Piezoelectric constitutive relations

Background on Tensors

→ Need for "geometrical objects" called Tensors.

Example

$$J = \sigma E \quad \text{— (i)}$$

J → scalar; σ → scalar; E → scalar

⇓ or,

$$\vec{J} = \sigma \vec{E} \quad \text{— (ii)}$$

$\vec{J}$ → vector; σ → second rank tensor; $\vec{E}$ → vector

Useful : Can help quantify material behavior in detail in situations arising in nature!

e.g., $\vec{E} = (0, 0, E_z)$ may cause $\vec{J} = (J_x, 0, 0)$, when

$$\sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & 0 \\ \sigma_{31} & \sigma_{32} & 0 \end{pmatrix}$$

Notation: $J_i = \sigma_{ij} E_j$. — (iii)

or, $J = \sigma E$ (context specific)

Piezoelectricity

$$X = d E$$

X → induced strain (second rank); d → Piezoelectric coefficient (third rank); E → Electric field (first rank)

$X_{ij} = d_{ijk} E_k$.

More appropriately, $X_{jk} = d_{ijk} E_i$. — (iv)

The d tensor can be conceived of comprising three layers of symmetrical matrices.

1st layer $(i=1)$ $\begin{pmatrix} d_{111} & d_{112} & d_{113} \\ d_{121} & d_{122} & d_{123} \\ d_{131} & d_{132} & d_{133} \end{pmatrix}$

2nd layer $(i=2)$ $\begin{pmatrix} d_{211} & d_{212} & d_{213} \\ d_{221} & d_{222} & d_{223} \\ d_{231} & d_{232} & d_{233} \end{pmatrix}$ — (v)

3rd layer $(i=3)$ $\begin{pmatrix} d_{311} & d_{312} & d_{313} \\ d_{321} & d_{322} & d_{323} \\ d_{331} & d_{332} & d_{333} \end{pmatrix}$

27 components!

In general

$$x_{kl} = d_{ikl} E_i + M_{ijkl} E_i E_j$$

&

$$x_{kl} = g_{ikl} P_i + Q_{ijkl} P_i P_j$$

— (vi)

where, E_i : electric field

P_i : polarization

d_{ikl} & g_{ikl} : piezoelectric coefficient matrices.

M_{ijkl} & Q_{ijkl} : electrostrictive coefficient matrices.

81 components!