forces
  - Forces act throughout the body of fluid and distributed over entire mass
  - Caused by external factors: gravitational, electro magnetic, or electro static fields
- Surface forces
  - Forces exerted its surroundings through direct contact at the surface.
  - Two components:
    - Normal force - component along the normal to the area
    - Shear force - along the plane of the area.

Fluid Statics

Normal Stresses in a Stationary Fluid

Assumption: Fluid is at rest

- No shear stresses and tensile stresses
- only normal forces - compressive in nature



Equations of static equilibrium

$$\Sigma F_x = \sigma_x \left(\frac{\Delta y \Delta z}{2}\right) - \sigma_n \Delta A cos\alpha = 0.$$

$$\Sigma F_y = \sigma_y \left(\frac{\Delta z \Delta x}{2}\right) - \sigma_n \Delta A cos\beta = 0.$$

$$\Sigma F_z = \sigma_z \left(\frac{\Delta x \Delta y}{2}\right) - \sigma_n \Delta A cos\gamma - \frac{\rho g}{6}(\Delta x \Delta y \Delta z) = 0.$$

$\Sigma F_x, \Sigma F_y, \Sigma F_z$ - net forces acting on fluid element
$cos\alpha, cos\beta, cos\gamma$ - direction cosines of the normal to the inclined plane

Fluid Statics

Normal Stresses in a Stationary Fluid

$$\Delta A cos\alpha = \frac{\Delta y \Delta z}{2}$$

$$\Delta A cos\beta = \frac{\Delta x \Delta y}{2}$$

$$\Delta A cos\gamma = \frac{\Delta y \Delta x}{2}$$

Upon equating and substituting,

$$\sigma_x = \sigma_y = \sigma_z = \sigma_n \tag{1}$$

Pascal's Law

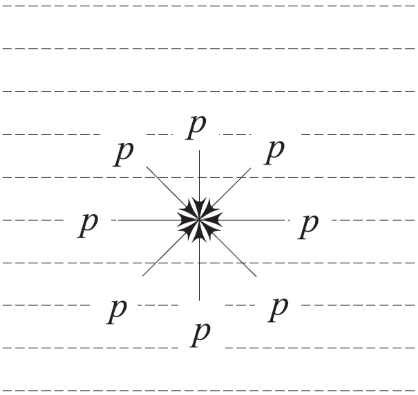
Pascal's law

Normal stress at any point in a fluid at rest are

- Directed towards the point from all directions
- Equal magnitude

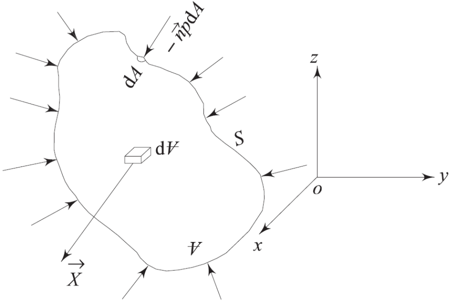
$$\sigma_x = \sigma_y = \sigma_z = -p \qquad (2)$$

where p is a scalar quantity, defined as hydrostatic/fluid static/thermodynamic pressure



Basic Equation of Fluid Statics

How pressure field varies across space coordinates ?



Surface - *s*

Volume - *v*

Forces

- Body forces, $\vec{F}_B$
- Surface forces, $\vec{F}_S$

Basic Equation of Fluid Statics

$$\vec{F}_B + \vec{F}_S = 0$$

$$\iiint_v \rho \vec{X} d\forall + \iint_S -\vec{\eta} P dA = 0 \qquad (3)$$

Gauss Divergence theorem: Surface integral to volume integral

$$\iint_S -\vec{\eta} P dA = \iiint_\forall -\nabla . P d\forall \qquad (4)$$

$$\iiint_\forall \rho \vec{X} d\forall + \iiint_\forall -\nabla . P dv = 0 \rightarrow \iiint_\forall \left(\rho \vec{X} - \nabla . P \right) dv = 0 \qquad (5)$$

Eq. (5) is valid for any elementary volume

$$\rho \vec{X} - \nabla . P = 0 \rightarrow \rho \vec{X} = \nabla . P \qquad (6)$$

Basic Equation of Fluid Statics

$$\rho\vec{x} = \nabla.P$$

Variation in pressure w.r.t. space coordinates

$$\nabla P = \vec{i}\frac{\partial p}{\partial x} + \vec{j}\frac{\partial p}{\partial y} + \vec{k}\frac{\partial p}{\partial z} \tag{7}$$

$$\rho\vec{X} = \vec{i}\rho X_x + \vec{j}\rho X_y + \vec{k}\rho X_z \tag{8}$$

$$\begin{aligned} \frac{\partial p}{\partial x} &= \rho X_x \\ \frac{\partial p}{\partial y} &= \rho X_y \\ \frac{\partial p}{\partial z} &= \rho X_z \end{aligned} \tag{9}$$

Gravity is the only body force, acting in z-direction. Hence, $\frac{\partial p}{\partial x} = 0$; $\frac{\partial p}{\partial y} = 0$; $\frac{\partial p}{\partial z} = -\rho g$

Basic Equation of Fluid Statics

$$\frac{\partial p}{\partial z} = -\rho g \rightarrow \frac{dp}{dz} = -\rho g$$

Integration cannot be done, until and unless we know the nature of the fluid: i.e change in density w.r.t. temperature or pressure.

Special Cases: (Refer to class notes for derivations and more details)

- Iso-density fluid: $P = P_o + \rho g(z_o - z)$
- Isothermal fluid: $P = P_o \exp\left[\frac{\rho_o g}{P_o}(z_o - z)\right]$

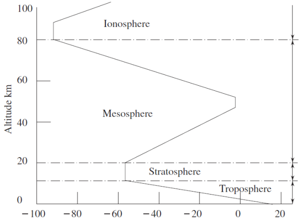
Basic Equation of Fluid Statics

Variation in Temperature

Non-isothermal Fluid

Variation in temperature w.r.t height: $T = T_o - \alpha z$

To=288 k at MSL=0, Lapse rate, α=6.5K/km; Z is Altitude in km



Pressure at any point, z: $P = P_o \left(1 - \frac{\alpha z}{T_o}\right)^{g/R\alpha}$

THANK YOU !!