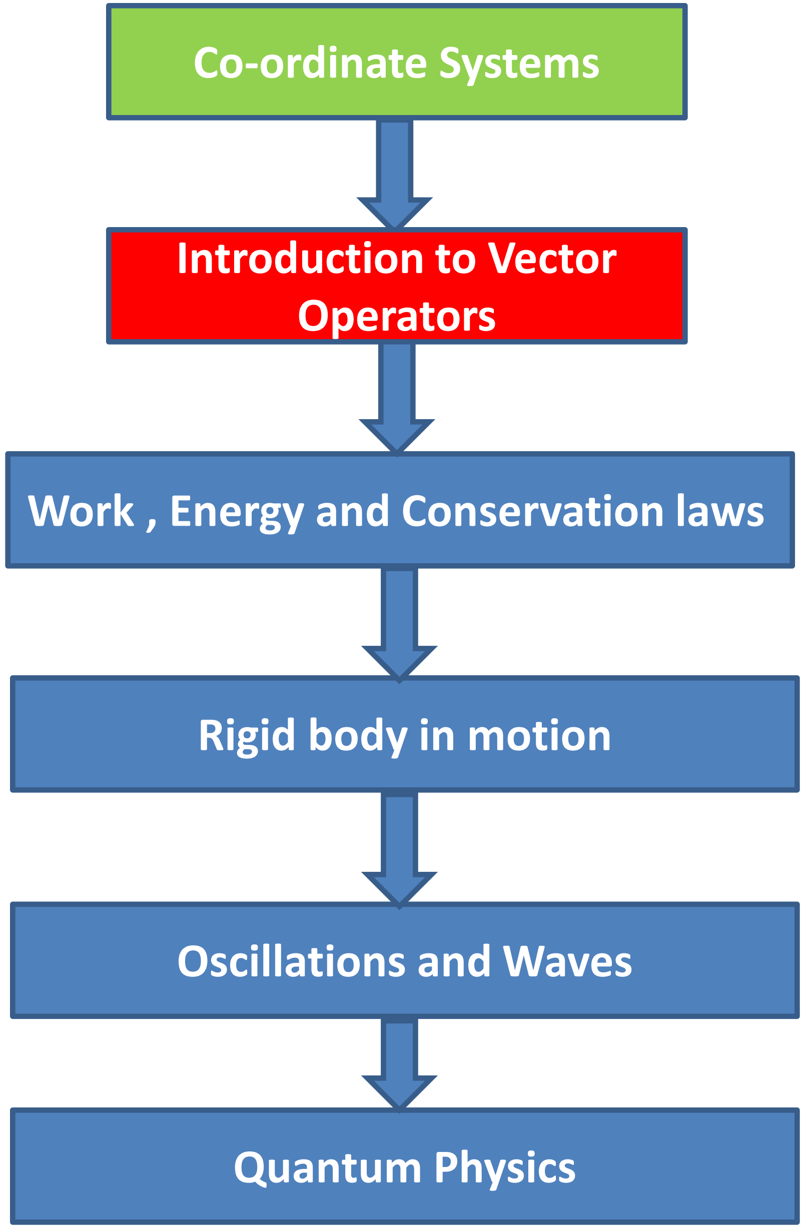
Highlights of the course



Chapter-2: Introduction to Vector Operators

Gradient, Divergence and Curl

Maxwell's Equations in Free Space

(in the absence of matter, $\rho = 0$, $\mathbf{J} = 0$, $\mathbf{P} = 0$, $\mathbf{M} = 0$)

Equations	Differential/Point Form
Gauss' Law for Electricity	$\nabla \cdot \mathbf{E} = 0$
Gauss' Law for Magnetism	$\nabla \cdot \mathbf{H} = 0$
Ampere's Law	$\nabla \times \mathbf{H} = \varepsilon_0 \dfrac{\partial \mathbf{E}}{\partial t}$
Faraday's Law of Induction	$\nabla \times \mathbf{E} = -\mu_0 \dfrac{\partial \mathbf{H}}{\partial t}$

Navier-Stokes Equations

Continuity Equation

$$\nabla \cdot \vec{V} = 0$$

Momentum Equations

$$\rho \frac{D\vec{V}}{Dt} = -\nabla p + \rho \vec{g} + \mu \nabla^2 \vec{V}$$

Total derivative =

$$\rho[\frac{\partial V}{\partial t} + (V.\nabla)V]$$

Change of velocity with time

Convective term

Pressure gradient: Fluid flows in the direction of largest change in pressure.

Body force term: External forces, that act on the fluid (gravitational force or electromegnetic).

Diffusion term: For a Newtonian fluid, viscosity operates as a diffusion of momentum.

The Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\Psi = i\hbar\frac{\partial \Psi}{\partial t}$$

The operator $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$ is the Laplacian in Cartesian coordinates.

Ψ is the wavefunction.

V is the potential.

$\hbar$ is the Planck constant divided by 2π.

The particle mass is represented by m.

$$\nabla \times \nabla \times \mathbf{E} = -\nabla^2 \mathbf{E} =$$

$$= -\mu \frac{\partial}{\partial t}(\nabla \times \mathbf{H}) \quad \text{(substitute in Ampere's Law)}$$

$$= -\mu \frac{\partial}{\partial t}\left(\frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}\right) \quad \text{(J is zero because source free region)}$$

$$= -\mu\varepsilon \frac{\partial}{\partial t}\left(\frac{\partial \mathbf{E}}{\partial t}\right)$$

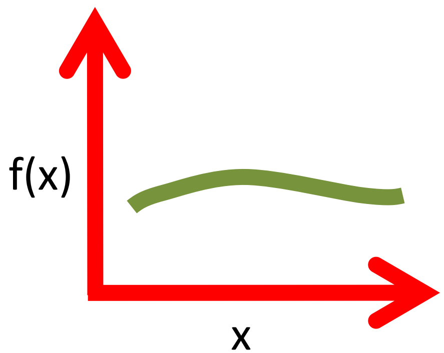
www.maxwells-equations.com

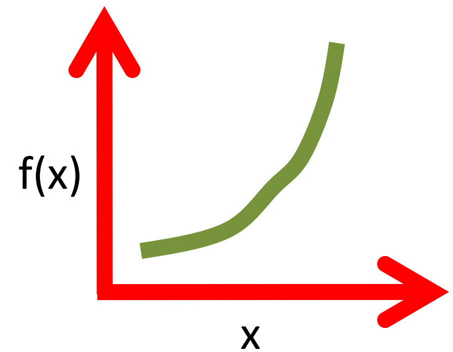
$$\Rightarrow \boxed{\nabla^2 \mathbf{E} = \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}} \quad \text{[The Vector Wave Equation]}$$

Many many more to learn in Science and Engineering ....

Ordinary Derivatives

Ordinary Derivatives





?

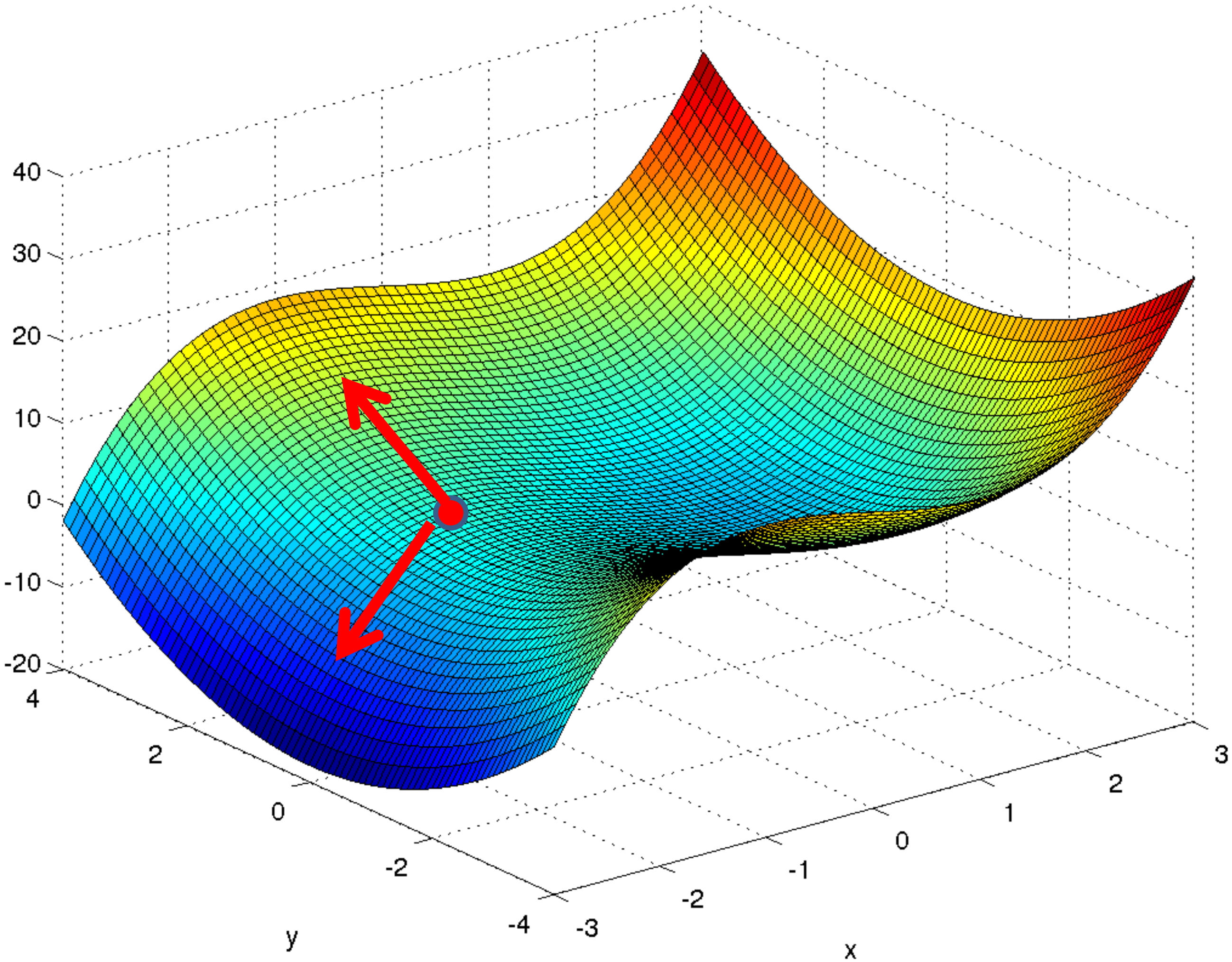
$$d\vec{r} = dr\hat{e}_r + r\,d\hat{e}_r$$

$$df = \left(\frac{df(x)}{dx}\right)dx$$

RECOLLECT (By Taylor Series) for Plane polar

$$d\hat{e}_r(\theta) = \frac{d\hat{e}_r(\theta)}{d\theta}\,d\theta$$

Derivative of a function in 3D



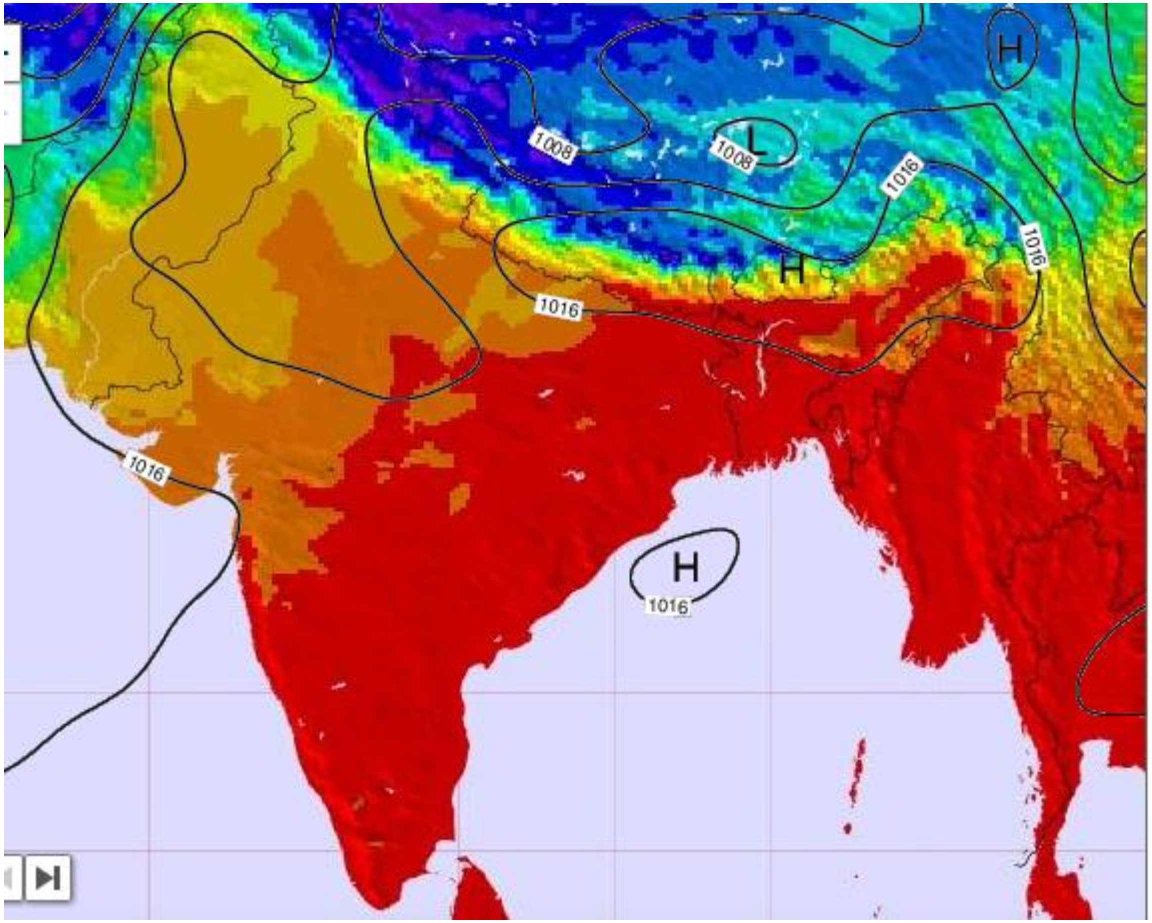
Derivative is directional!

Scalar Field Function

A function of space whose value at each point is a scalar quantity

Example: Temperature

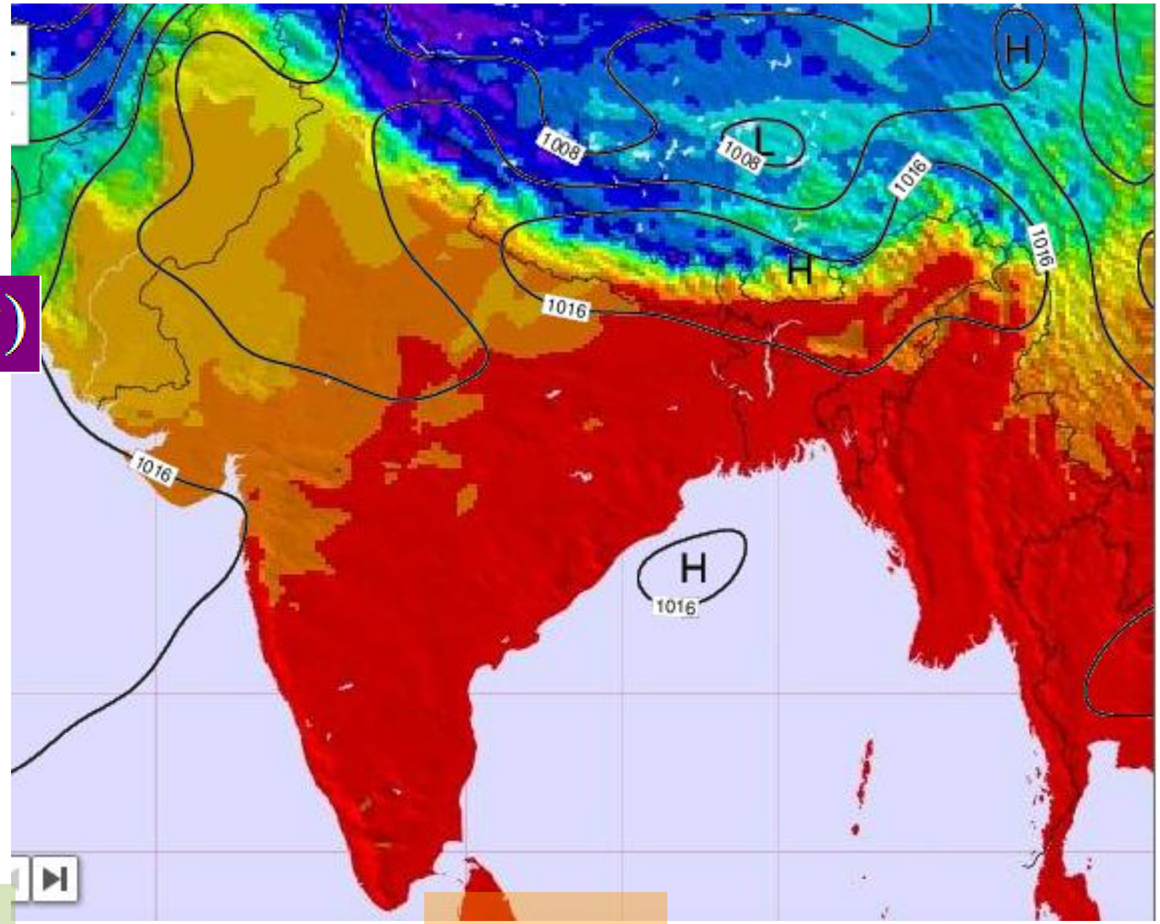
$$T(x, y, z)$$



Gradient of a scalar function

$$T(x, y, z)$$

$$T(x+dx, y+dy, z+dz)$$

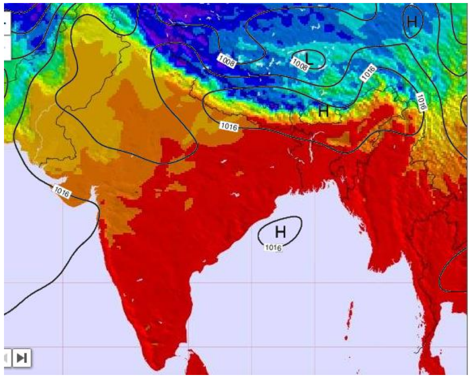


$$\frac{\partial T(x, y, z)}{\partial x}$$

Partial derivative

Example $\hat{e}_r(\theta, \phi)$ **in spherical polar**

Gradient of a function



$$df = \left(\frac{df(x)}{dx} \right) dx$$

$$dT = \frac{\partial T}{\partial x} dx + \frac{\partial T}{\partial y} dy + \frac{\partial T}{\partial z} dz$$

$$T(x, y, z)$$

$$dT = \left(\frac{\partial T}{\partial x} \hat{e}_x + \frac{\partial T}{\partial y} \hat{e}_y + \frac{\partial T}{\partial z} \hat{e}_z \right) \bullet \left(dx\hat{e}_x + dy\hat{e}_y + dz\hat{e}_z \right)$$

$$dT = \vec{\nabla} T \bullet d\vec{l}$$

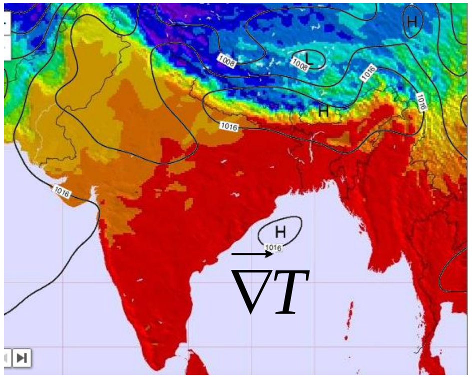
$$\vec{\nabla} T = \left(\frac{\partial T}{\partial x} \hat{e}_x + \frac{\partial T}{\partial y} \hat{e}_y + \frac{\partial T}{\partial z} \hat{e}_z \right)$$

Gradient

Gradient: Geometrical interpretation

$$dT = \vec{\nabla} T \bullet \vec{dl}$$

$$dT = \left|\vec{\nabla} T\right| \left|\vec{dl}\right| \cos\theta$$



$T(x, y, z)$

dT is maximum when gradient $\vec{\nabla} T$ points in the direction of $\vec{dl}$.

Maximum increase in the temperature occurs when gradient and the $\vec{dl}$ are parallel to each other

An example

Variation of temperature in a region is given by the function $T(x,y) = x^2 + y^2$
Find whether the unit vector $\hat{l} = \frac{\hat{e}_x + \hat{e}_y}{\sqrt{2}}$ points in the direction of maximum

increase of temperature at location (1,1).

$$\vec{\nabla} T(x,y) = \hat{e}_x \frac{\partial T}{\partial x} + \hat{e}_y \frac{\partial T}{\partial y} = 2x\hat{e}_x + 2y\hat{e}_y$$

$$\text{Given } \hat{l} = \frac{\hat{e}_x + \hat{e}_y}{\sqrt{2}}$$

$$\cos\theta = \frac{\vec{\nabla} T(x,y).\hat{l}}{|\vec{\nabla} T(x,y)||\hat{l}|} = \frac{2x + 2y}{\sqrt{4x^2 + 4y^2} \times \sqrt{2}}$$

At location(1,1)

$$\cos\theta = \frac{4}{\sqrt{16}} = 1$$

Yes the given unit vector
Points in the increasing
Direction of temperature

Another example

Find the gradient of $r = \sqrt{x^2 + y^2 + z^2}$

$$\vec{\nabla} f = \left(\frac{\partial f}{\partial x} \hat{e}_x + \frac{\partial f}{\partial y} \hat{e}_y + \frac{\partial f}{\partial z} \hat{e}_z \right) = \frac{\partial r}{\partial x} \hat{e}_x + \frac{\partial r}{\partial y} \hat{e}_y + \frac{\partial r}{\partial z} \hat{e}_z$$

$$= \frac{1}{2} \frac{2x}{\sqrt{x^2 + y^2 + z^2}} \hat{e}_x + \frac{1}{2} \frac{2y}{\sqrt{x^2 + y^2 + z^2}} \hat{e}_y + \frac{1}{2} \frac{2z}{\sqrt{x^2 + y^2 + z^2}} \hat{e}_z$$

$$= \frac{\vec{r}}{|r|} = \hat{r}$$

What would it mean for the gradient $\vec{\nabla} f$ **to vanish?**

Stationary point of *f(x,y,z)*

$\vec{\nabla}$: Vector operator (The del operator)

$$\vec{\nabla} = \left(\hat{e}_x \frac{\partial}{\partial x} + \hat{e}_y \frac{\partial}{\partial y} + \hat{e}_z \frac{\partial}{\partial z} \right)$$

Three ways the operator $\vec{\nabla}$: can act:

$\vec{\nabla} f$ (The gradient)

$\vec{\nabla} \bullet \vec{V}$ (The divergence)

$\vec{\nabla} \times \vec{V}$ (The curl)

The divergence and curl operates on a VECTOR FIELD .

Examples of vector fields are Velocity of fluid flow , Electric field, Magnetic field etc

Vector Field

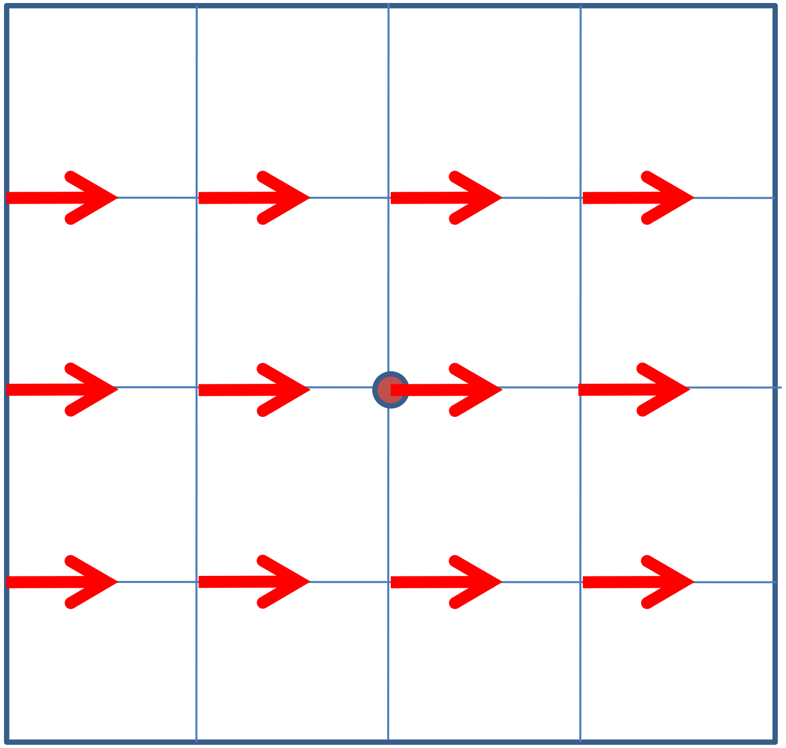
- A vector field in the plane is an assignment of a vector to every point (x,y) in the plane.
- Algebraically a vector field is a function of (x, y) whose value is a vector
- i.e. **F(x, y)** is a vector field if

$$\vec{F}(x,y) = M(x,y)\hat{e}_x + N(x,y)\hat{e}_y$$

Lets see some examples……………

Vector Field

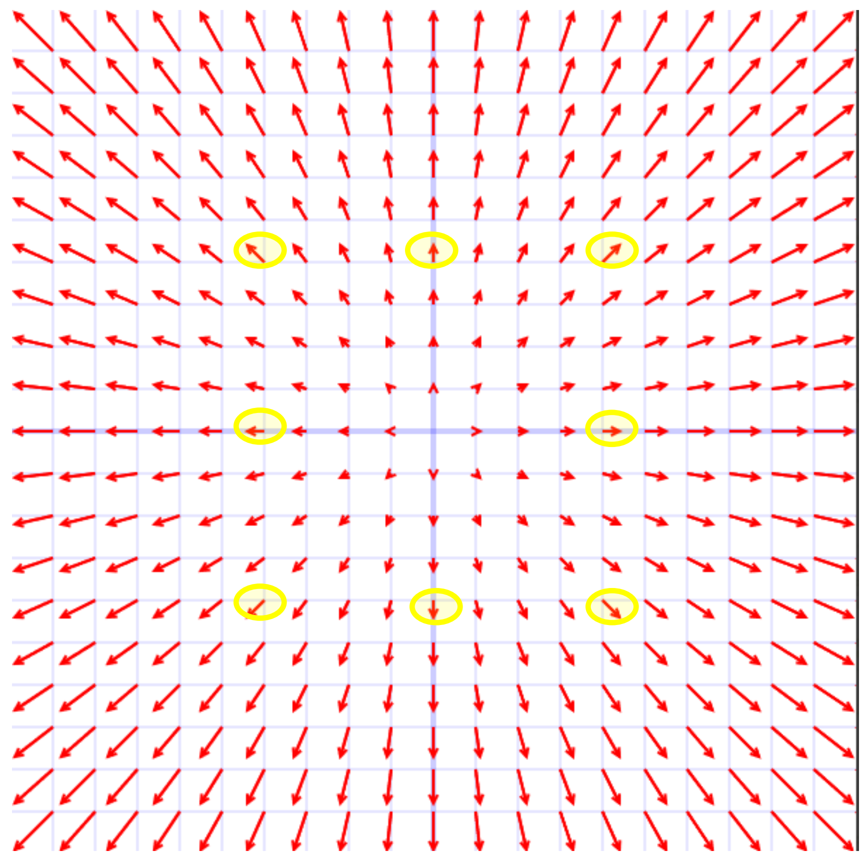
$$\vec{A} = 0.5\hat{e}_x$$



Vector Field

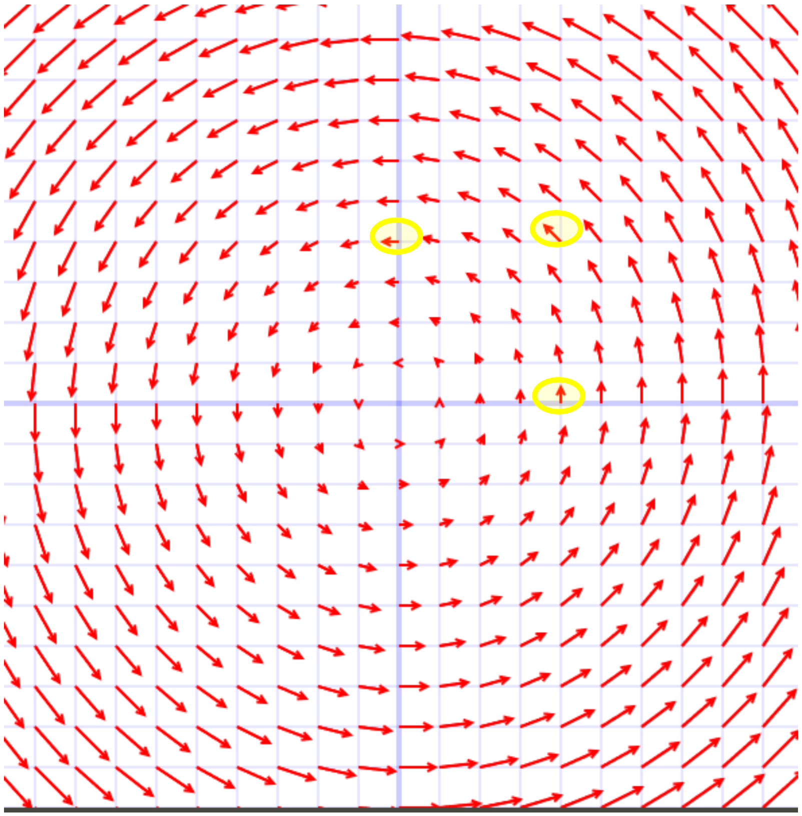
$$\vec{A} = 1\hat{e}_x + 1\hat{e}_y :$$

$$\vec{A} = x\hat{e}_x + y\hat{e}_y :$$



Vector Field

$$\overrightarrow{A} = -y\hat{e}_x + x\hat{e}_y :$$



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Announcement

First Quiz on 16th April (Saturday, 10:AM)

Online Quiz, 50 minutes duration

Maximum marks: 10

Nature: Descriptive