PH515: Lecture Note: ~~Therm~~ Thermo-electric effects. Date: 12/01/2015. jb.

→ Thermo-electric effect: Direct conversion of temperature difference to voltage and vice-a-versa.

<u>Seebeck effect</u>: - temperature gradient across a conductor gives rise to an electric field

- usually measured in reference to other material by forming a bi-metallic junction

- discovered in 1821 by German physicist Thomas Johann Seebeck.
  - discovered that a compass needle would be deflected by a closed loop formed by two different metals joined in two places, with a temp difference between the junctions

- General equation. - Seebeck effect is described locally by the creation of an electromotive field

$$E_{emf} = -S\,\nabla T$$

where S is the seebeck coefficient, (also known as thermopower), ∇T is the gradient in temperature.

<u>Peltier effect</u>: - A current flow through a junction ~~of~~ between two conductors A and B, may result in the generation or removal of heat at the junction. Heat generated at the junction per unit time is

$$\dot{Q} = (\Pi_A - \Pi_B)\, I.$$

Π = peltier coefficient

Important Note: Total heat is not only determined by Peltier effect, Joule heating and thermal gradient effects may also contribute.

How can thermo electric effects be used in micro devices -

→ If dimensions of a device are reduced, then

Mass reduces as third power of dimension l.

Surface area decreases as square of l.

i.e. the surface to volume or surface to mass ratio is very large for micro devices.

→ Because of small mass, microdevices can be heated up more quickly with less energy consumption.

→ Due to large surface area/mass ratio, cooling down can also be quickly.

Now if an isotropic rigid body with a coefficient of thermal extension α_{th} is heated up by a temperature change ΔT, it extends to all direction by the strain ε_{th}.

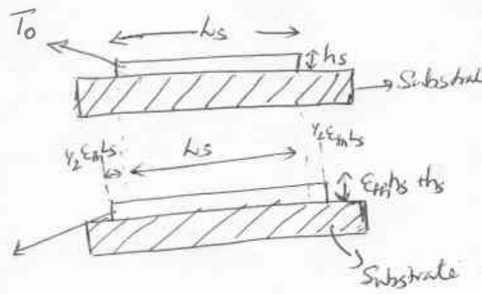
i.e. $\varepsilon_{th} = \alpha_{th} \Delta T$, α_{th} is a material constant.

If a pressure load p is acting on the heated body against the direction of thermal extension, the strain generated by the pressure according to Hooke's law needs to be added.

$$\boxed{\varepsilon_{th} = \alpha_{th} \Delta T - \frac{p}{Y}}$$

, where Y = Young's modulus of the heated body.

The deflections of the rigid body in the direction of its length L_s and thickness h_s under the action of compressive forces F_x and F_z are derived as



$$d_z = \alpha_{th} h_s \Delta T - \frac{p}{Y} h_s$$

$$= \alpha_{th} h_s \Delta T - \left[\frac{F_z}{L_s b_s Y} - \frac{\nu_s F_x}{h_s b_s Y}\right] h_s$$

$$d_z = \alpha_{th} h_s \Delta T - \frac{h_s}{L_s b_s} \cdot \frac{F_z}{Y} + \frac{\nu_s}{\;} F_x/Y$$

$$d_x = \alpha_{th} L_s \Delta T - \frac{L_s}{h_s b_s}\frac{F_x}{EY} + \frac{\nu_s}{b_s}\frac{F_z}{Y}$$

Now the force can be calculated from the above eqn. For instance if deflection and force are acting in z-direction only, then

$$F_z = ?$$

$$\alpha_{th} L_s \Delta T = \frac{\nu_s}{b_s}\frac{F_z}{Y}$$

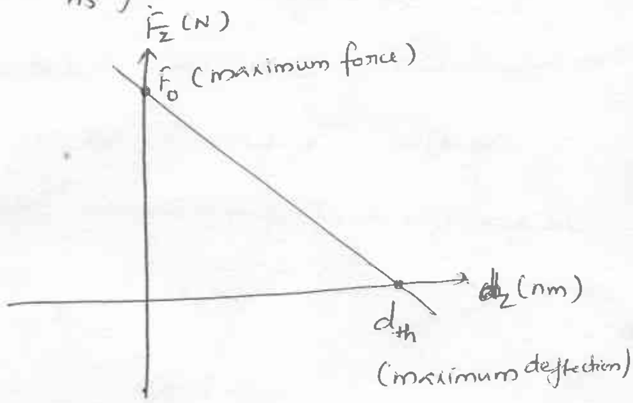
$$d_z = \alpha_{th} h_s \Delta T - \frac{h_s}{L_s b_s}\frac{F_z}{Y}$$

$$= \frac{L_s b_s Y \alpha_{th} h_s \Delta T - h_s F_z}{L_s b_s Y}$$

$$L_s b_s Y d_z = L_s b_s Y \alpha_{th} h_s \Delta T - h_s F_z$$

$$L_s b_s Y \frac{d_z}{h_s} = L_s b_s Y \alpha_{th} \Delta T - F_z$$

$$F_z = L_s b_s Y \left(\alpha_{th} \Delta T - \frac{d_z}{h_s}\right)$$



Electrostatic forces:

~~→ Simplest approach - energy method~~

Parallel plate capacitors

A capacitor consists of two electrodes mounted at a certain distance. When the electrodes are charged, they are attracted by electrostatic Coulomb forces. The potential energy U stored in the capacitor is

$$U = \tfrac{1}{2} C V^2 ,$$ where C is the capacitance and V the voltage

The Capacitance C for a parallel plate capacitor is given by

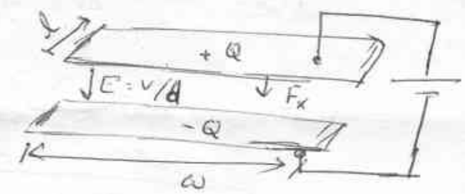
$$C = \frac{\varepsilon_0 \varepsilon_r A}{d},$$ where ε_0 - permittivity of free space

ε_r - relative permittivity of the material

A = Area of the electrode

d = the distance between the electrodes.

Hence $U = \frac{1}{2} \cdot \varepsilon_0 \varepsilon_r \cdot \frac{A}{d} \cdot V^2$



From the potential energy, the capacitive force can be calculated as the derivative of energy. The capacitive forces transform voltage directly into a movement and they are insensitive to temperature changes.

$$F_{cap} = \frac{\partial U}{\partial x} = -\frac{1}{2} \cdot \varepsilon_0 \varepsilon_r \cdot \frac{A}{d^2} \cdot V^2 .$$

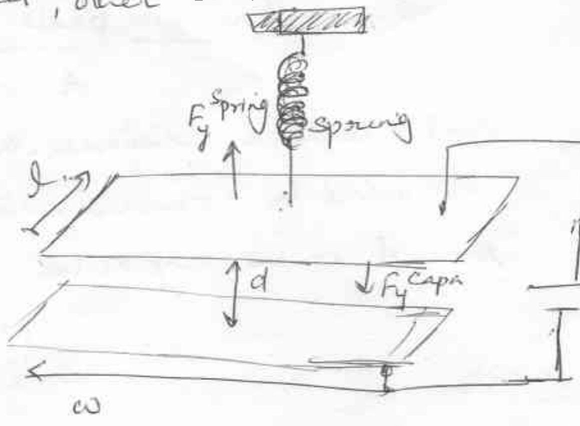
i.e. force $F \propto \frac{1}{d^2}$, large force when 'd' is smaller.

$$|F_{cap}| = \frac{1}{2}\left(\varepsilon_0 \varepsilon_r \frac{A}{d^2}\right) V^2 .$$

Electrostatic - Mechanical force balance:

→ Consider a parallel plate capacitor

→ One plate (electrode) is fixed, other connected to a spring.



~~Now~~ When the forces due to the spring & the capacitive force are in balance

$$F_{cap} = F_{spring}$$

$$\frac{1}{2}\frac{\varepsilon_0 \varepsilon_r A}{(d-y)^2} V^2 = k \cdot y \qquad A = w \cdot l$$

$$y = \frac{\varepsilon_0 \varepsilon_r A}{(d-y)^2 \cdot 2k} V^2 = \frac{\varepsilon_0 \varepsilon_r \cdot w \cdot V^2}{2 \cdot k} \cdot \frac{L}{(d-y)^2}$$

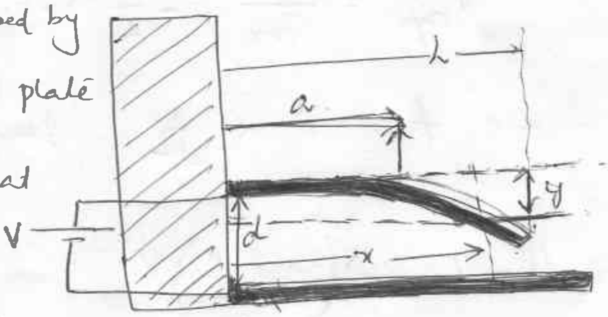
Now let $\Delta = y/d$, normalized displacement.

$$\Delta = \frac{\varepsilon_0 \cdot \varepsilon_r \cdot w \cdot V^2}{2 \cdot k} \frac{L}{d^2} \cdot \frac{1}{(1-\Delta)^2}$$

$$\boxed{\Delta \cdot (1-\Delta)^2 = \frac{\varepsilon_0 \cdot \varepsilon_r \cdot w \cdot V^2}{2 \cdot k} \frac{L}{d^2}}$$

Application: Electrostatic displacement of cantilever.

→ Capacitor configuration formed by cantilever beam and fixed plate



→ Cantilever beam supported at one end only

→ a force F applied at a point distance 'a' from the support, which results in the displacement of 'y' at position 'x'.

For a beam submitted to a pure transverse bending moment M the deflection y is given by

$$\frac{d^2y}{dx^2} = \frac{M}{YI}$$, where Y: Young's modulus for the material of the beam

I: ~~Seal~~ Moment of inertia for the beam cross-section

$= \frac{1}{12} \cdot \omega \cdot t^3$, t: thickness of beam, ω: width of beam

For a cantilever subjected to a point load normal to the surface, the moment is

$$M = F(L - x)$$

$$\frac{d^2y}{dx^2} = \frac{F(L-x)}{YI}$$

Integrate twice with respect to x, we get

$$\cancel{y(x) = \frac{F}{6EI}\begin{cases} x^2(3a-x), & x < a \\ a^2(3x-a), & x > a \end{cases}}$$

$$y = -\frac{F}{YI}\left(\frac{1}{2}Lx^2 - \frac{1}{6}x^3 + Ax + B\right)$$

~~wat~~ Applying the boundary conditions at $x = 0$

(1) deflection is null $y(0) = 0$, $\Rightarrow B = 0$

(2). slope is null $\left.\frac{dy}{dz}\right|_{z=0} = 0 \Rightarrow A = 0$.

$$\Rightarrow \quad y(x) = -\frac{F}{YI}\left(\frac{L}{2}x^2 - \frac{1}{6}x^3\right)$$

$$= -\frac{Fx^2}{6YI}(3a - x) \quad \text{for } x < a$$

$$= -\frac{F \cdot a^2}{6YI}(3x - a) \quad \text{for } x > a$$

$$y(x) = \frac{F}{6YI}\begin{cases} x^2(3a-x) & \text{for } x < a \\ a^2(3x-a) & \text{for } x > a \end{cases}$$

→ tip deflection $(x = L)$ due to δF at position $x = a$

$$\delta y_{tip} = \frac{\delta F}{6 Y \cdot I} a^2 (3 \cdot L - a)$$

→ total tip deflection $y_{tip} = \int_0^L \delta y_{tip}$

$$= \int_0^L \frac{dF}{6YI} x^2 (3L - x)$$

Now Force $F = \frac{\varepsilon_0 \varepsilon_r}{2} \frac{V^2}{d^2} \cdot A$. for $d \to$ the separation between plates

$$\delta F = \frac{\varepsilon_0 \varepsilon_r}{2} \left[\frac{V}{d - y(x)} \right]^2 \underbrace{\omega \cdot dx}_{area}.$$

Now the displacement y at the point of force application x is

$$y(x) = \frac{dF}{6YI} x^2 (3x - x)$$

$$y(x) = \frac{dF}{3YI} x^3$$

$$\Rightarrow dF = \frac{\varepsilon_0 \varepsilon_r}{2} \left[\frac{V}{d - \frac{dF}{3YI} \cdot x^3} \right]^2 \omega \cdot dx.$$

$$\Rightarrow y_{tip} = \int_0^L \left\{ \frac{dF}{6 \cdot Y \cdot I} x^2 (3L - x) \right\}$$

$y(x) = \left(\frac{x}{L}\right)^2 \cdot y_{tip}$

Now assume $y(x) = \left(\frac{x}{L}\right)^2 y_{tip}$ (parabolic bending of beam)

$$dF = \frac{\varepsilon_r \varepsilon_0}{2} \left[\frac{V}{d - (x/L)^2 y_{tip}} \right]^2 \cdot \omega \cdot dx$$

$$y_{tip} = \frac{\varepsilon_r \varepsilon_0}{2} \cdot \frac{\omega \cdot V^2}{6 \cdot Y \cdot I} \cdot \int_0^L \frac{3 \cdot L \cdot x^2 - x^3}{\left[d - (y_{tip}/L^2) \cdot x^2 \right]^2} \Big\} dx$$

Let $y_{tip} = Y$ inside the integral

$$Y_{tip} = \frac{\varepsilon_r \cdot \varepsilon_0 \cdot w \cdot V^2}{2 \cdot 6 \cdot Y \cdot I} \int_0^L \left\{ \frac{3 \cdot L \cdot x^2 - x^3}{\left[d - (Y/L^2) x^2 \right]^2} \right\} dx.$$

integrals.

$$- \int \left\{ \frac{x^2}{(a^2 - x^2)^2} \right\} dx = \frac{x}{2(a^2 - x^2)} - \frac{1}{4a} \cdot \ln \left| \frac{a+x}{a-x} \right|$$

$$- \int \left\{ \frac{x^3}{|a^2 - x^2|^2} \right\} dx = \frac{a^2}{2(a^2 - x^2)} + \frac{1}{2} \ln \left| a^2 - x^2 \right|$$

$$Y_{tip} = \frac{\varepsilon_r \varepsilon_0 \cdot w \cdot V^2 \cdot L^4}{2 \cdot 6 \cdot Y \cdot I \cdot (y_{tip})^2} \left[\frac{3L^2 - (d \cdot L^2 / y_{tip})}{2 \cdot \{ (d \cdot L^2 / y_{tip}) - L^2 \}} + \frac{1}{2} - \frac{3L}{4 \sqrt{d \cdot L^2 / y_{tip}}} \cdot \ln \left| \frac{\sqrt{d \cdot L^2 / y_{tip}} + L}{\sqrt{d \cdot L^2 / y_{tip}} - L} \right| + \frac{1}{2} \cdot \ln \left| \frac{d \cdot L^2 / y_{tip}}{d \cdot L^2 / y_{tip} - L^2} \right| \right]$$

Let $\Delta = \frac{Y_{tip}}{d}$ = normalized displacement.

$$\Rightarrow \Delta^3 = \frac{\varepsilon_r \varepsilon_0 w}{12 \cdot Y \cdot I} \cdot \frac{V^2 L^4}{d^3} \left[\frac{\Delta}{1 - \Delta} + \frac{1}{2} \ln \left| \frac{1}{1-\Delta} \right| - \frac{3}{4} \sqrt{\Delta} \ln \left| \frac{1 + \sqrt{\Delta}}{1 - \sqrt{\Delta}} \right| \right]$$

$$\Delta^3 \cdot \left[\frac{\Delta}{1-\Delta} + \frac{1}{2} \ln \left| \frac{1}{1-\Delta} \right| - \frac{3}{4} \sqrt{\Delta} \ln \left| \frac{1+\sqrt{\Delta}}{1-\sqrt{\Delta}} \right| \right]^{-1} = \frac{\varepsilon_r \varepsilon_0 w}{12 \cdot Y \cdot I} \cdot \frac{L^4}{d^3} \cdot V^2$$