Back to Piezoelectricity

For a third-rank tensor, the transformation due to a change in coordinate system is represented by,

$$\boxed{d'_{ijk} = a_{il}\, a_{jm}\, a_{kn}\, d_{lmn}} \quad \text{— (viii)}$$

Example

When the crystal has a four-fold axis along z-axis, the transformation matrix is given by

$$T_4 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

$$d_{123} = d_{132} \;\&\; d_{213} = d_{231}$$

$$\Rightarrow d_{111} = d_{222} = d_{112} = d_{121} = d_{221} = d_{211}$$
$$= d_{212} = d_{122} = d_{331} = d_{313} = d_{133} = d_{323}$$
$$= d_{332} = d_{323} = d_{233} = d_{312} = d_{321} = 0;$$
$$d_{333} = 0; \quad d_{311} = d_{322}; \quad d_{113} = d_{131} = d_{223} = d_{232}$$
$$\& \; d_{123} = d_{132} = -d_{213} = -d_{231}.$$

∴ d tensor:

1st layer
$$\begin{pmatrix} 0 & 0 & d_{131} \\ 0 & 0 & d_{123} \\ d_{131} & d_{123} & 0 \end{pmatrix}$$

2nd layer
$$\begin{pmatrix} 0 & 0 & -d_{123} \\ 0 & 0 & d_{131} \\ -d_{123} & d_{131} & 0 \end{pmatrix}$$

3rd layer
$$\begin{pmatrix} d_{311} & 0 & 0 \\ 0 & d_{311} & 0 \\ 0 & 0 & d_{333} \end{pmatrix}$$

} 4 indep. elements only.

Reduction

d_{ijk} is symmetric in j & k.

∴ 27 independent coefficient

⇓

18 independent coefficient

second & third suffix can be replaced by a single suffix.

Table

Tensor / Matrix	11	22	33	23,32	31,13	12,21
	1	2	3	4	5	6

∴ Eqn. (v.) takes the form,

$$\begin{pmatrix} d_{11} & \frac{1}{2}d_{16} & \frac{1}{2}d_{15} \\ \frac{1}{2}d_{16} & d_{12} & \frac{1}{2}d_{14} \\ \frac{1}{2}d_{15} & \frac{1}{2}d_{14} & d_{13} \end{pmatrix}$$

$$\begin{pmatrix} d_{21} & \frac{1}{2}d_{26} & \frac{1}{2}d_{25} \\ \frac{1}{2}d_{26} & d_{22} & \frac{1}{2}d_{24} \\ \frac{1}{2}d_{25} & \frac{1}{2}d_{24} & d_{23} \end{pmatrix} \quad \text{——— (ix.)}$$

$$\begin{pmatrix} d_{31} & \frac{1}{2}d_{36} & \frac{1}{2}d_{35} \\ \frac{1}{2}d_{36} & d_{32} & \frac{1}{2}d_{34} \\ \frac{1}{2}d_{35} & \frac{1}{2}d_{34} & d_{33} \end{pmatrix}$$

In order to maintain consistency, strain tensor can be written as

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{pmatrix} \longrightarrow \begin{pmatrix} x_1 & \frac{1}{2}x_6 & \frac{1}{2}x_5 \\ \frac{1}{2}x_6 & x_2 & \frac{1}{2}x_4 \\ \frac{1}{2}x_5 & \frac{1}{2}x_4 & x_3 \end{pmatrix} \quad \text{——— (x.)}$$

∴ Eqn. (iv.) becomes,

$$x_j = d_{ij} E_i \quad (i = 1, 2, 3;\ j = 1, 2, \ldots, 6).$$

——— (xi.)

Eqn (vi.) becomes,

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} d_{11} & d_{21} & d_{31} \\ d_{12} & d_{22} & d_{32} \\ d_{13} & d_{23} & d_{33} \\ d_{14} & d_{24} & d_{34} \\ d_{15} & d_{25} & d_{35} \\ d_{16} & d_{26} & d_{36} \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix} + \begin{pmatrix} M_{11} & M_{21} & M_{31} & M_{41} & M_{51} & M_{61} \\ M_{12} & M_{22} & M_{32} & M_{42} & M_{52} & M_{62} \\ M_{13} & M_{23} & M_{33} & M_{43} & M_{53} & M_{63} \\ M_{14} & M_{24} & M_{34} & M_{44} & M_{54} & M_{64} \\ M_{15} & M_{25} & M_{35} & M_{45} & M_{55} & M_{65} \\ M_{16} & M_{26} & M_{36} & M_{46} & M_{56} & M_{66} \end{pmatrix} \begin{pmatrix} E_1^2 \\ E_2^2 \\ E_3^2 \\ E_2 E_3 \\ E_3 E_1 \\ E_1 E_2 \end{pmatrix}.$$

Problem: $BaTiO_3$ has a tetragonal crystal symmetry (point group 4mm) at ambient temperatures. Determine the nature of strain induced in the material when an electric field is applied along: (a.) crystallographic c-axis, (b.) crystallographic a-axis

(Uchino, FE devices).

Sol^n. The matrix equation for point group 4mm becomes,

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} = \begin{pmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & d_{33} \\ 0 & d_{15} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix}.$$

s.t.,
$d_{31} < 0$,
$d_{33} > 0$, &
$d_{15} > 0$.

$\Rightarrow x_1 = x_2 = d_{31} E_3.$

$x_3 = d_{33} E_3.$

$x_4 = d_{15} E_2.$

$x_5 = d_{15} E_1.$

$x_6 = 0.$

Case (a.) Elongation in c-direction & contraction along a- and b-directions.

Case (b.) Shear strain $x_5 (\equiv 2x_{31}) = d_{15} E_1$ is induced.