## PH103 (Physics-I)

## Tutorial-V (September 20, 2018)

1. A mass is dropped from a point directly above the equator. Consider the moment when the object has fallen a distance $d$. If we consider only the centrifugal force, then the correction to $g_{\text {eff }}$ at this point (relative to the release point) is an increase by $\omega^{2} d$. There is, however, also a second-order Coriolis effect. What is the sum of these corrections?
2. A uniform thin rod of length $L$ and mass $M$ is pivoted at one end. The pivot is attached to the top of a car accelerating at rate $a_{0}$.
(a) What is the equilibrium value of angle $\theta$ between the rod and the top of the car?
(b) Suppose that the rod is displaced by a small angle $\phi$ from equilibrium. What is its motion for small $\phi$ ?
3. A high speed hydrofoil races across the ocean at the equator at a speed of $200 \mathrm{miles} / \mathrm{hr}$. Let the acceleration due to gravity for an observer at rest on the earth be $g$. Find the fractional change in gravity $\frac{\Delta g}{g}$ measured by a passenger on the hydrofoil when the hydrofoil heads in the following directions:
(a) East
(b) West
(c) South
(d) North
4. A particle of mass $m$ is located at $x=2, y=0, z=3$.
(a) Obtain the moment of inertia tensor relative to the origin.
(b) If the particle undergoes pure rotation about the z-axis through a small angle $\beta$, show that the moment of inertia tensor is unchanged to first order in $\beta$ if $\beta \ll 1$.
5. A wheel is at one end of an axle of length $l$. The other end of the axle is suspended from a string of length $L^{\prime}$. The wheel is set into motion so that it executes uniform precession in the horizontal plane. The wheel has mass $M$ and the moment of inertia about its center of mass is $I_{0}$. Its spin angular velocity is $\omega_{\mathrm{s}}$. Find the angle $\beta$ that the string makes with the vertical. [Note: Neglect the masses of the axle and the string and assume that $\beta$ is so small that approximations like $\sin \beta \approx \beta$ are justified.]
6. "The rotational motion of a rigid body is stable about the axis about which the moment of inertia is either a maximum or a minimum". Prove this! [Hint: Use conservation of energy and angular momentum. You may also prove it using Euler's equations discussed in class.]
7. Four masses lie at the points shown on a rigid isosceles right triangle with hypotenuse length $4 a$. The mass at the right angle is $3 m$, and the other three masses are $m$. Label them $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$, as shown. Assume that the object is floating freely in outer space. Mass C is struck with a quick blow, directed into the page. Let the impulse have magnitude $\int F d t=P$. What are the velocities of all the masses immediately after the blow?

