CE 213 - Fluid Mechanics

Viscosity and Surface Tension

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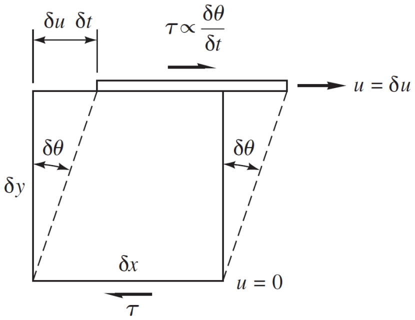
Learning Objectives

- Viscosity
- Classification of fluids
- Surface tension and fe special cases

Viscosity

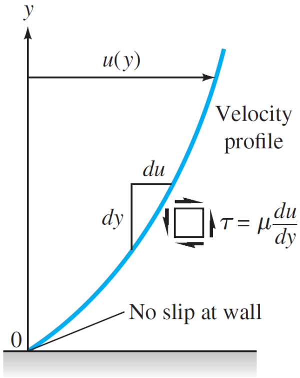
- Can be manifested only when the fluid is in motion
- Quantitative measure of a fluid's resistance to flow
- Determines the fluid strain rate that is generated by a given applied shear stress



$$\text{Viscosity, } \tau \propto \frac{\delta\theta}{\delta t} \quad (1)$$

$$\tan\theta = \frac{\delta u \delta t}{\delta y} \quad (2)$$

Viscosity



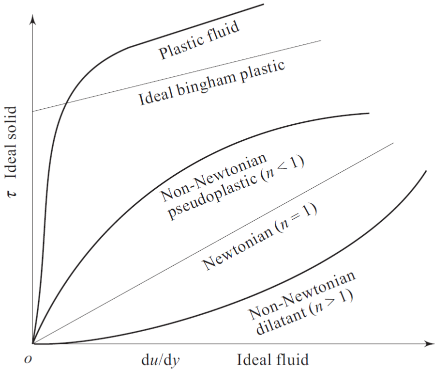
In the limit of infinitesimal changes, this becomes a relation between shear strain rate and velocity gradient

$$\frac{d\theta}{dt} = \frac{du}{dy} \tag{3}$$

$$\text{Viscosity, } \tau = \mu \frac{du}{dy} \tag{4}$$

Viscosity

Newtonian and Non-Newtonian Fluids



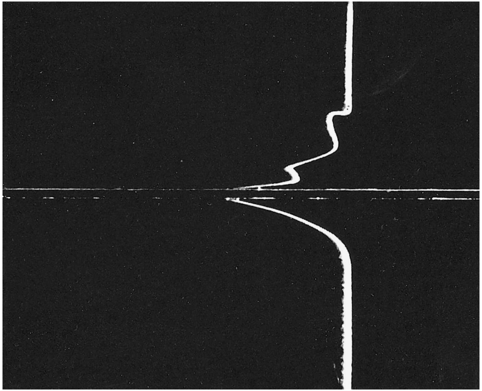
Ostwald-de-waele model/ Power law model

$$\tau = m\left(\frac{du}{dy}\right)^{n} \tag{5}$$

$$\tau = m\left(\frac{du}{dy}\right)^{n-1}\frac{du}{dy} \tag{6}$$

m- flow behaviour index
n - flow consistency index

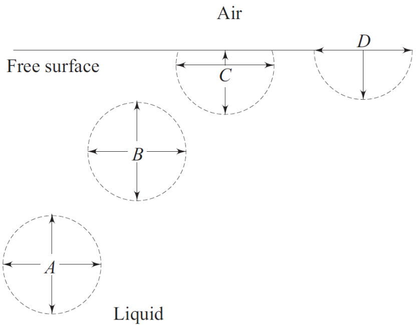
No-slip Condition



- The velocity of the fluid at the solid surface relative to the solid is 0
- Friction and interaction between the solid and the liquid

The no-slip condition at solid surfaces should not be confused with the wetting of surfaces by liquids.

Surface Tension



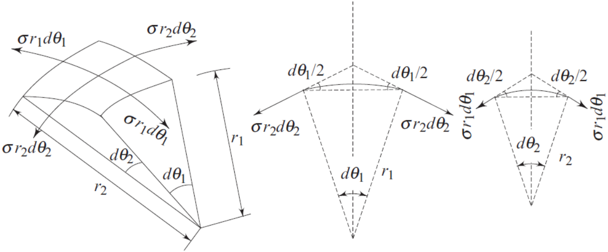
$$\text{Surface tension, } \sigma = \frac{F}{L} \qquad (7)$$

Points to remember

- That surface tension should occur only when there is a surface of separation between two fluids
- Surface tension of the liquid is not the property of the liquid itself, it is a binary property of the liquid and the gas.
- Interfacial tension

Surface Tension

Case 1: Water on One Side and Air on another Side



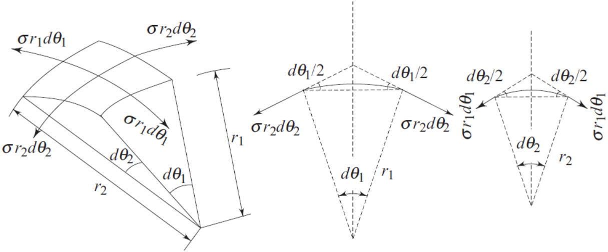
p_i and p_o = Pressure on its convex and concave sides respectively

$$2\sigma r_2 d\theta_2 \sin(\frac{d\theta_1}{2}) + 2\sigma r_1 d\theta_1 \sin(\frac{d\theta_2}{2}) = (p_i - p_o) r_1 r_2 d\theta_1 d\theta_2 \tag{8}$$

$$p_i - p_o = \Delta p = \sigma \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \tag{9}$$

Surface Tension

Case 2: Air on both sides of a thin liquid film



$$2x2\sigma r_2 d\theta_2 \sin(\frac{d\theta_1}{2}) + 2x2\sigma r_1 d\theta_1 \sin(\frac{d\theta_2}{2}) = (p_i - p_o) r_1 r_2 d\theta_1 d\theta_2 \tag{10}$$

$$p_i - p_o = \Delta p = 2x\sigma\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \tag{11}$$

Surface Tension

Special Cases

Case i: Spherical Liquid Drop

- The radius of curvature is same in all directions
- $r_1 = r_2 = r$ i.e. the radius of the drop
- Interacting with air from only one side

$$\Delta p = \sigma \left(\frac{1}{r} + \frac{1}{r} \right) = \frac{2\sigma}{r}. \tag{12}$$

Case ii: Spherical Bubble

- The radius of curvature is same in all directions
- $r_1 = r_2 = r$
- Interacting with air from both sides (inside the bubble and outside the bubble).

$$\Delta p = 2\sigma \left(\frac{1}{r} + \frac{1}{r} \right) = \frac{4\sigma}{r}. \tag{13}$$

Surface Tension
Special Cases

Case iii: Cylindrical Liquid Jet

- The radius of curvature is r is one direction and ∞ in another direction
- $r_1 = r$ and $r_2 = \infty$

$$\Delta p = \sigma \left(\frac{1}{r} + \frac{1}{\infty} \right) = \frac{\sigma}{r}. \tag{14}$$

THANK YOU !!