PH101 (Physics-I) **Tutorial-III (August 11, 2014)**

[Motion]*

1. A projectile of mass m is fired from the origin at speed v_0 and angle θ. It is attached to the origin by a spring with spring constant k and relaxed length zero.
(a) Find $x(t)$ and $y(t)$.
(b) Verify that for small $\omega \equiv \sqrt{\frac{k}{m}}$, the trajectory reduces to normal projectile motion.
(c) Verify that for large ω, the trajectory reduces to simple harmonic motion, *i.e.*, oscillatory motion along a line (at least before the projectile smashes back into the ground!).
(d) Physically interpret "small " and "large ".
(e) What value should ω take so that the projectile hits the ground when it is moving straight downward?

2. For small oscillations, the period of a pendulum is approximately $T \approx 2\pi\sqrt{\frac{l}{g}}$ independent of the amplitude, θ_0. In class we used a perturbative approach for estimating the corrections to T when amplitude θ_0 becomes large. In this tutorial problem, an alternate method for solving the same problem is illustrated.
(a) Using $dt = \frac{dx}{v}$, show that the exact expression for T is

$$T = \sqrt{\frac{8l}{g}} \int_0^{\theta_0} \frac{\mathrm{d}\theta}{\sqrt{\cos\theta - \cos\theta_0}}$$

(b) Making use of the identity $\cos\phi = 1 - 2\sin^2\frac{\phi}{2}$, write T in terms of sines [why!]. Make a suitable change of variables,

$$\sin x \equiv \frac{\sin\frac{\theta}{2}}{\sin\frac{\theta_0}{2}}$$

Now expand the integrand in powers of θ_0 and evaluate the resulting integrals to show that

$$T \approx 2\pi\sqrt{\frac{l}{g}}\left(1 + \frac{\theta_0^2}{16} + \cdots\right)$$

3. Consider the Atwood's machine shown in the figure. Note that the axle of the bottom pulley has two strings attached to it. Obtain the accelerations of the masses.

*Note: Please follow the strategies for "Problem Solving" explained in the class.

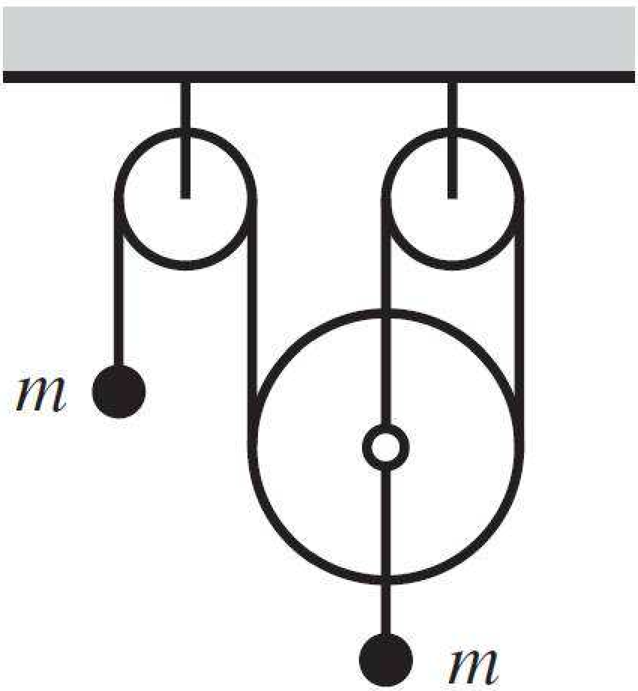

Figure 1: An Atwood's machine.