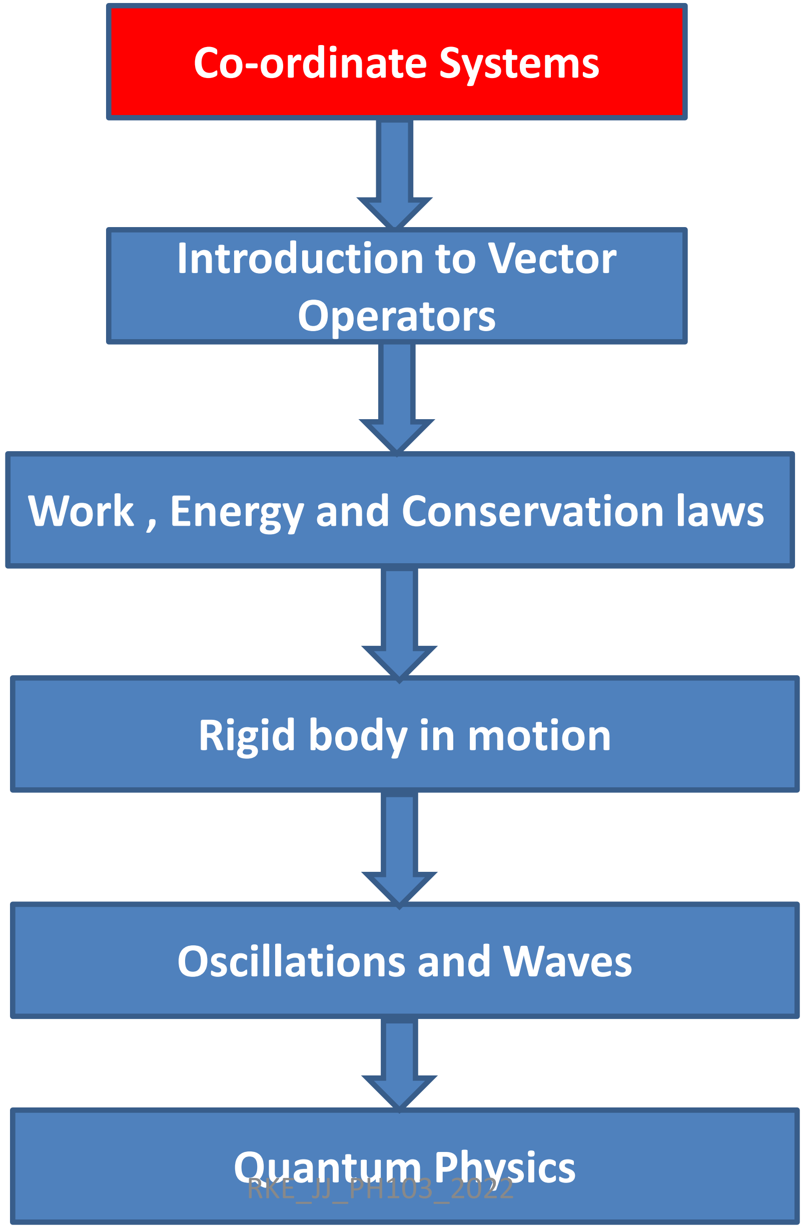
Highlights of the course



Chapter 1

Co-ordinate Systems

Introduction to

DIFFERENT CO-ORDINATE SYSTEMS

INFINITESIMAL LINE, AREA AND VOLUME ELEMENTS

Text Books for Chapter 1

- **Textbooks:**

- D. Kleppner and R. J. Kolenkow, An introduction to Mechanics, Tata McGraw-Hill, New Delhi, 2000.

- P. C. Deshmukh, Foundations of Classical Mechanics, Cambridge University Press, 2019

RECAP

Velocity in polar co-ordinates

Velocity in Polar Coordinates

$$\vec{r} = r\hat{e}_r$$

$$\vec{V} = \frac{d(r\hat{e}_r)}{dt} = \frac{dr}{dt}\hat{e}_r + r\frac{d\hat{e}_r}{dt}$$

$$\frac{d\hat{e}_r}{dt} = \frac{d\hat{e}_r}{d\theta}\frac{d\theta}{dt}$$

$$\frac{d\hat{e}_r}{dt} = \hat{e}_\theta \frac{d\theta}{dt}$$

$$\vec{V} = \frac{d(r\hat{e}_r)}{dt} = \frac{dr}{dt}\hat{e}_r + r\frac{d\theta}{dt}\hat{e}_\theta$$

$\frac{d\hat{e}_r}{d\theta}$ Geometrical and mathematical interpretation

Derivatives of unit vectors in Polar Coordinates

$$\frac{d\hat{e}_r}{d\theta} = \hat{e}_\theta$$

What about $\dfrac{d\hat{e}_r}{dr}$?

$$\hat{e}_r = \hat{e}_x \cos(\theta) + \hat{e}_y \sin(\theta)$$

$$\frac{d\hat{e}_r}{dr} = 0$$

Derivatives of unit vectors in Polar Coordinates

$$\frac{d\hat{e}_\theta}{dr} = ?$$

$$\frac{d\hat{e}_\theta}{d\theta} = ?$$

$$\hat{e}_\theta = -\hat{e}_x \sin(\theta) + \hat{e}_y \cos(\theta)$$
$$\hat{e}_r = \hat{e}_x \cos(\theta) + \hat{e}_y \sin(\theta)$$

Derivative of unit vectors

Try Geometrical consideration also

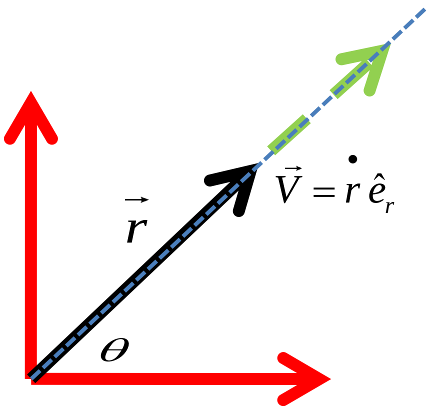
$$\frac{d\hat{e}_\theta}{d\theta} = -\hat{e}_r$$
$$\frac{d\hat{e}_\theta}{dr} = 0$$

$$\frac{d\hat{e}_r}{d\theta} = \hat{e}_\theta$$
$$\frac{d\hat{e}_r}{dr} = 0$$

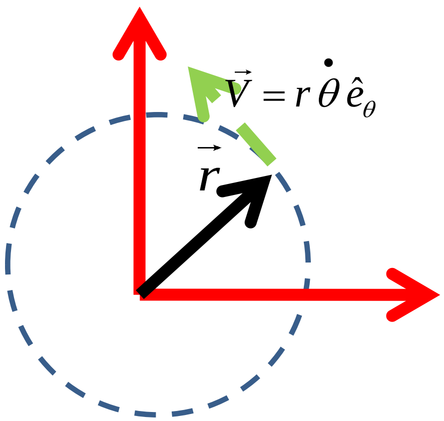
Back to Velocity……

$$\vec{V} = \frac{d(r\hat{e}_r)}{dt} = \frac{dr}{dt}\hat{e}_r + r\frac{d\theta}{dt}\hat{e}_\theta$$

Case 1: θ is constant



Case 2: r is constant



Acceleration in polar coordinates

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{d\left(\dot{r}\,\hat{e}_r + r\,\dot{\theta}\,\hat{e}_\theta\right)}{dt}$$

$$\frac{d\hat{e}_r}{dt} = \dot{\theta}\,\hat{e}_\theta$$

$$\vec{a} = \left(\ddot{r} - r\,\dot{\theta}^2\right)\hat{e}_r + \left(r\,\ddot{\theta} + 2\,\dot{r}\,\dot{\theta}\right)\hat{e}_\theta$$

$$\frac{d\hat{e}_\theta}{dt} = \dot{\theta}\,\frac{d\hat{e}_\theta}{d\theta} = -\dot{\theta}\,\hat{e}_r$$

Acceleration in polar coordinates

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

- $\ddot{r}$: Linear acceleration in the radial direction
- $r\dot{\theta}^2$: Centripetal acceleration in the radial direction
- $r\ddot{\theta}$: Linear acceleration in the tangential direction
- $2\dot{r}\dot{\theta}$: Coriolis acceleration

Points to ponder!

Does a constant radial velocity ($\dot{r}$) ensures a zero radial acceleration ?

Does a constant angular velocity ($\dot{\theta}$) means zero angular acceleration ?

Acceleration in radial and tangential components separately

$$\vec{V} = \dot{r}\,\hat{e}_r + r\,\dot{\theta}\,\hat{e}_\theta$$

$$\vec{V} = \vec{V}_r + \vec{V}_\theta$$

Acceleration due to change in radial direction alone….

$$\frac{d\vec{V}_r}{dt} = \frac{d(\dot{r}\,\hat{e}_r)}{dt} = \ddot{r}\,\hat{e}_r + \dot{r}\,\dot{\theta}\,\hat{e}_\theta$$

Acceleration due to change in tangential direction alone...

$$\frac{d\vec{V}_\theta}{dt} = \frac{d(r\,\dot{\theta}\,\hat{e}_\theta)}{dt} = -r\,\dot{\theta}^2\,\hat{e}_r + \left(\dot{r}\,\dot{\theta} + r\,\ddot{\theta}\right)\hat{e}_\theta$$

Acceleration in polar coordinates

$$\vec{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{e}_\theta$$

Linear acceleration in the radial direction

Centripetal acceleration in the radial direction

Linear acceleration in the tangential direction

Coriolis acceleration

$$\frac{d\vec{V}_r}{dt} = \ddot{r}\,\hat{e}_r + \dot{r}\,\dot{\theta}\,\hat{e}_\theta$$

Acceleration due to change in radial direction alone….

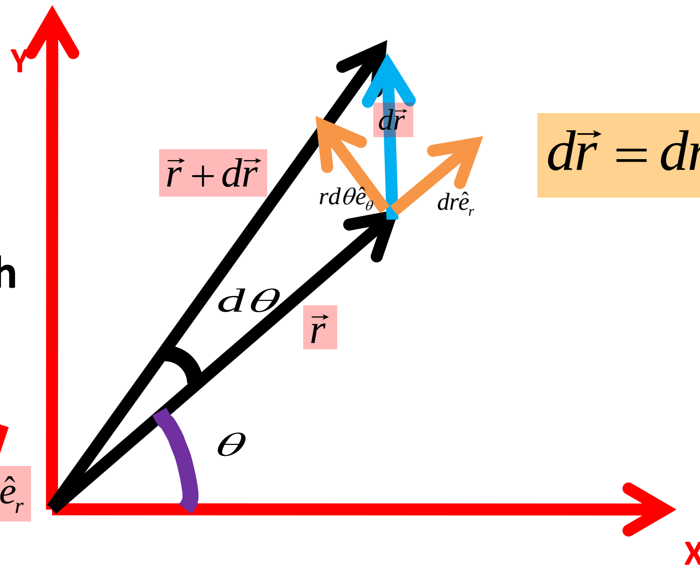
$$\frac{d\vec{V}_\theta}{dt} = -r\dot{\theta}^2\,\hat{e}_r + \left(\dot{r}\,\dot{\theta} + r\ddot{\theta}\right)\hat{e}_\theta$$

Acceleration due to change in tangential direction alone…

Infinitesimal line element in polar co-ordinates

Infinitesimal line element in plane polar coordinates

$$\vec{r} = r\hat{e}_r$$



$$d\vec{r} = dr\hat{e}_r + rd\theta\hat{e}_\theta$$

Mathematical approach

$$\vec{r}+\Delta\vec{r} = (r+\Delta r)(\hat{e}_r+\Delta\hat{e}_r)$$

$$\vec{r}+\Delta\vec{r} = r\hat{e}_r + r\Delta\hat{e}_r + \Delta r\hat{e}_r + \cancel{\Delta r\Delta\hat{e}_r}$$

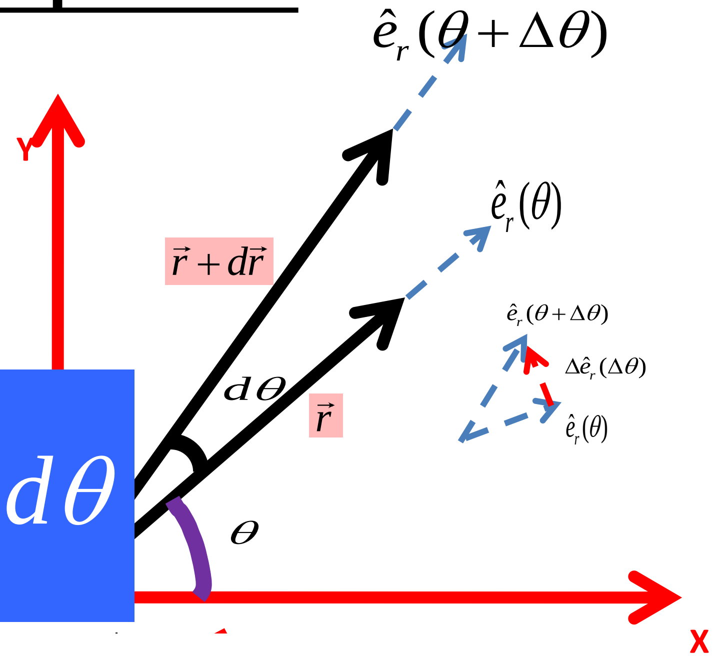
$$\Delta\vec{r} = \Delta r\hat{e}_r + r\Delta\hat{e}_r$$

$$d\vec{r} = dr\hat{e}_r + rd\hat{e}_r$$

Taylor series expansion

$$d\vec{r} = dr\hat{e}_r + r d\hat{e}_r$$?

$$d\hat{e}_r = \Delta\hat{e}_r(\Delta\theta)$$
$$\Delta\theta \rightarrow 0$$



$$d\vec{r} = dr\hat{e}_r + r\frac{d\hat{e}_r}{d\theta}d\theta$$

$$f(x_0 + \Delta x) =$$

r series expansion

$$\hat{e}_r(\theta+\Delta\theta) = \hat{e}_r(\theta) + \frac{d\hat{e}_r}{d\theta}\Delta\theta$$

$$\hat{e}_r(\theta+\Delta\theta) - \hat{e}_r(\theta) = \frac{d\hat{e}_r}{d\theta}\Delta\theta$$

In the limits $\Delta\theta \rightarrow 0$, $d\hat{e}_r = \frac{d\hat{e}_r}{d\theta}d\theta$

Infinitesimal line element in plane polar coordinates

$$d\vec{r} = dr\hat{e}_r + r\left(\frac{d\hat{e}_r}{d\theta}d\theta\right)$$

$$\frac{d\hat{e}_r}{d\theta} = \hat{e}_\theta$$

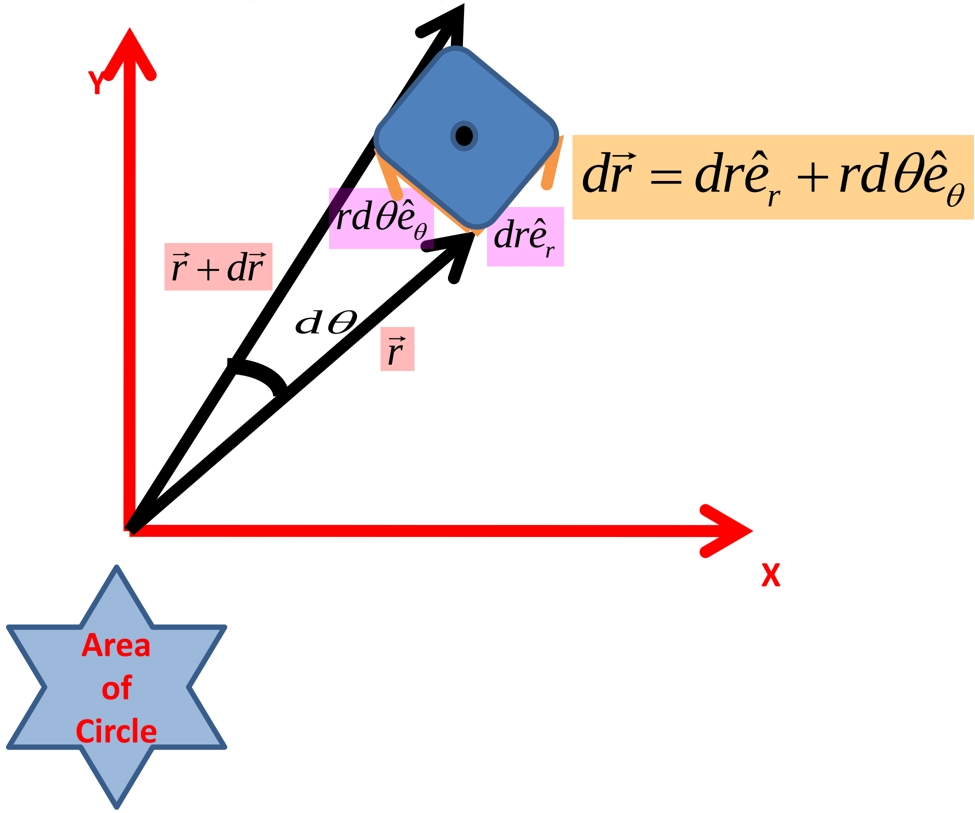
$$d\vec{r} = dr\hat{e}_r + rd\theta\hat{e}_\theta$$

Elemental area in plane polar coordinates

Elemental area in plane polar coordinates

Elemental area:

$$d\vec{A} = r\,dr\,d\theta\,\hat{n}$$



Get ready for the Tutorial ……