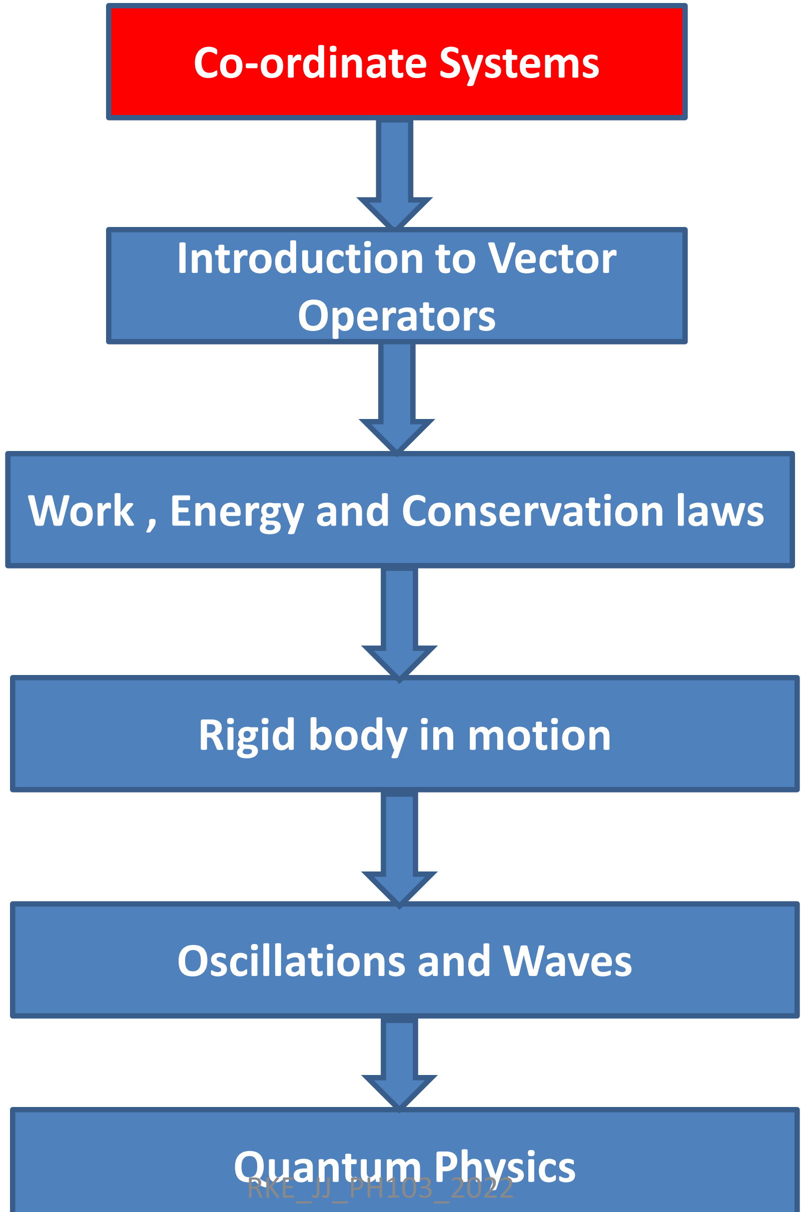
# Highlights of the course



# 1. Co-ordinate Systems (Continued…)

**Choice of Co-ordinate systems**

**Cartesian Co-ordinates:** Line, area and volume element

**Plane-Polar Co-ordinates:** Unit vectors, transformations, Rate of change, velocity and acceleration, Line element, area element

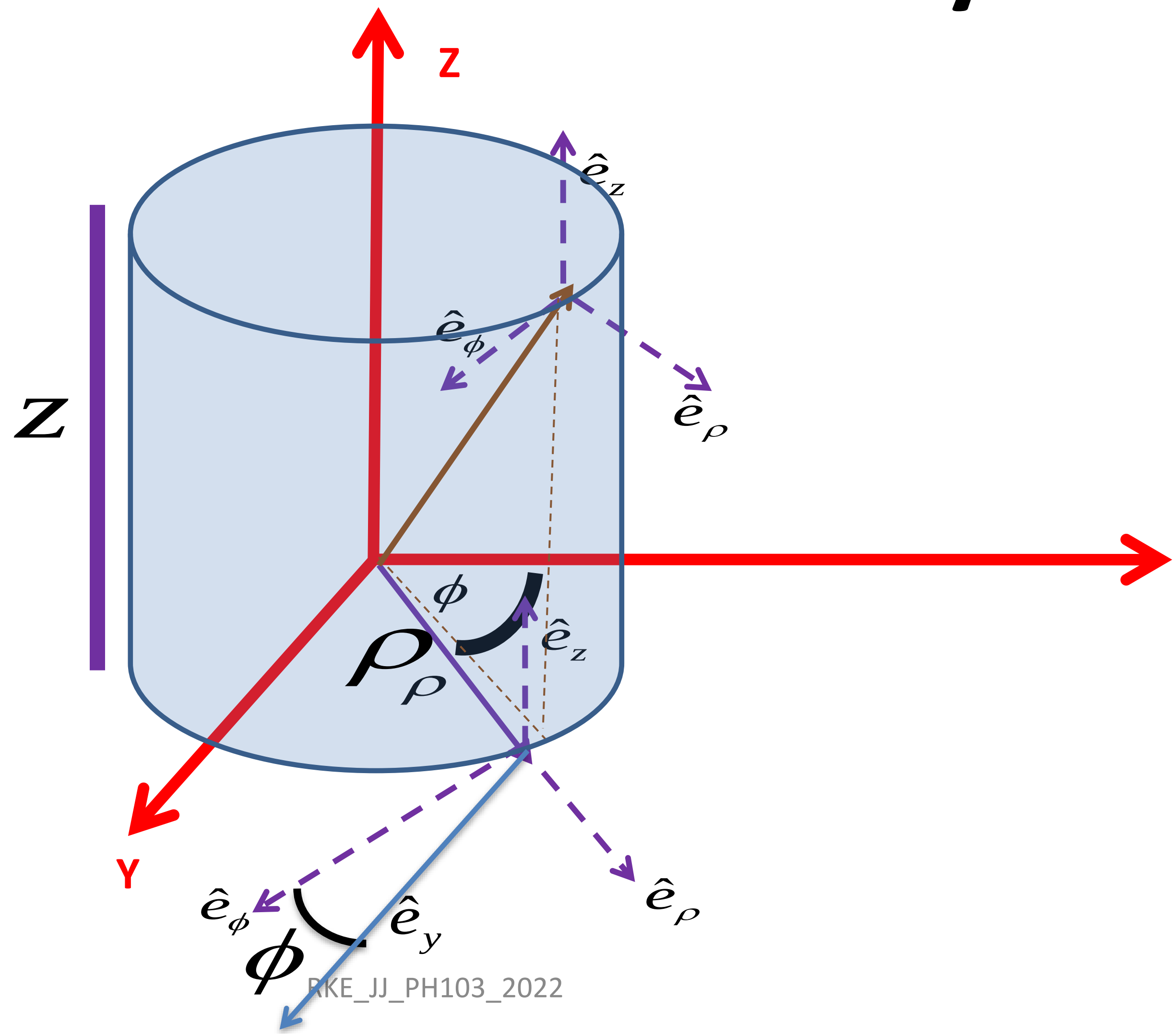
**Lec 0-2 & Tut1**

**Cylindrical Co-ordinates:** Unit vectors and its transformations, Rate of change, velocity and acceleration, line, area and volume element

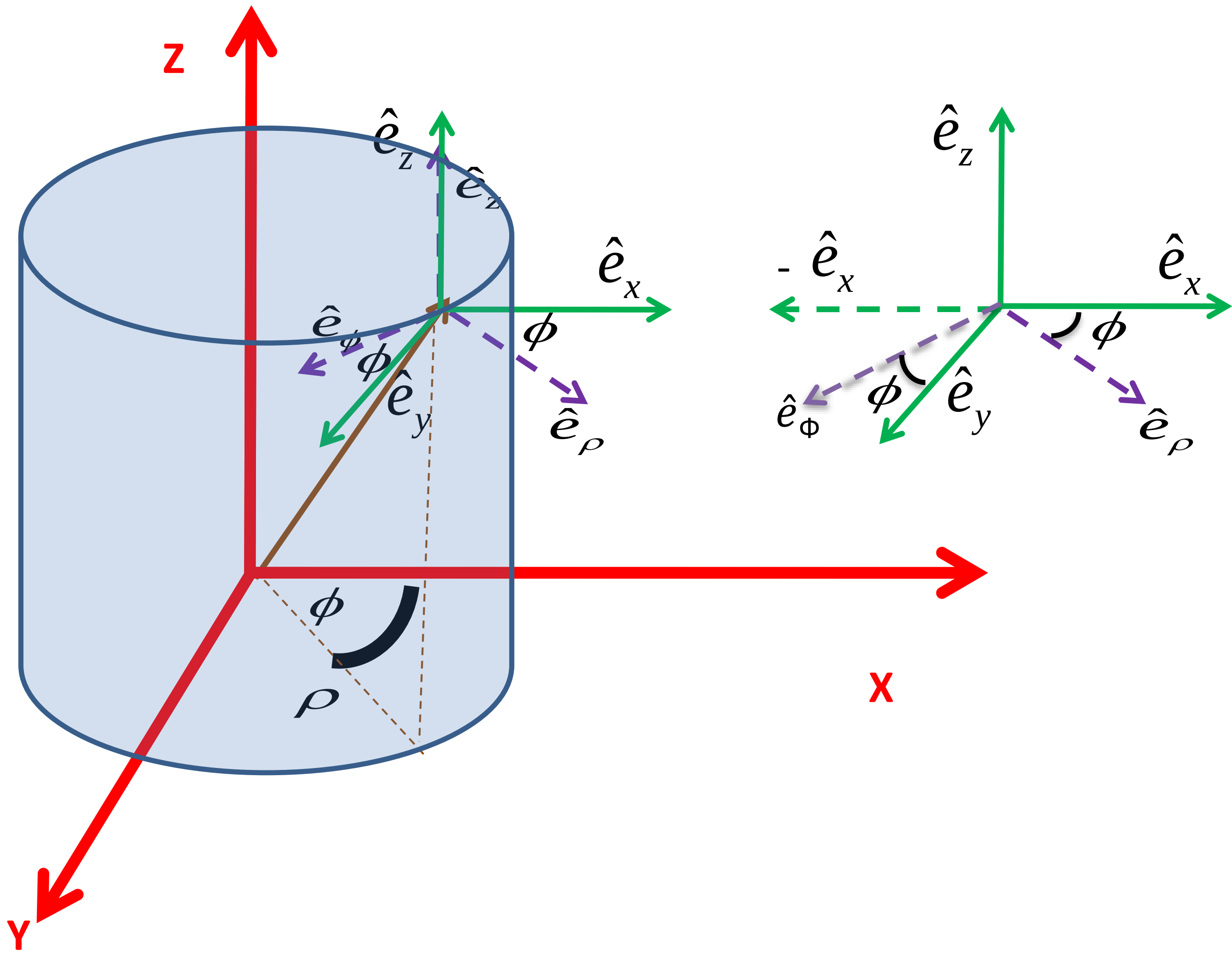
**Spherical Polar Co-ordinates:** Unit vectors and its transformations, Rate of change, velocity and acceleration, line, area and volume element

# Cylindrical Coordinate System

# Cylindrical Coordinate System



# Transformation of unit vectors




$$\hat{e}_\rho = \hat{e}_x \cos(\phi) + \hat{e}_y \sin(\phi) + 0\hat{e}_z$$
$$\hat{e}_\phi = -\hat{e}_x \sin(\phi) + \hat{e}_y \cos(\phi) + 0\hat{e}_z$$
$$\hat{e}_z = \hat{e}_z$$

$$\hat{e}_x = \hat{e}_\rho \cos(\phi) - \hat{e}_\phi \sin(\phi) + 0\hat{e}_z$$
$$\hat{e}_y = \hat{e}_\rho \sin(\phi) + \hat{e}_\phi \cos(\phi) + 0\hat{e}_z$$
$$\hat{e}_z = \hat{e}_z$$

# Derivatives of unit vectors

$$\frac{d\hat{e}_\rho}{d\phi} = \hat{e}_\phi$$

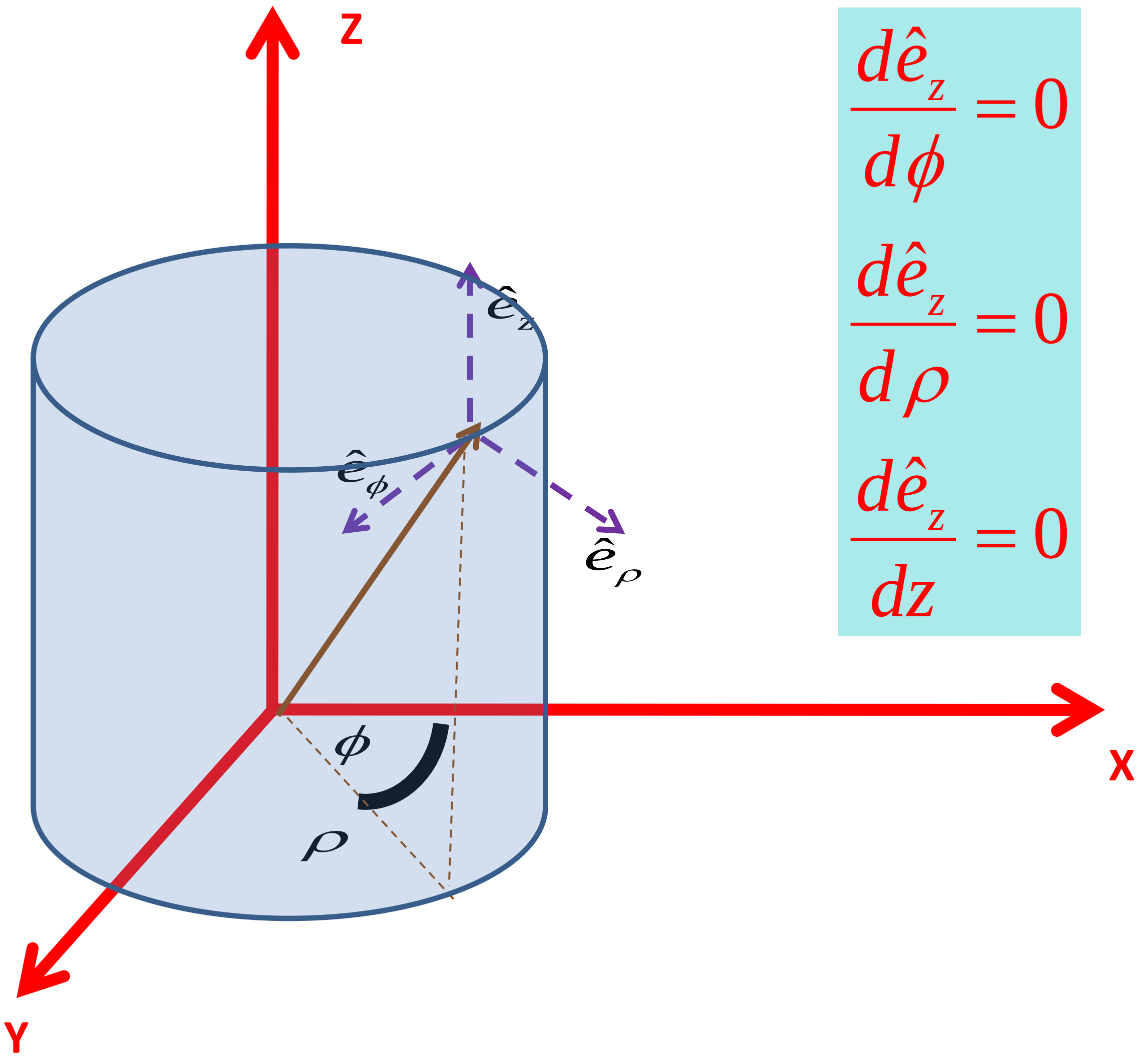
$$\frac{d\hat{e}_\rho}{d\rho} = 0$$

$$\frac{d\hat{e}_\rho}{dz} = 0$$

$$\frac{d\hat{e}_\phi}{d\phi} = -\hat{e}_\rho$$

$$\frac{d\hat{e}_\phi}{d\rho} = 0$$

$$\frac{d\hat{e}_\phi}{dz} = 0$$

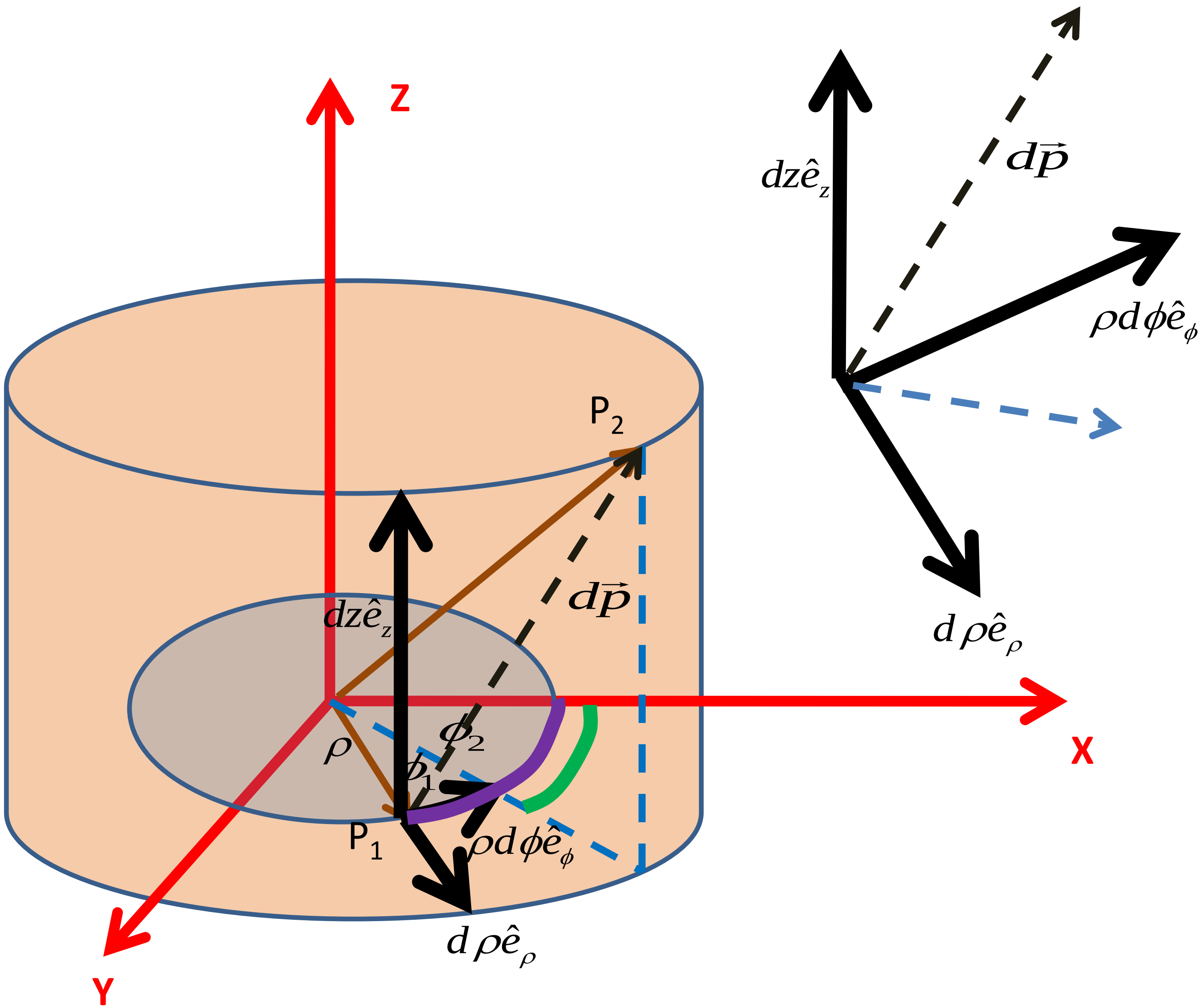



$$\frac{d\hat{e}_z}{d\phi} = 0$$

$$\frac{d\hat{e}_z}{d\rho} = 0$$

$$\frac{d\hat{e}_z}{dz} = 0$$

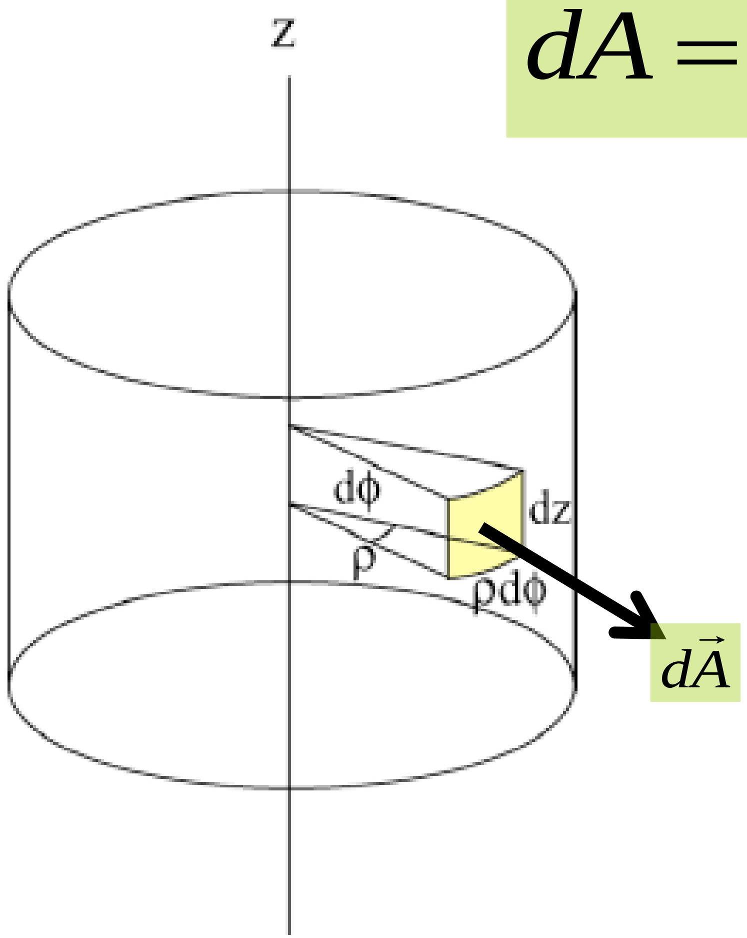
# Infinitesimal line element




$$d\vec{p} = d\rho\hat{e}_\rho + \rho d\phi\hat{e}_\phi + dz\hat{e}_z$$

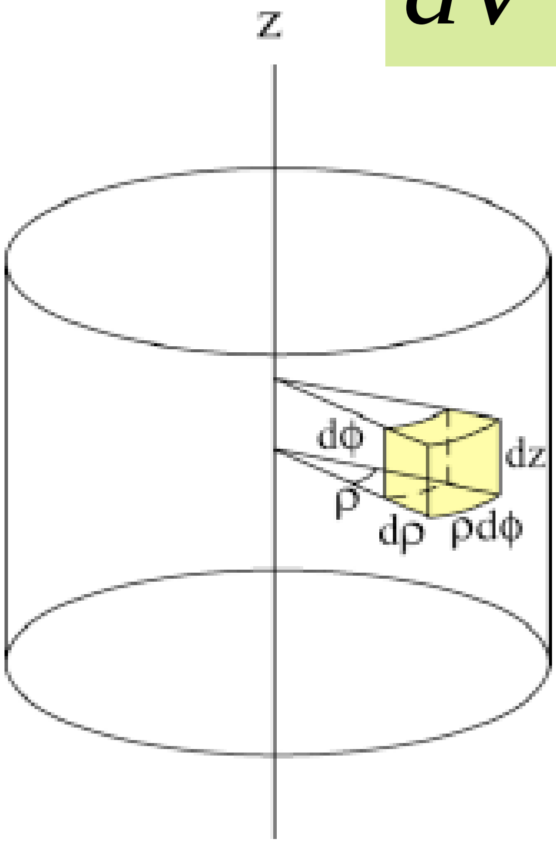
# Infinitesimal area element

$$d\vec{A} = \rho d\phi dz \hat{e}_\rho$$



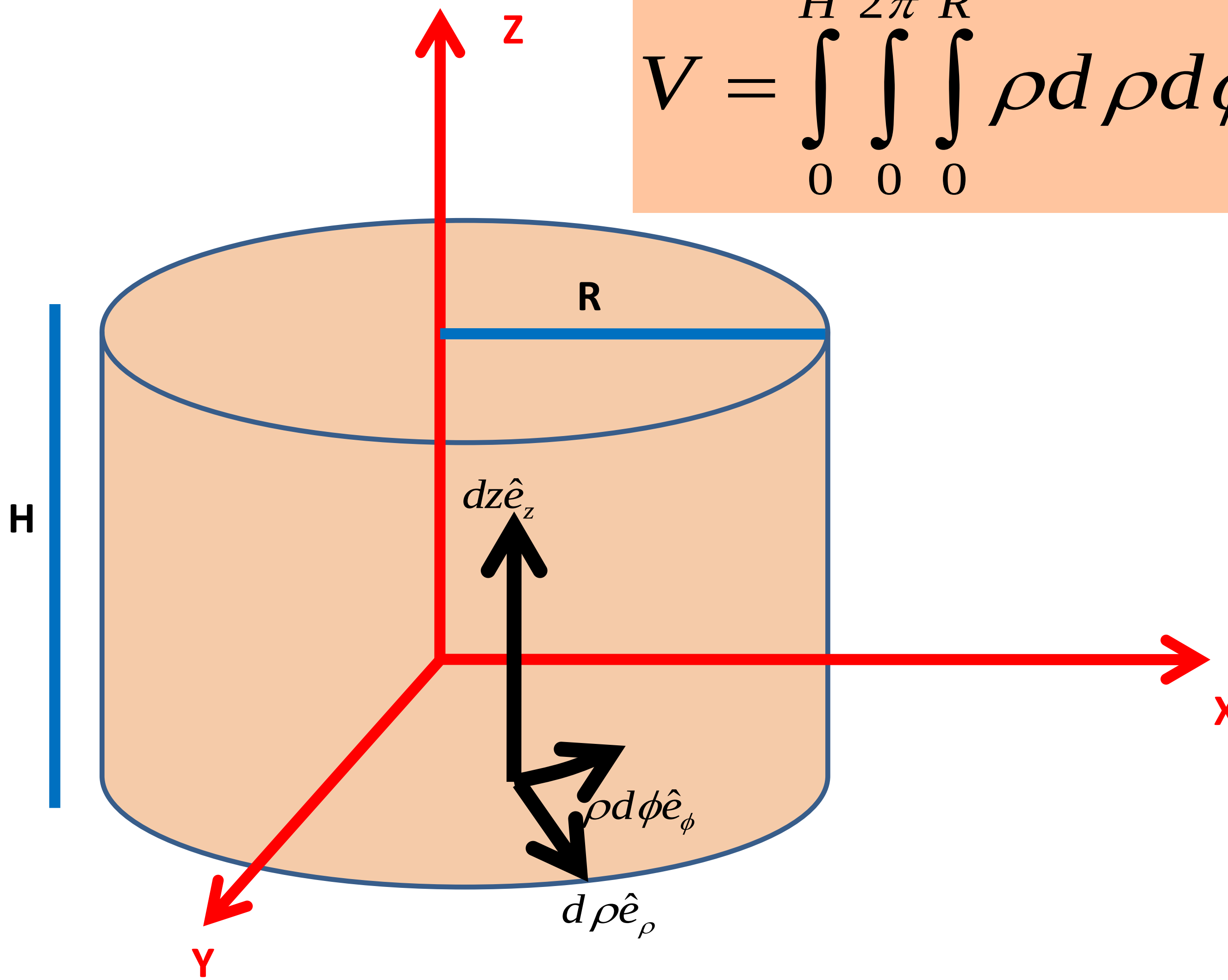
# Infinitesimal Volume element

$$dV = \rho d\rho d\phi dz$$



# Volume of cylinder

$$V = \int_0^H \int_0^{2\pi} \int_0^R \rho d\rho d\phi dz$$




$$d\vec{p} = d\rho\hat{e}_\rho + \rho d\phi\hat{e}_\phi + dz\hat{e}_z$$



$$dV = \rho d\rho d\phi dz$$

# Velocity in cylindrical polar co-ordinates

$$\vec{r} = \rho\hat{e}_\rho + z\hat{e}_z$$

$$\vec{V} = \frac{d(\rho\hat{e}_\rho + z\hat{e}_z)}{dt}$$

$$\frac{d\hat{e}_\rho}{d\phi} = \hat{e}_\phi$$

$$\frac{d\hat{e}_\rho}{d\rho} = 0$$

$$\frac{d\hat{e}_\rho}{dz} = 0$$

$$\frac{d\hat{e}_z}{d\theta} = 0$$

$$\frac{d\hat{e}_z}{d\rho} = 0$$

$$\frac{d\hat{e}_z}{dz} = 0$$

# Acceleration in cylindrical polar co-ordinates

$$\vec{r} = \rho\hat{e}_{\rho} + z\hat{e}_{z}$$

$$\vec{V} = \dot{\rho}\,\hat{e}_{r} + \rho\,\dot{\phi}\,\hat{e}_{\phi} + \dot{z}\,\hat{e}_{z}$$

$$\vec{a} = \left(\ddot{\rho} - \rho\,\dot{\phi}^{2}\right)\hat{e}_{\rho} + \left(\rho\,\ddot{\phi} + 2\,\dot{\rho}\,\dot{\phi}\right)\hat{e}_{\phi} + \ddot{z}\,\hat{e}_{z}$$

## Plane polar

$$(r,\theta)$$

$$\vec{r} = r\hat{e}_r$$

$$d\vec{r} = dr\hat{e}_r + rd\theta\hat{e}_\theta$$

$$\hat{e}_r = \hat{e}_x\cos(\theta) + \hat{e}_y\sin(\theta)$$
$$\hat{e}_\theta = -\hat{e}_x\sin(\theta) + \hat{e}_y\cos(\theta)$$

$$\vec{V} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

## Cylindrical polar

$$(\rho,\phi,z)$$

$$\vec{r} = \rho\hat{e}_\rho + z\hat{e}_z$$

$$d\vec{r} = d\rho\hat{e}_\rho + \rho d\phi\hat{e}_\phi + dz\hat{e}_z$$

$$\hat{e}_\rho = \hat{e}_x\cos(\phi) + \hat{e}_y\sin(\phi) + 0\hat{e}_z$$
$$\hat{e}_\phi = -\hat{e}_x\sin(\phi) + \hat{e}_y\cos(\phi) + 0\hat{e}_z$$
$$\hat{e}_z = \hat{e}_z$$

$$\vec{V} = \dot{\rho}\,\hat{e}_\rho + \rho\dot{\phi}\,\hat{e}_\phi + \dot{z}\,\hat{e}_z$$

$$\vec{a} = \left(\ddot{\rho} - \rho\dot{\phi}^2\right)\hat{e}_\rho + \left(\rho\ddot{\phi} + 2\dot{\rho}\dot{\phi}\right)\hat{e}_\phi + \ddot{z}\,\hat{e}_z$$