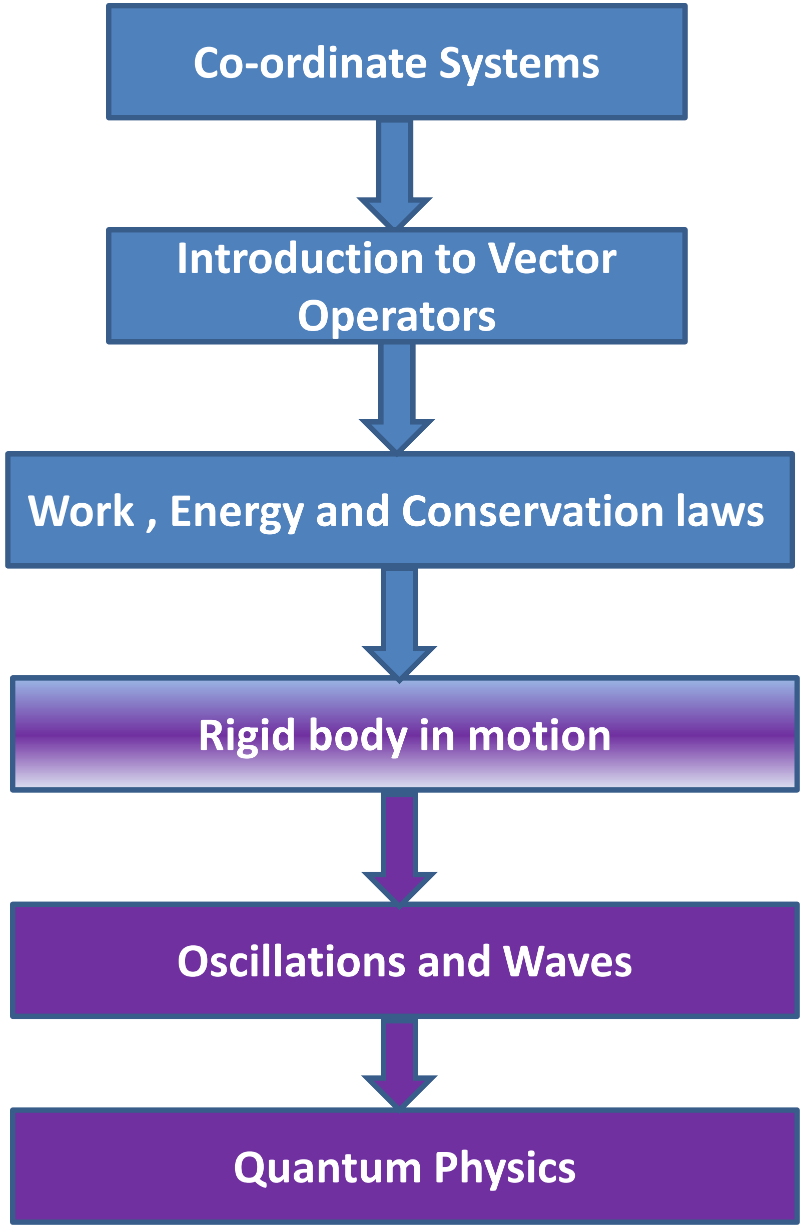
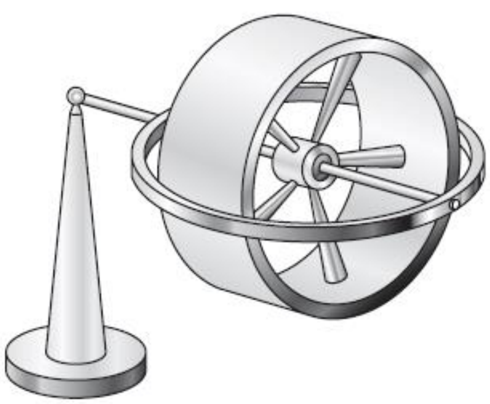
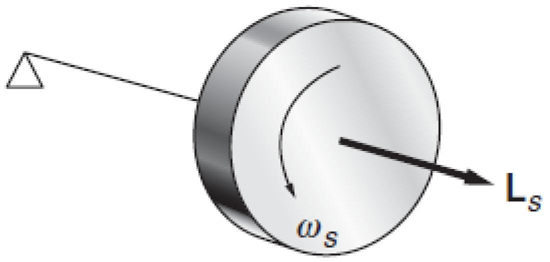
Highlights of the course



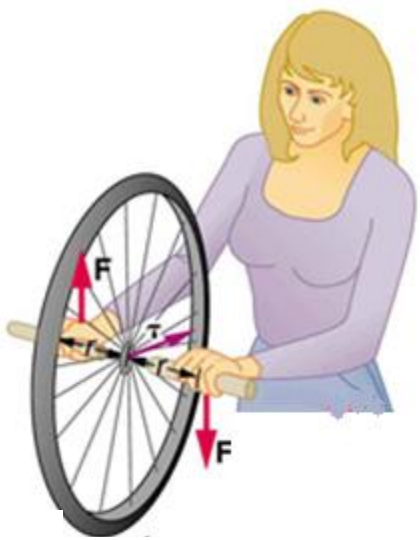
Gyroscope

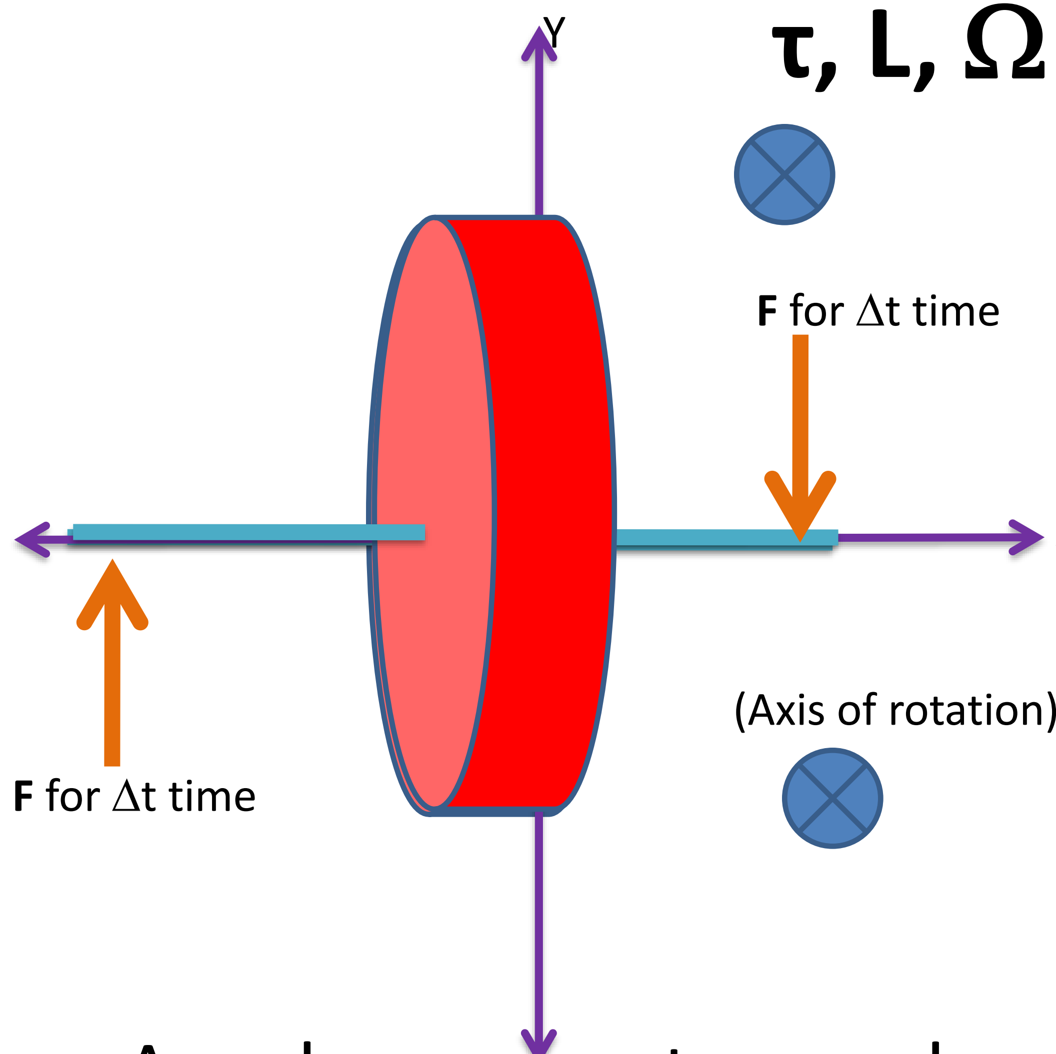




Thought Experiment!

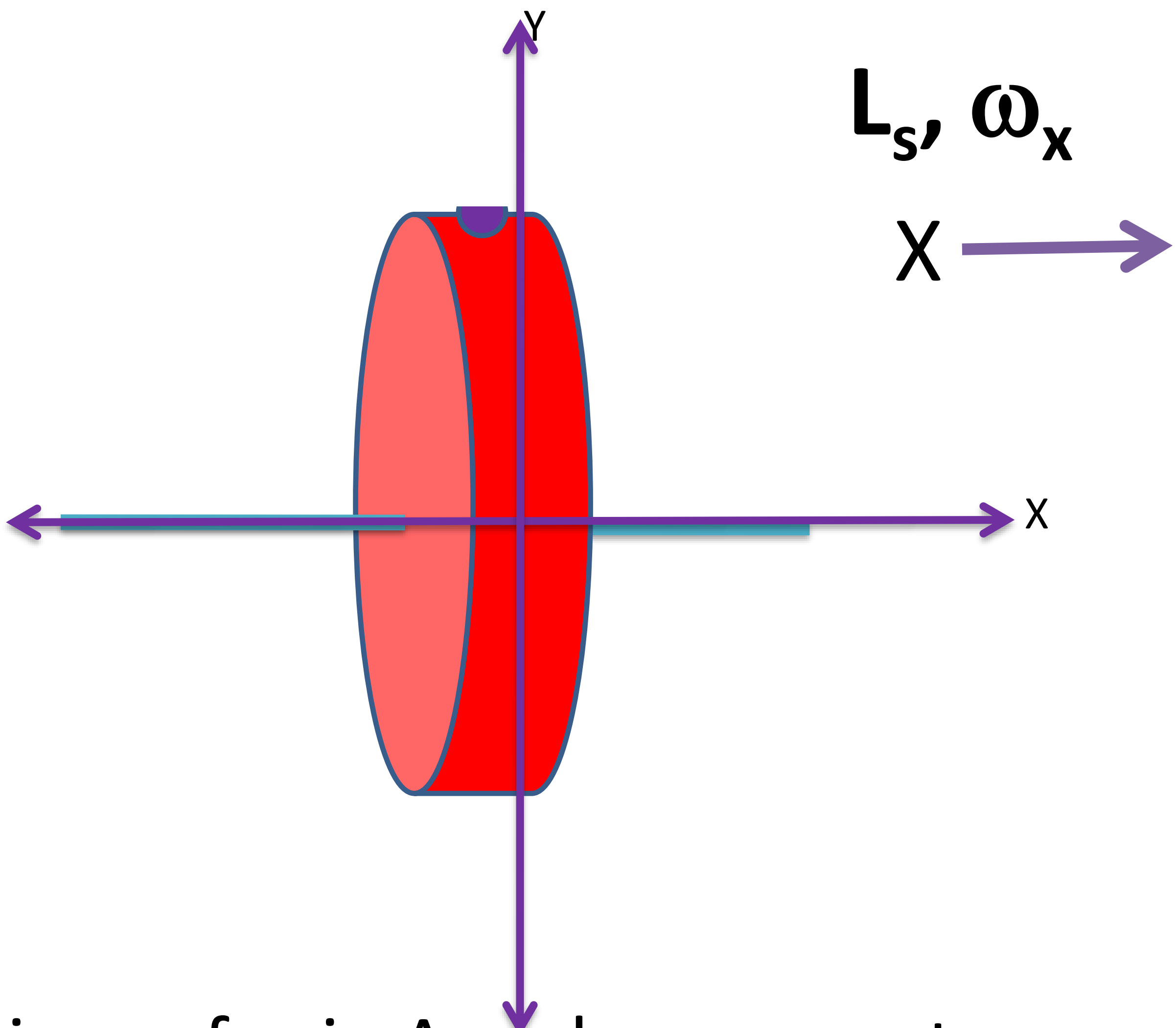
Disk rotated by twist in outer space





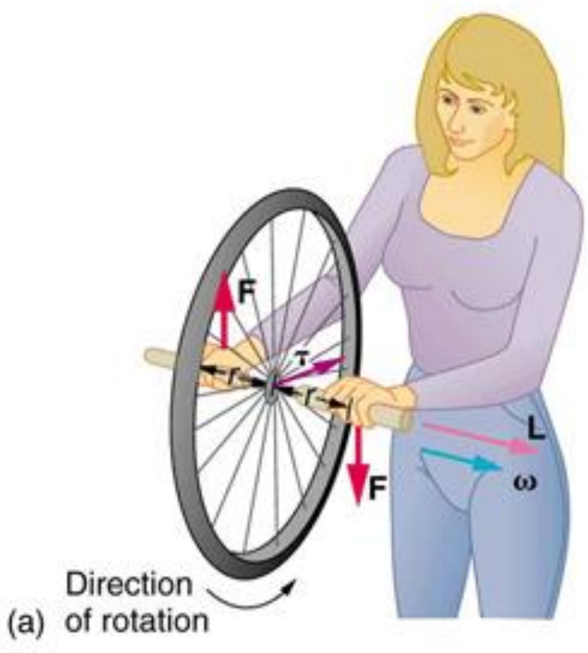
Directions of Torque, Angular momentum and Angular velocity?

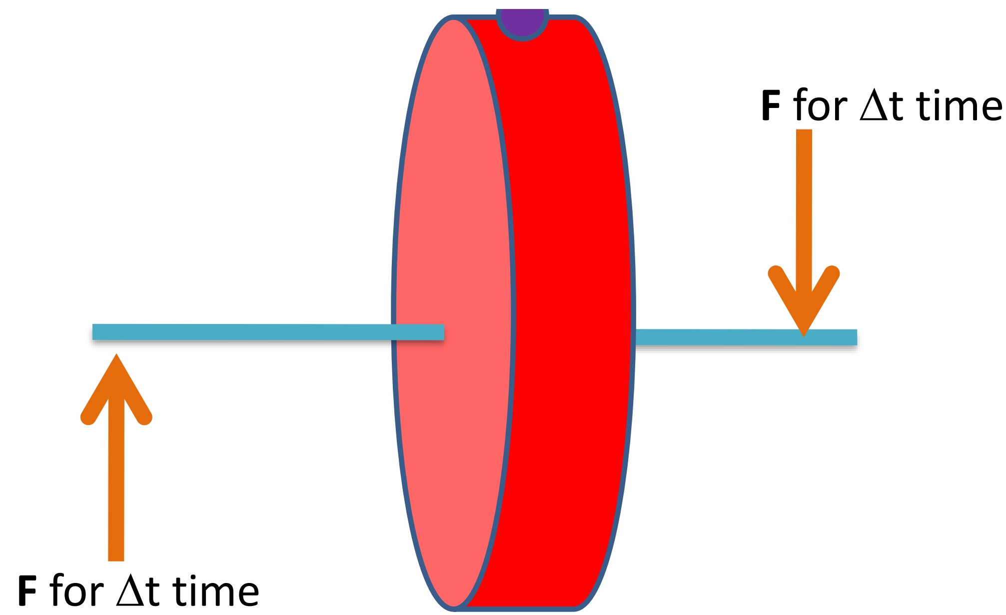
Disk spinning in outer space



Directions of spin Angular momentum and Angular velocity?

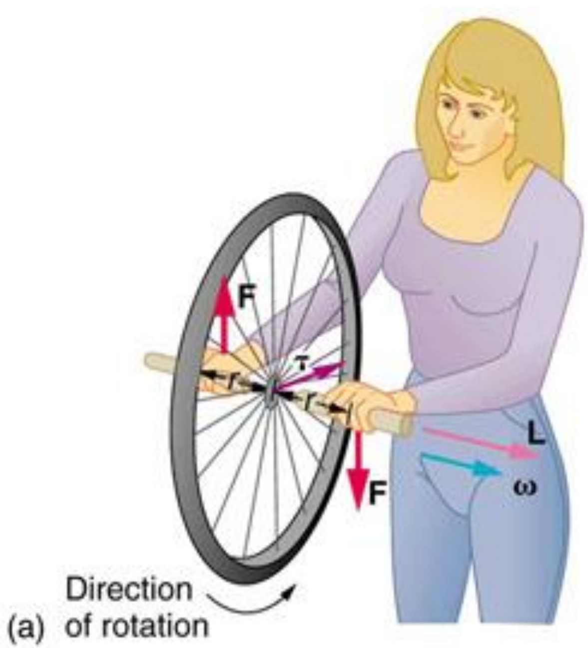
Disk (spin + twist) in outer space?

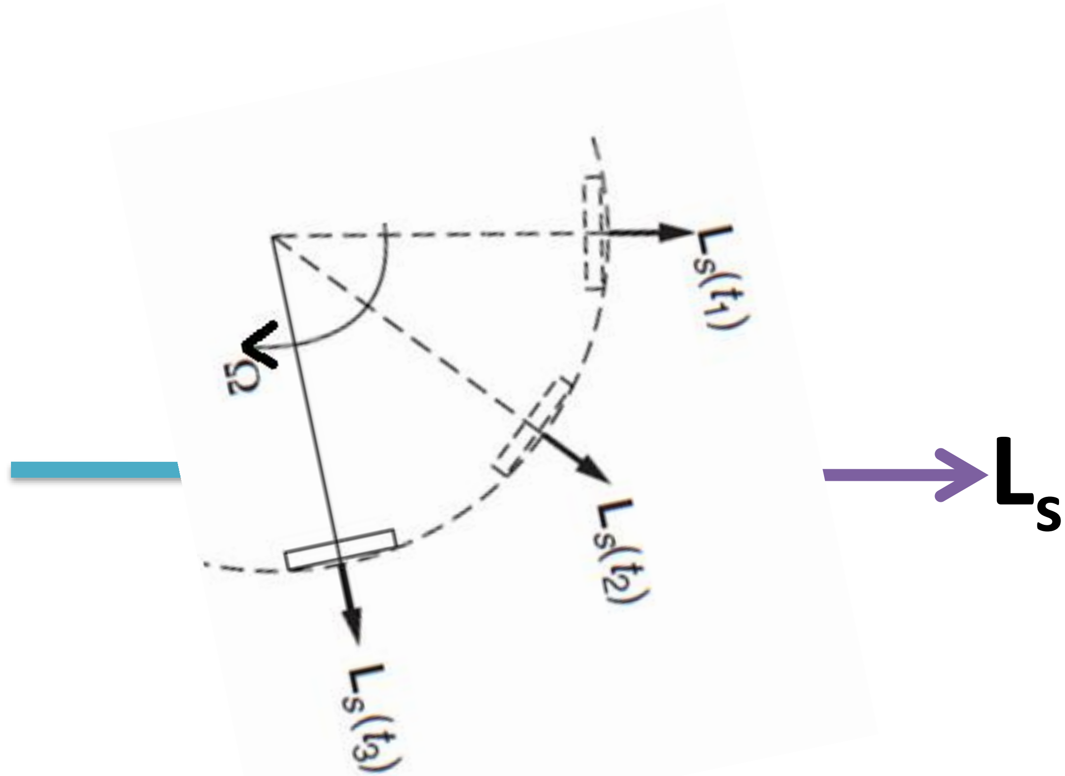




How will be the motion of Disk?
Spin and Rotate?

Disk (spin + twist) in outer space?



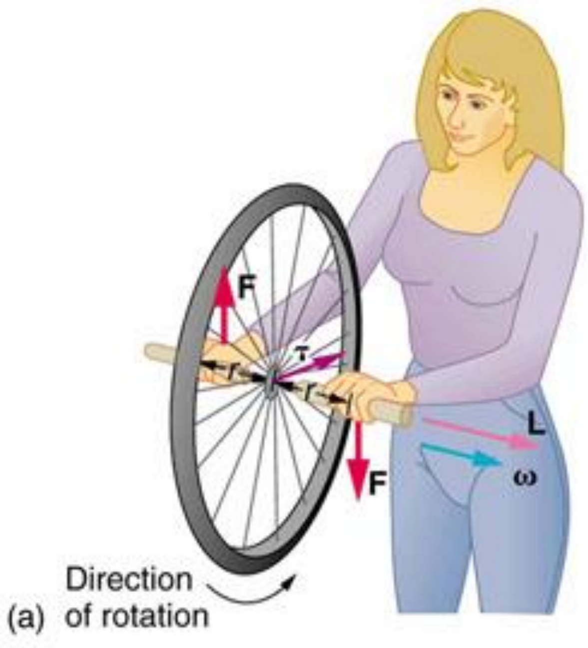


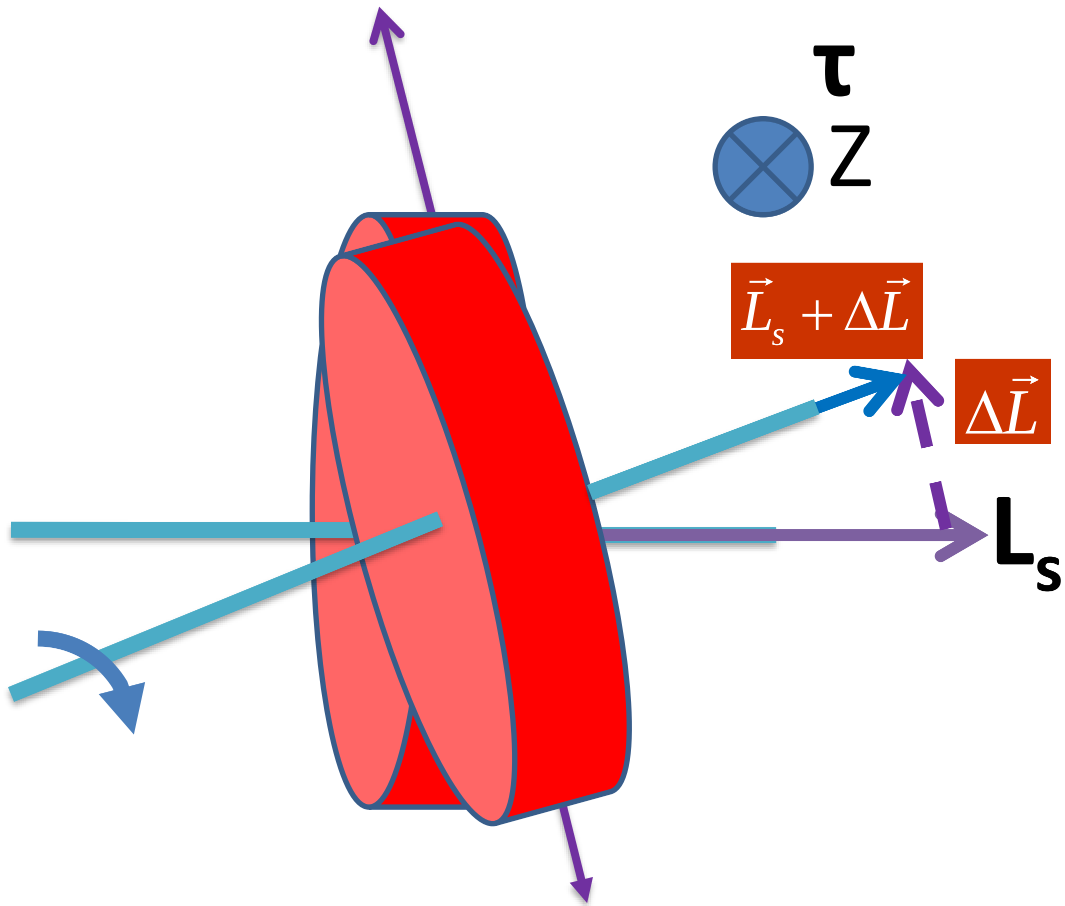
For the disk to spin and rotate, there must be an external torque (dL_s/dt) always, which will change the spin angular momentum.

Spin and rotation will not happen simultaneously!

Counter Intuitive!

Disk (spin + twist) in outer space?



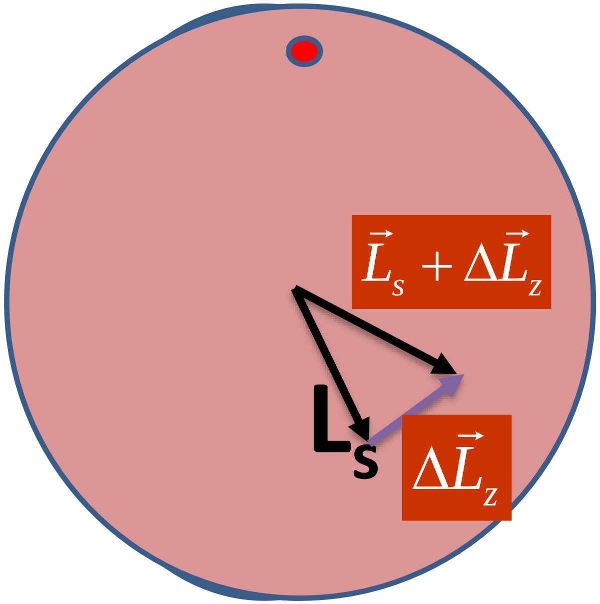


$$\Delta\vec{L} = \vec{\tau}\Delta t$$

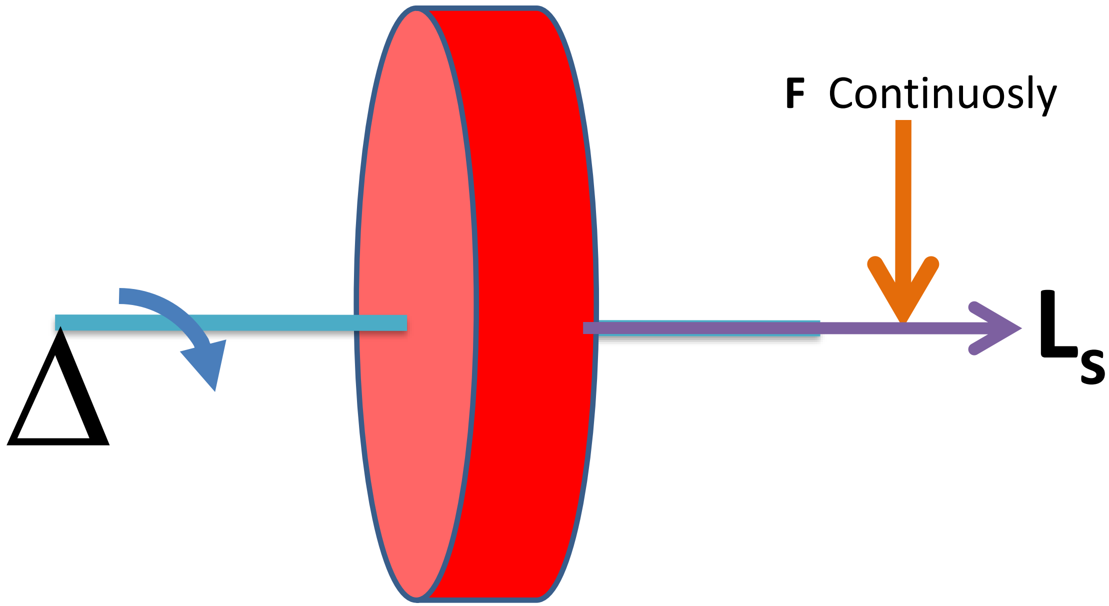
Disk orients in the direction of torque, whereas the force was applied in the y direction!

Counter Intuitive!

View along X-axis side

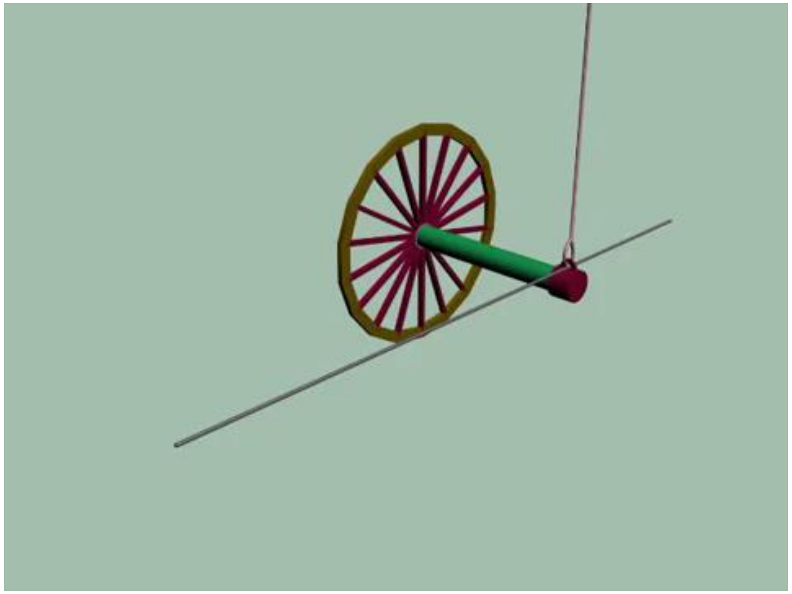


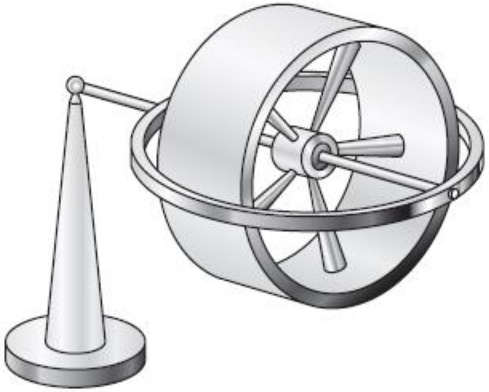
Disk (spin + twist) when continuous torque applied externally?

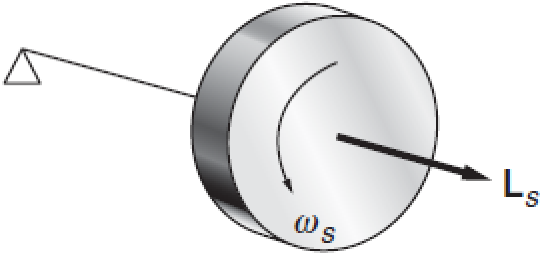


How will be the motion of Disk?
Spin and Rotate?

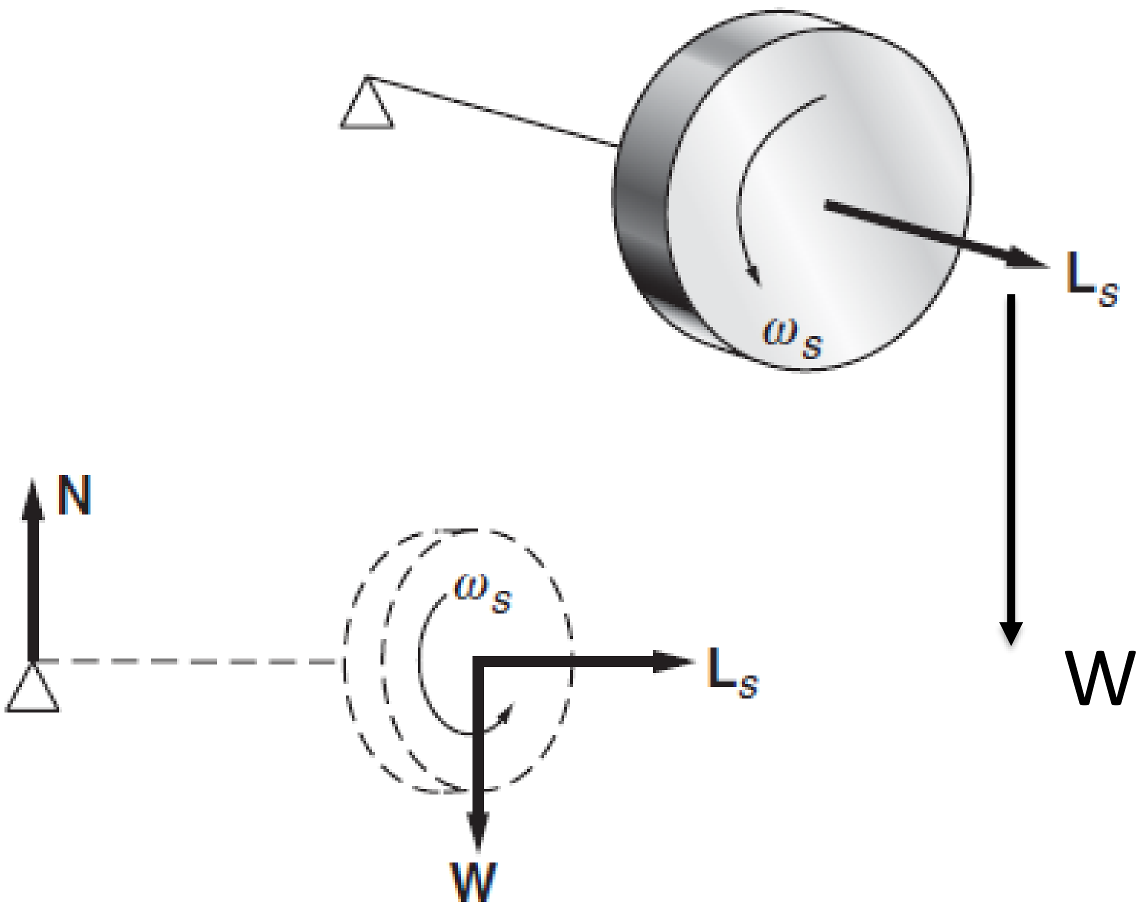
Gyroscope





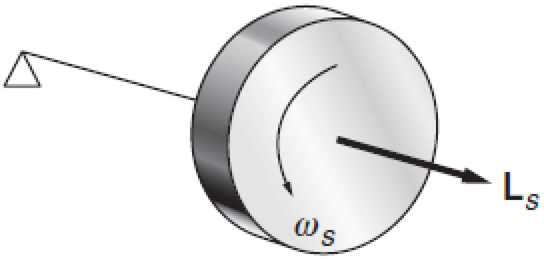


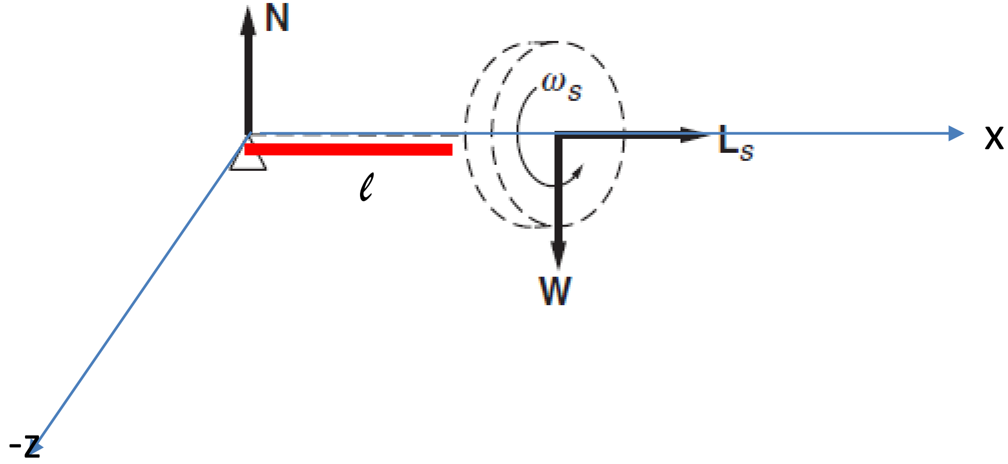
Gyroscope Precision



Gyroscope Precision

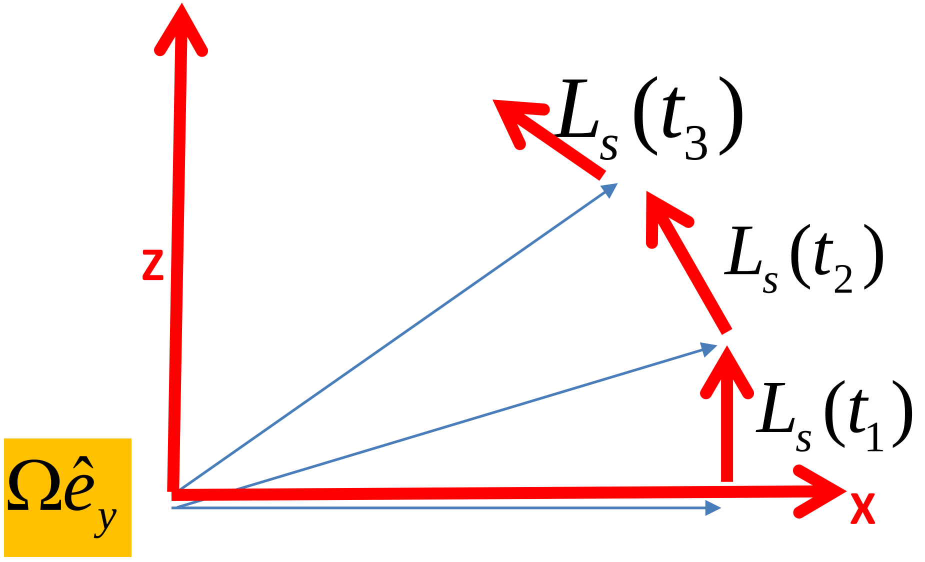
$$\tau = lW$$



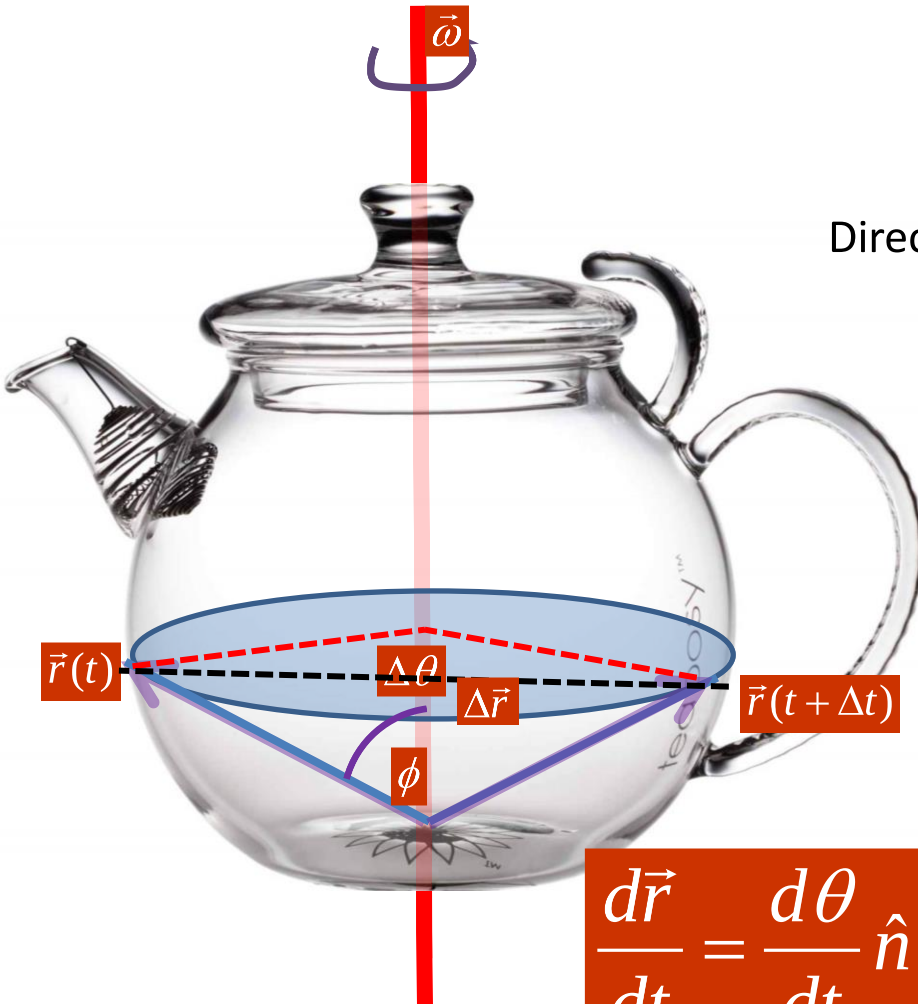


TOP VIEW

$$\left|\frac{dL_s}{dt}\right| = ?$$

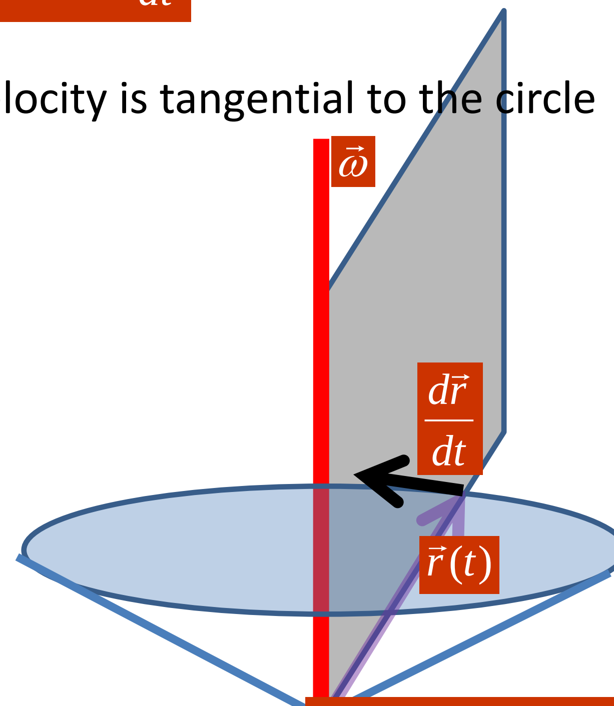


Vector nature **of angular velocity** and **angular momentum**



$$\left|\frac{dr}{dt}\right| = r\sin\phi\ \frac{d\theta}{dt}$$

Direction of velocity is tangential to the circle

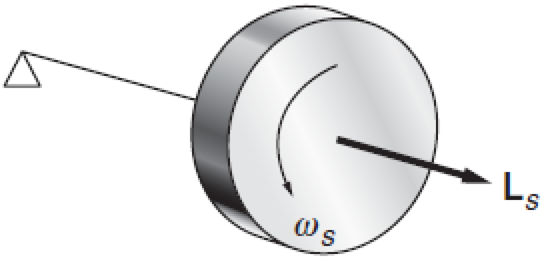


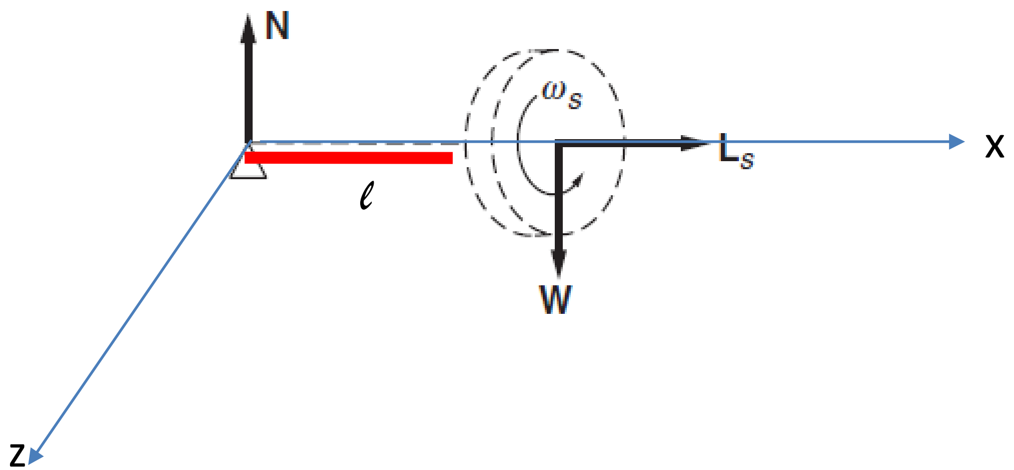
$$\frac{d\vec{r}}{dt} = \frac{d\theta}{dt}\hat{n}\times\vec{r} = \vec{\omega}\times\vec{r}$$

$$\frac{d\vec{A}}{dt} = \vec{\omega}\times\vec{A}$$

Gyroscope Precision

$$\vec{\tau} = l\hat{e}_x \times W(-\hat{e}_y)$$



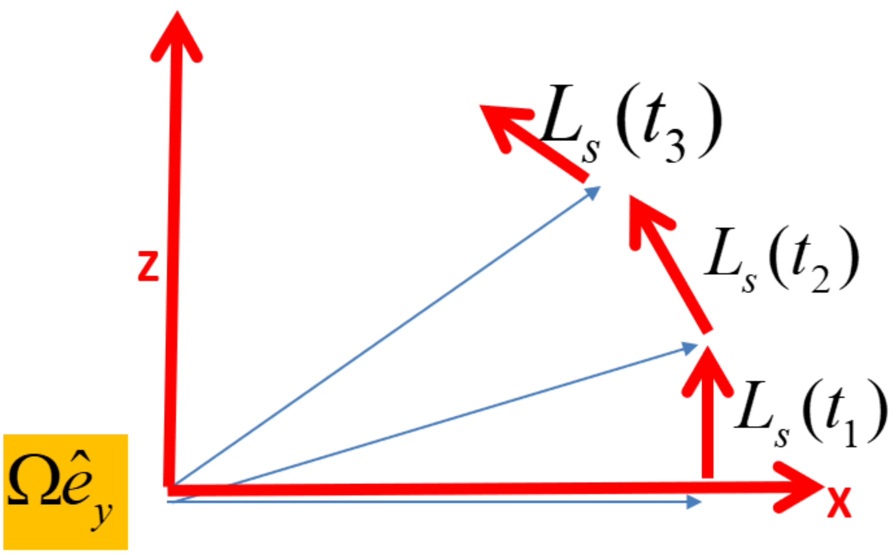


$$\frac{d\vec{A}}{dt} = \vec{\Omega} \times \vec{A}$$

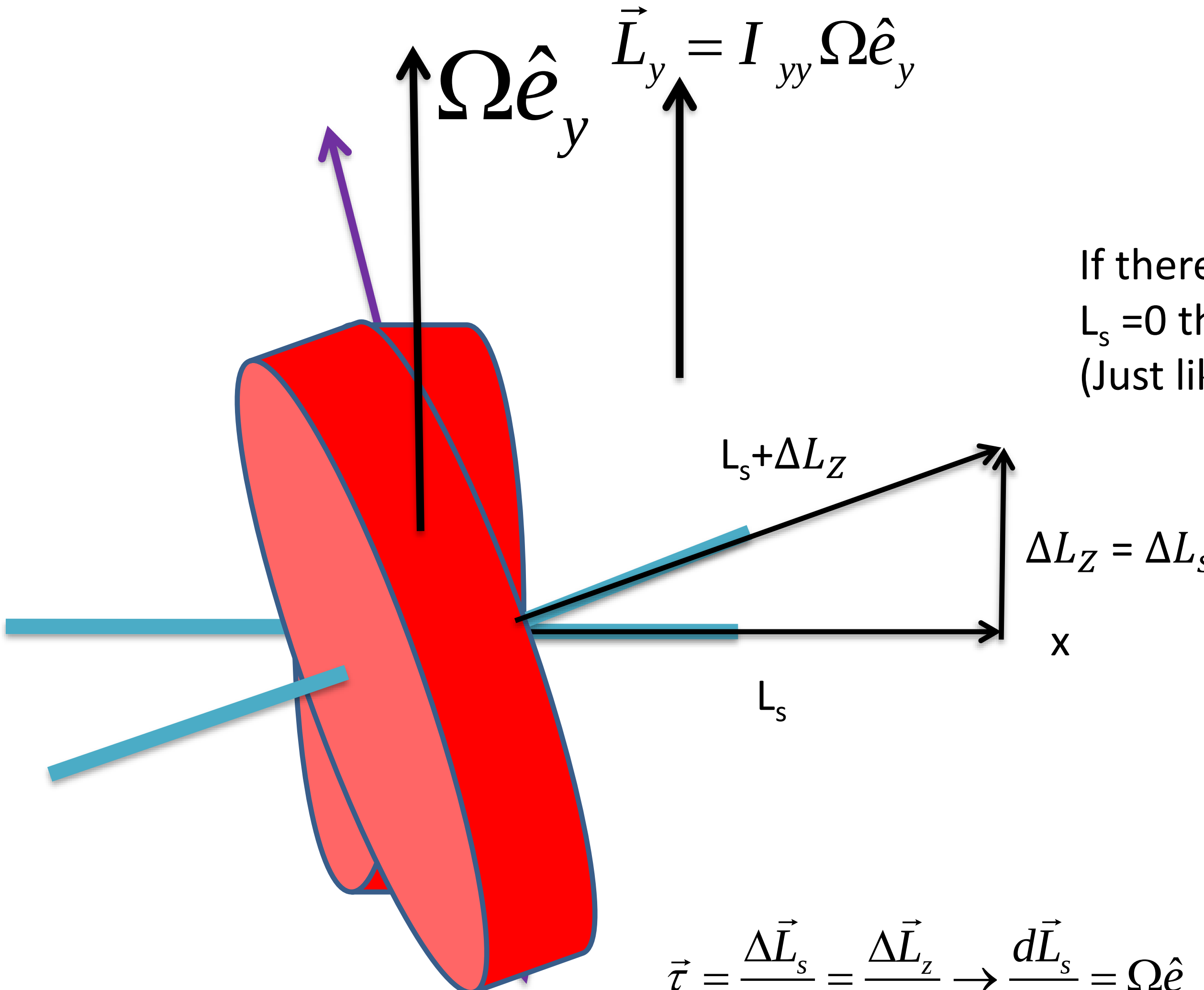
$$\frac{d\vec{L}_s}{dt} = \Omega\hat{e}_y \times L_s\hat{e}_x$$

$$\left|\frac{dL_s}{dt}\right| = \Omega\ L_s$$

TOP VIEW



The torque on the gyroscope must account for the change in L_s



If there were no spin
L_s =0 then $\Delta L_Z = \Delta L_s$=0
(Just like first case)

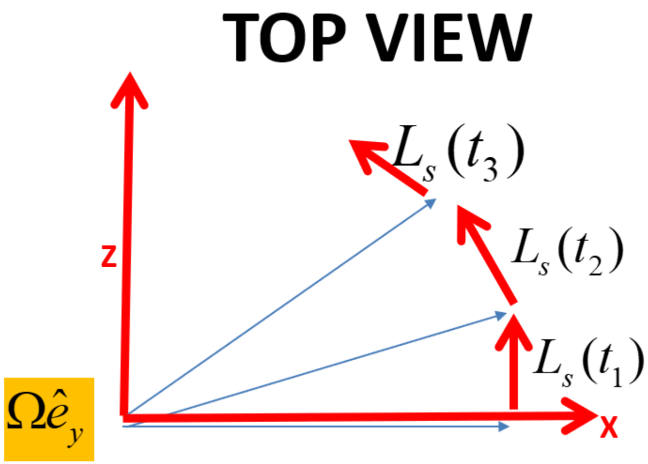
$$\vec{\tau} = \frac{\Delta\vec{L}_s}{\Delta t} = \frac{\Delta\vec{L}_z}{\Delta t} \rightarrow \frac{d\vec{L}_s}{dt} = \Omega\hat{e}_y \times L_s\hat{e}_x = \Omega L_s(-\hat{e}_z)$$

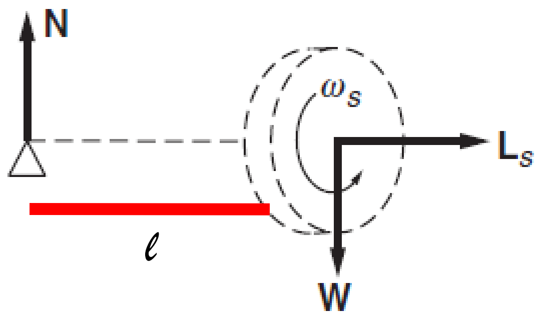
also

$$\vec{\tau} = l\hat{e}_x \times W(-\hat{e}_y) = lW(-\hat{e}_z)$$

$$\therefore lW = \Omega L_s$$

Gyroscope Precision



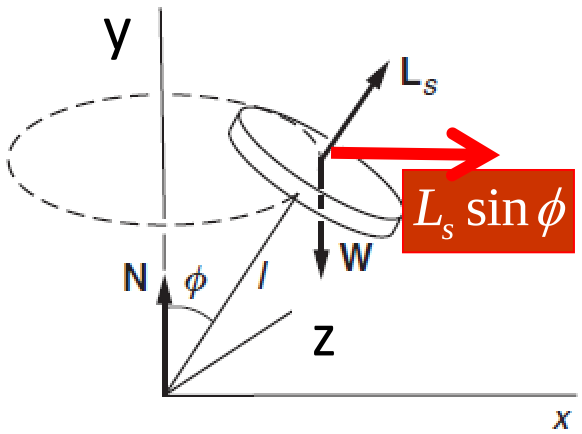


$$lW = \Omega L_s$$

$$lW = \Omega I_0 \omega_s$$

$$\Omega = \frac{lW}{I_0 \omega_s}$$

Gyroscope Precision



Consider a gyroscope in uniform precession with its axle at angle ϕ with vertical

The horizontal component of angular momentum is $L_s \sin\phi$

$$\left|\frac{dL_s}{dt}\right| = \Omega L_s \sin\phi$$

$$lW \sin\phi = \Omega L_s \sin\phi$$

$$\Omega = \frac{lW}{I_0 \omega_s}$$

The precessional velocity is independent of ϕ