CE 213 - Fluid Mechanics

Viscosity and Surface Tension

Bachu Anilkumar
Department of Civil and Environmental Engineering
IIT Patna
Learning Objectives

- Viscosity
- Classification of fluids
- Surface tension and fe special cases
Viscosity

- Can be manifested only when the fluid is in motion
- Quantitative measure of a fluid's resistance to flow
- Determines the fluid strain rate that is generated by a given applied shear stress

\[
\tau \propto \frac{\delta \theta}{\delta t} \quad (1)
\]

\[
\tan \theta = \frac{\delta u \delta t}{\delta y} \quad (2)
\]
Viscosity

In the limit of infinitesimal changes, this becomes a relation between shear strain rate and velocity gradient

\[ \frac{d\theta}{dt} = \frac{du}{dy} \]  

Viscosity, \( \tau = \mu \frac{du}{dy} \)
Viscosity
Newtonian and Non-Newtonian Fluids

Ostwald-de-waele model/ Power law model

\[ \tau = m \left( \frac{du}{dy} \right)^n \]  \hspace{1cm} (5)

\[ \tau = m \left( \frac{du}{dy} \right)^{n-1} \frac{du}{dy} \]  \hspace{1cm} (6)

- \( m \) - flow behaviour index
- \( n \) - flow consistency index
No-slip Condition

- The velocity of the fluid at the solid surface relative to the solid is 0
- Friction and interaction between the solid and the liquid

The no-slip condition at solid surfaces should not be confused with the wetting of surfaces by liquids.
Surface Tension

Surface tension, \( \sigma = \frac{F}{L} \)  

Points to remember:
- That surface tension should occur only when there is a surface of separation between two fluids.
- Surface tension of the liquid is not the property of the liquid itself, it is a binary property of the liquid and the gas.
- Interfacial tension.
Surface Tension
Case 1: Water on One Side and Air on another Side

\[ \sigma r_1 d\theta_1 + \sigma r_2 d\theta_2 = (p_i - p_o) r_1 r_2 d\theta_1 d\theta_2 \] (8)

\[ p_i - p_o = \Delta p = \sigma \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \] (9)

\( p_i \) and \( p_o \) = Pressure on its convex and concave sides respectively
Surface Tension
Case 2: Air on both sides of a thin liquid film

\[
2 \cdot 2 \sigma r_2 d \theta_2 \sin\left(\frac{d \theta_1}{2}\right) + 2 \cdot 2 \sigma r_1 d \theta_1 \sin\left(\frac{d \theta_2}{2}\right) = (p_i - p_o) r_1 r_2 d \theta_1 d \theta_2 \quad (10)
\]

\[
p_i - p_o = \Delta p = 2 \sigma \left(\frac{1}{r_1} + \frac{1}{r_2}\right) \quad (11)
\]
Surface Tension
Special Cases

Case i: Spherical Liquid Drop

- The radius of curvature is same in all directions
- \( r_1 = r_2 = r \) i.e. the radius of the drop
- Interacting with air from only one side

\[
\Delta p = \sigma \left( \frac{1}{r} + \frac{1}{r} \right) = \frac{2\sigma}{r}.
\]  
(12)

Case ii: Spherical Bubble

- The radius of curvature is same in all directions
- \( r_1 = r_2 = r \)
- Interacting with air from both sides (inside the bubble and outside the bubble).

\[
\Delta p = 2\sigma \left( \frac{1}{r} + \frac{1}{r} \right) = \frac{4\sigma}{r}.
\]  
(13)
Case iii: Cylindrical Liquid Jet

- The radius of curvature is $r$ in one direction and $\infty$ in another direction
- $r_1 = r$ and $r_2 = \infty$

$$\Delta \rho = \sigma \left( \frac{1}{r} + \frac{1}{\infty} \right) = \frac{\sigma}{r}. \quad (14)$$
THANK YOU !!