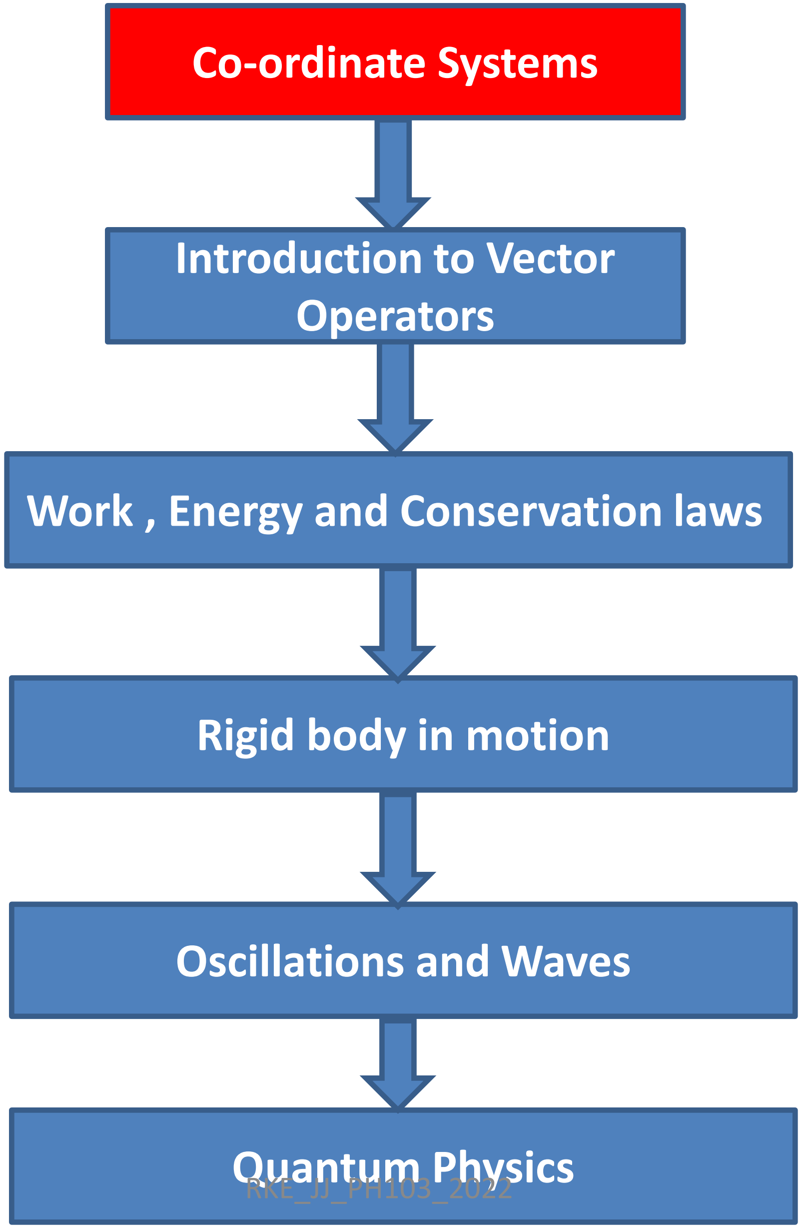
Highlights of the course



Chapter 1

Co-ordinate Systems

Introduction to

DIFFERENT CO-ORDINATE SYSTEMS

INFINITESIMAL LINE, AREA AND VOLUME ELEMENTS

Outline

Choice of Co-ordinate systems

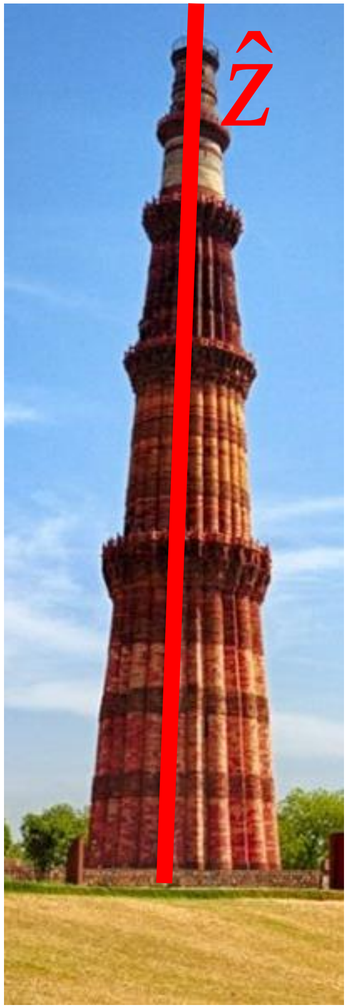
Cartesian Co-ordinates: Unit vectors, Infinitesimal line, area and volume element

Plane-Polar Co-ordinates: Unit vectors, transformations, Rate of change, velocity and acceleration, Infinitesimal line, area element

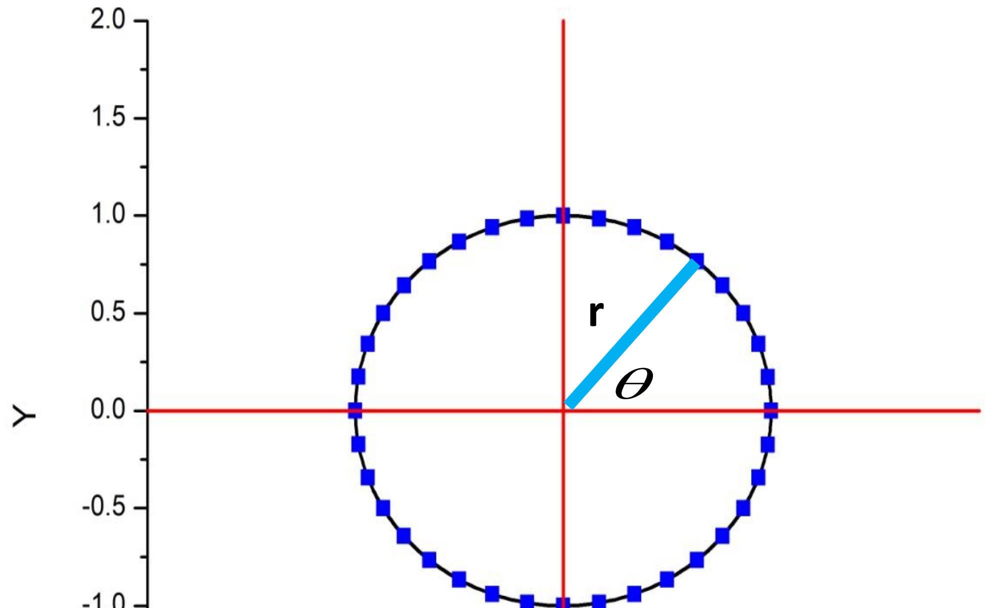
Cylindrical Co-ordinates: Unit vectors and its transformations, Rate of change, velocity and acceleration, Infinitesimal line, area and volume element

Spherical Polar Co-ordinates: Unit vectors and its transformations, Rate of change, velocity and acceleration, Infinitesimal line, area and volume element

Why do we need different coordinate system?



Why do we need different coordinate system?



X	Y
1	0
0.9397	0.34201
0.76606	0.64277
0.50003	0.86601
0	1

r	θ
1	0
	20
	40
	60
	90

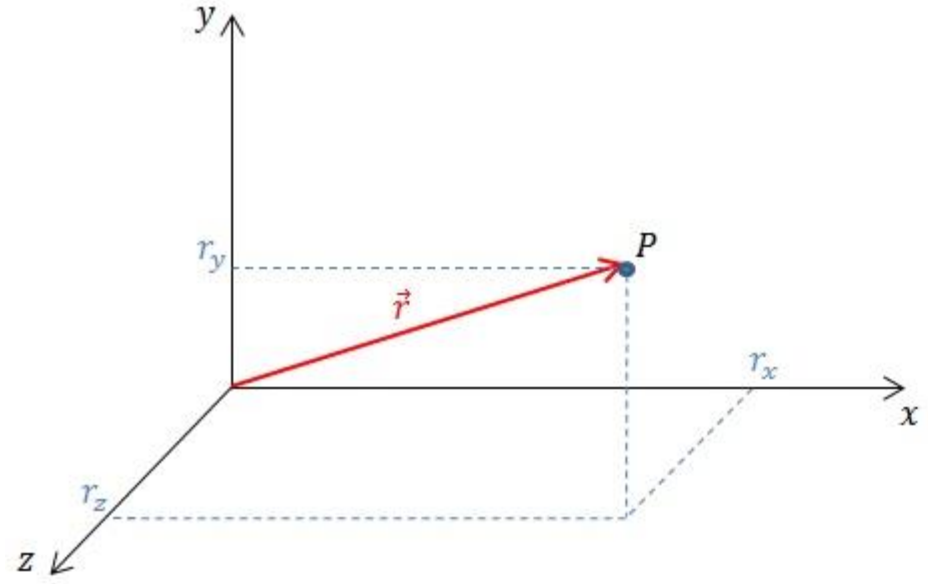
Proper choice of a coordinate system can vastly simplify a problem

Representing a vector in Cartesian Co-ordinate system

Cartesian Co-ordinates

A coordinate system consists of four basic elements:

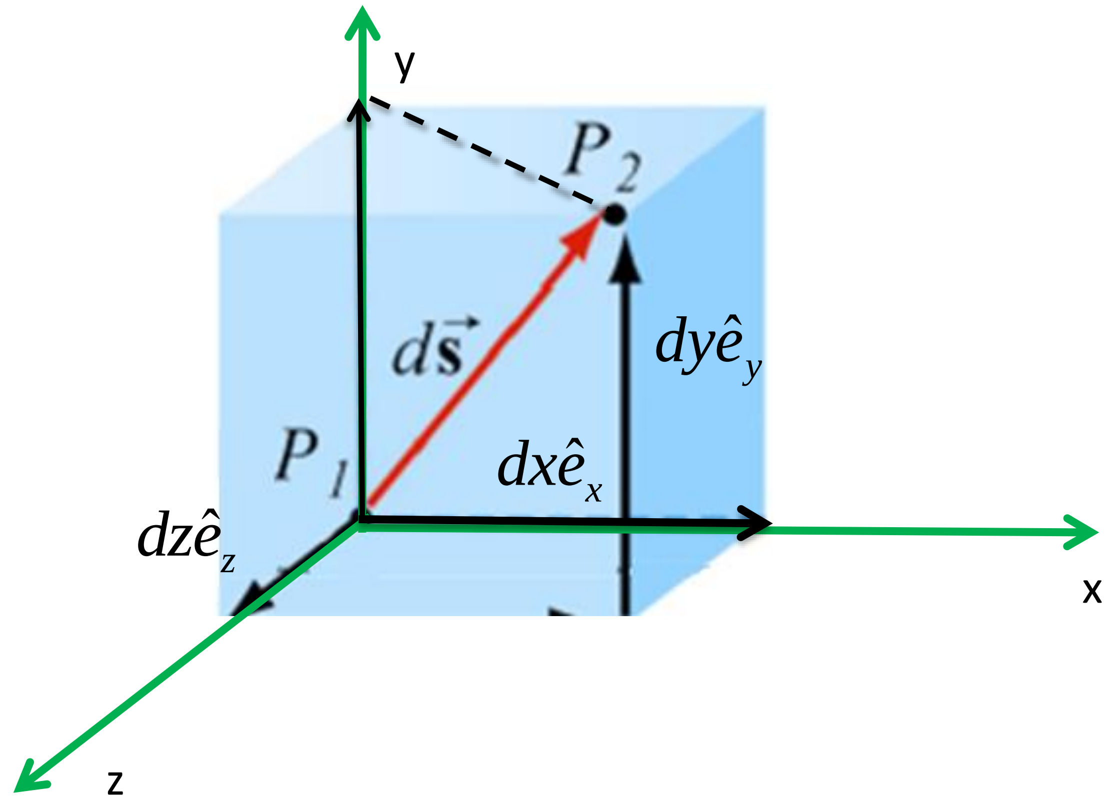
1) Choice of origin (2) Choice of axes (3) Choice of positive direction for each axis (4) Choice of unit vectors for each axis



$$\vec{r} = r_x \hat{e}_x + r_y \hat{e}_y + r_z \hat{e}_z$$

Infinitesimal (very small) line element in Cartesian Co-ordinate system

Cartesian Co-ordinates



Infinitesimal line element

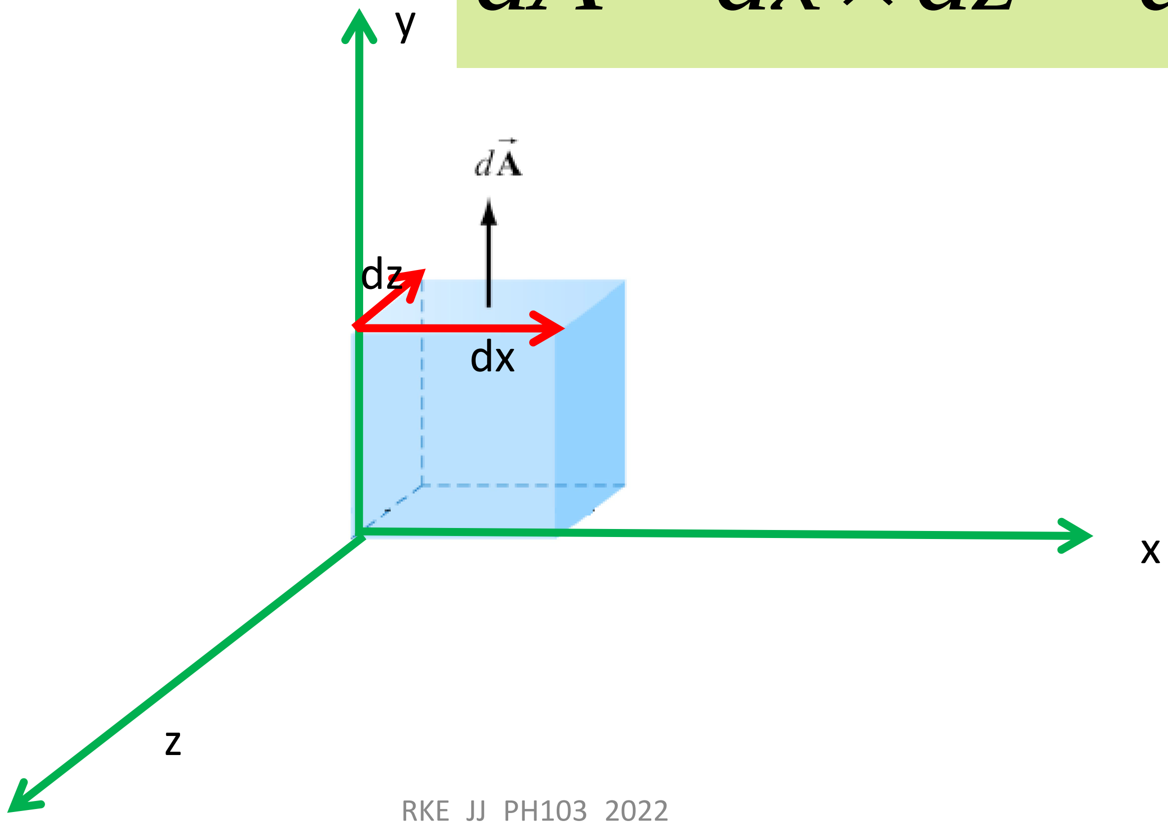
$$d\vec{s} = dx\hat{e}_x + dy\hat{e}_y + dz\hat{e}_z$$

Infinitesimal Area element in Cartesian Co-ordinate system

Cartesian Co-ordinates

Infinitesimal Area element

$$\vec{dA} = \overrightarrow{dx} \times \overrightarrow{dz} = dxdz\hat{e}_y$$

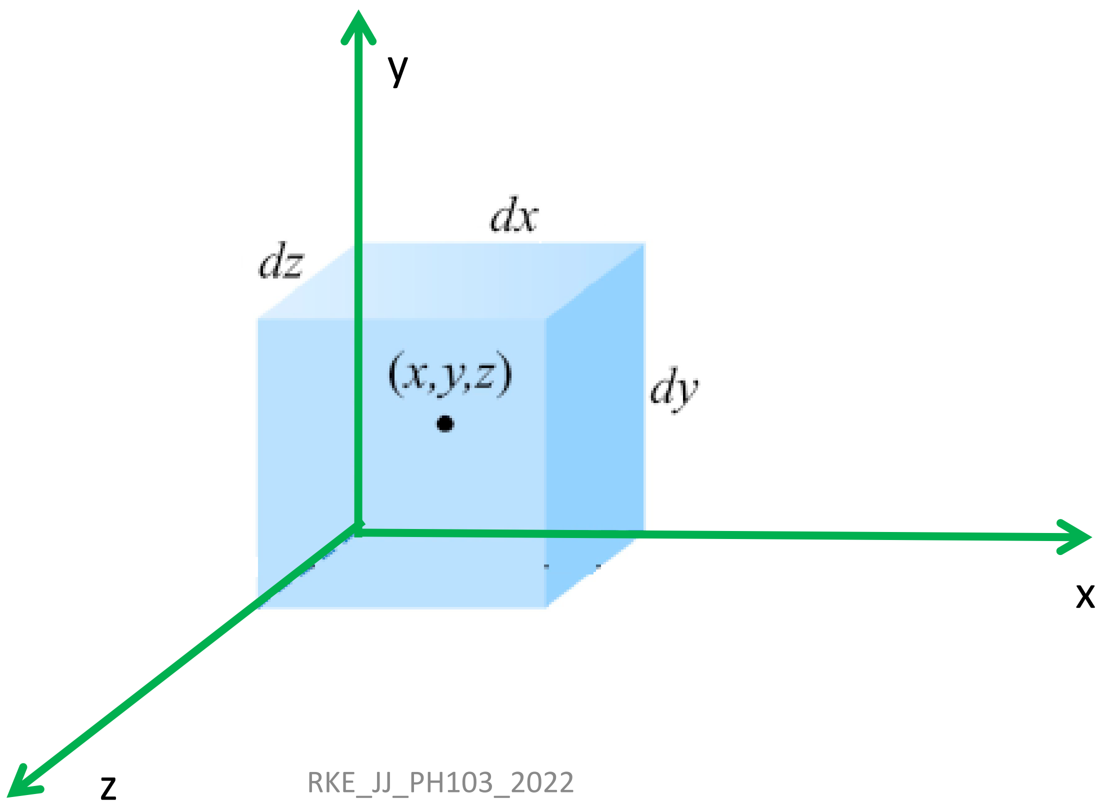


Infinitesimal Volume element in Cartesian Co-ordinate system

Cartesian Coordinates

Infinitesimal Volume element

$$dV = dxdydz$$



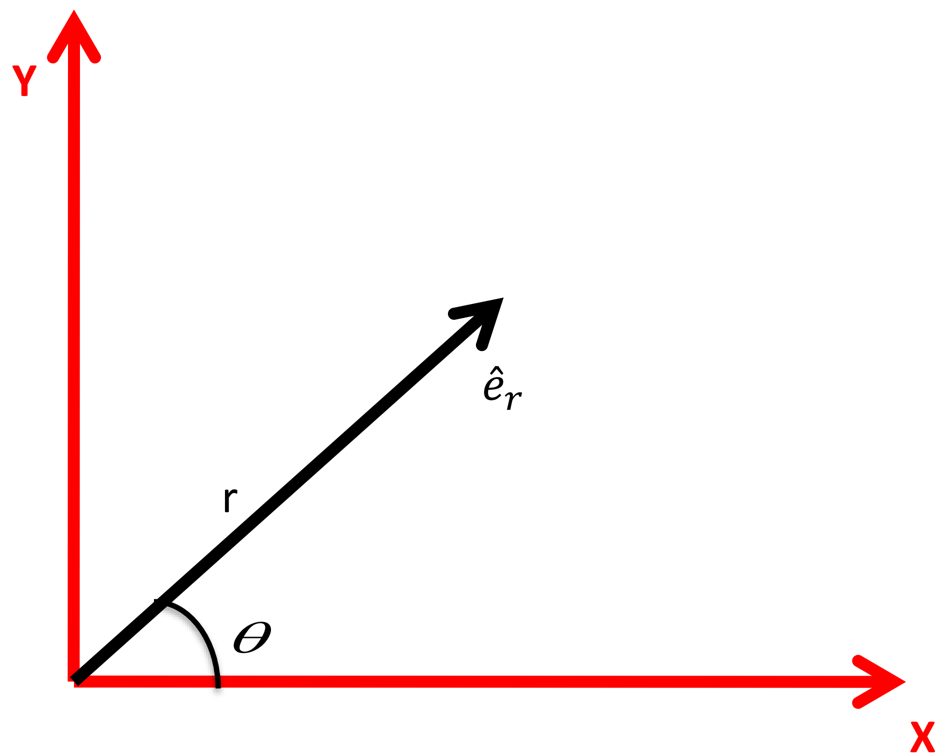
Polar co-ordinate system

Representation of a vector and unit vector transformations in polar co-ordinates

Representation of a Vector in Polar Coordinates

$$\vec{r} = r\hat{e}_r$$

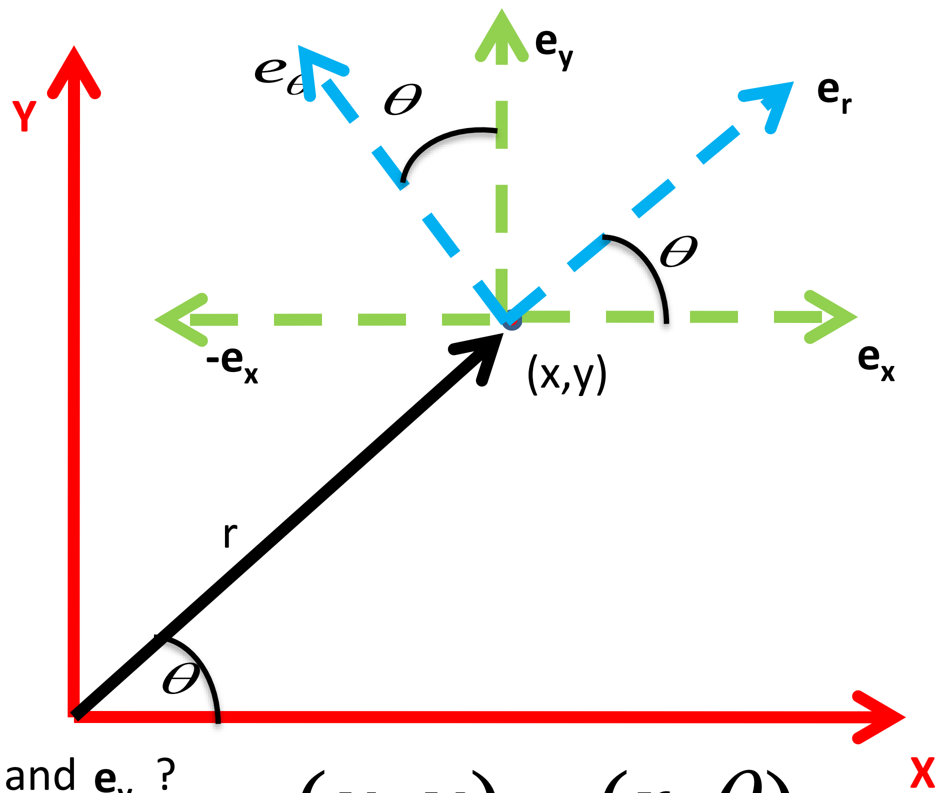
What is unit vector $\hat{e}_r$?



Unit vector representation in Polar Coordinates

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$

$$r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$



$$(x, y) \equiv (r, \theta)$$

What is $\mathbf{e}_r$ and e_θ in terms of $\mathbf{e}_x$ and $\mathbf{e}_y$?

$$\hat{e}_r = \hat{e}_x \cos(\theta) + \hat{e}_y \sin(\theta)$$
$$\hat{e}_\theta = -\hat{e}_x \sin(\theta) + \hat{e}_y \cos(\theta)$$

What is $\mathbf{e}_x$ and $\mathbf{e}_y$ in terms of $\mathbf{e}_r$ and e_θ ?

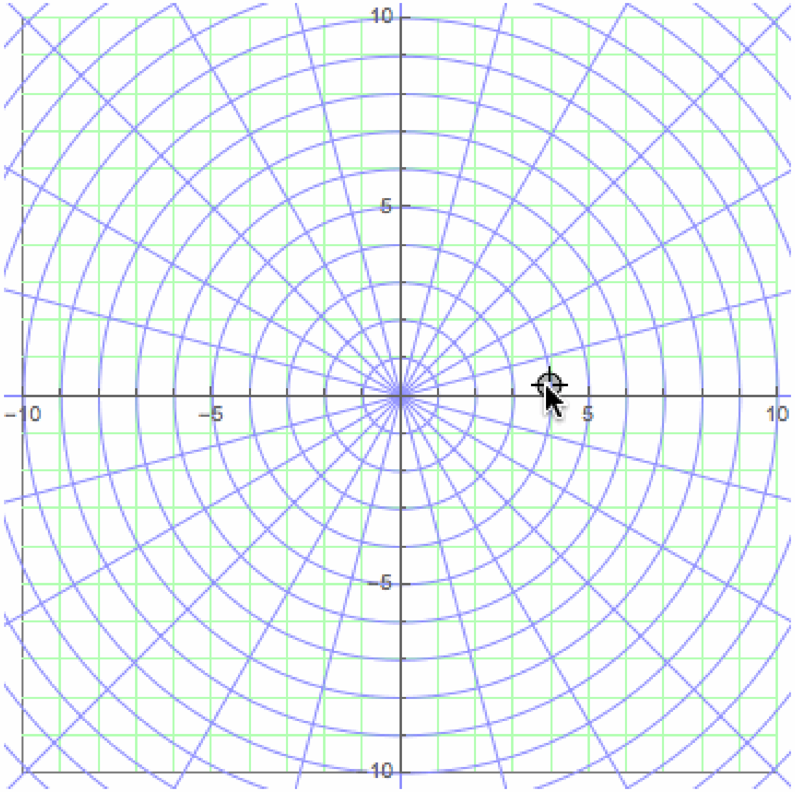
$$\hat{e}_x = \hat{e}_r \cos(\theta) - \hat{e}_\theta \sin(\theta)$$
$$\hat{e}_y = \hat{e}_r \sin(\theta) + \hat{e}_\theta \cos(\theta)$$

Verify that $\hat{e}_\Theta . \hat{e}_r = 0$

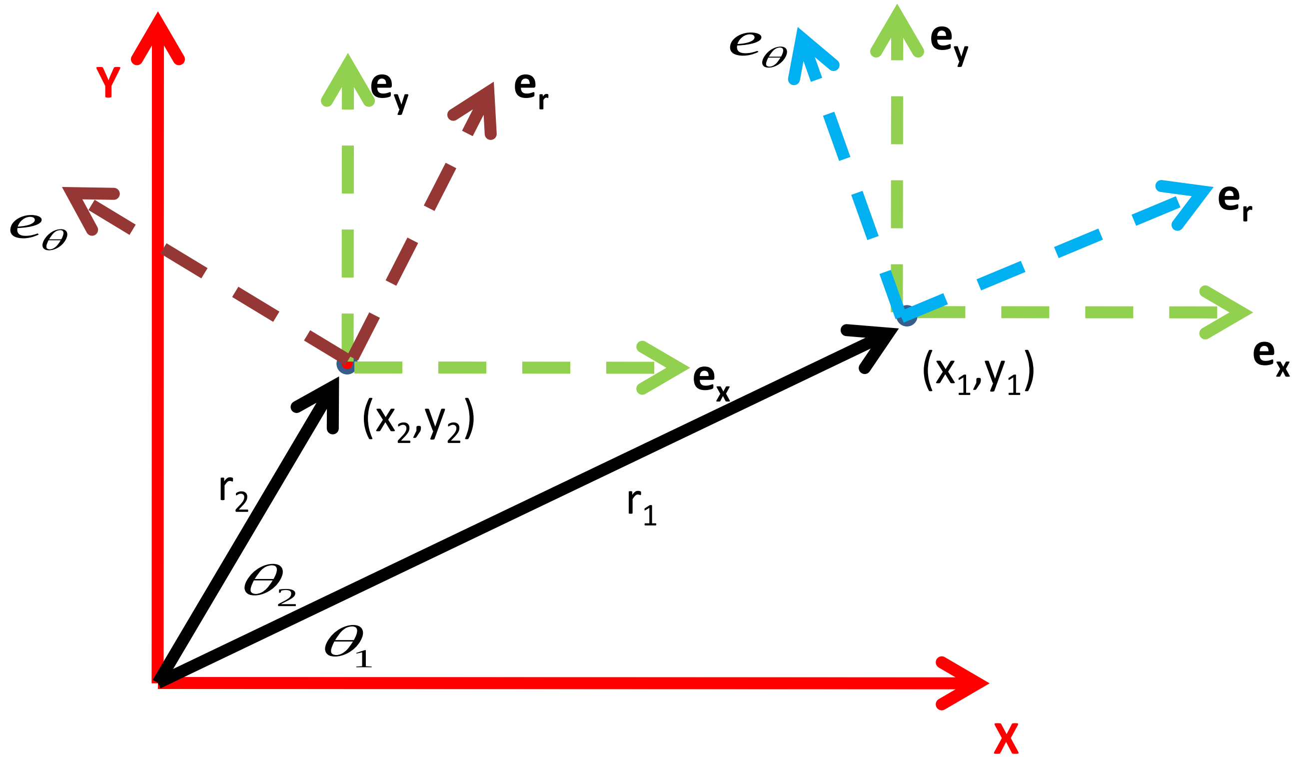
Above vector in Polar Co-ordinates is represented as

$$\vec{r} = r\hat{e}_r$$

Motion in polar co-ordinates, velocity and acceleration



Motion in Plane Polar Coordinates



Cartesian coordinate system: Constant unit vectors

Plane polar coordinate system: Varying unit vectors

Velocity and acceleration in polar co-ordinates

Velocity in Polar Coordinates

Velocity in Cartesian Coordinates: $\vec{V} = \frac{dx}{dt}\hat{e}_x + \frac{dy}{dt}\hat{e}_y$

$$\vec{r} = r\hat{e}_r$$

Velocity in Polar Coordinates: $$\vec{V} = \frac{d(r\hat{e}_r)}{dt} = \frac{dr}{dt}\hat{e}_r + r\frac{d\hat{e}_r}{dt}$$

?

$$\hat{e}_r = \hat{e}_x \cos(\theta) + \hat{e}_y \sin(\theta)$$

$$\hat{e}_\theta = -\hat{e}_x \sin(\theta) + \hat{e}_y \cos(\theta)$$

?

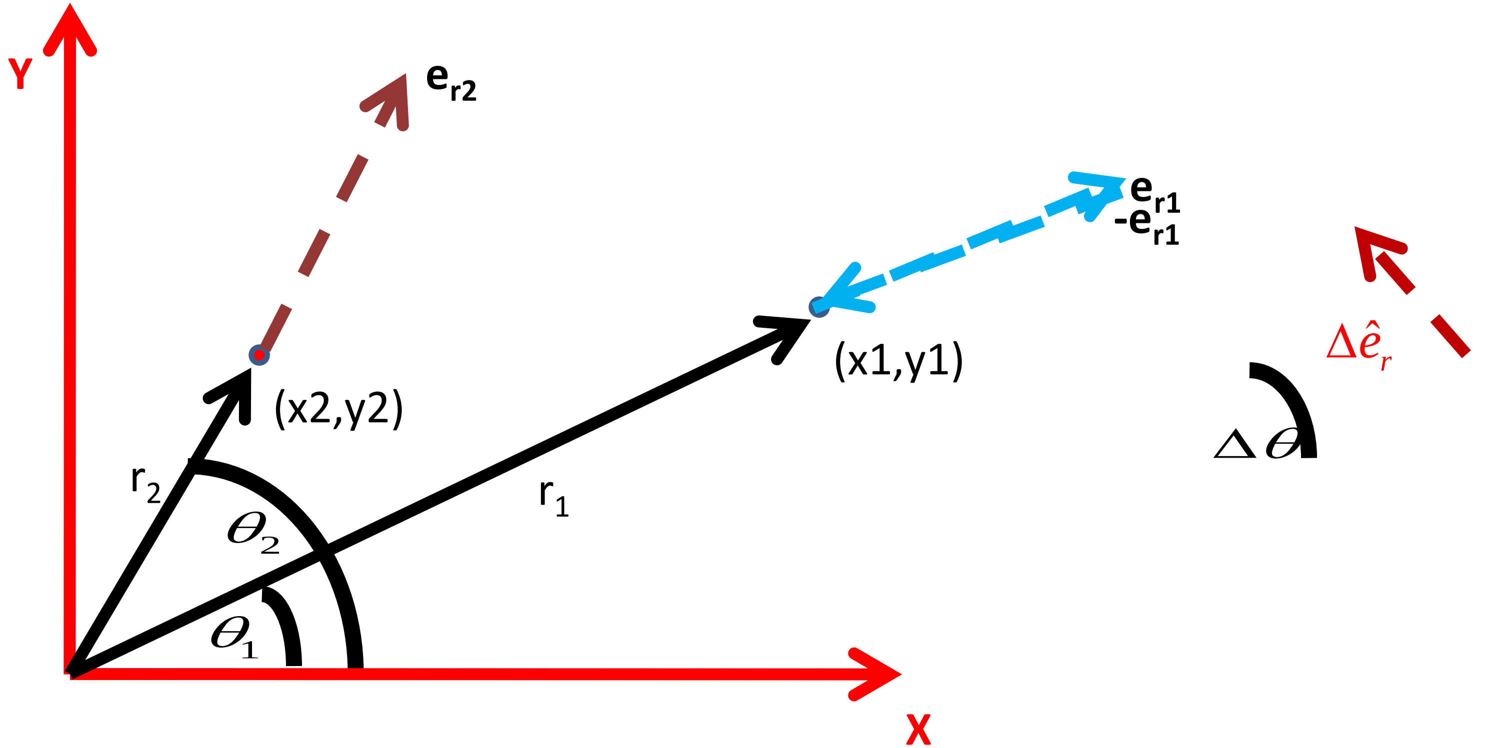
$$\frac{d\hat{e}_r}{dt} = \frac{d\hat{e}_r}{d\theta}\frac{d\theta}{dt}$$

$$\frac{d\hat{e}_r}{d\theta} = -\hat{e}_x \sin(\theta) + \hat{e}_y \cos(\theta) = \hat{e}_\theta$$

$$\frac{d\hat{e}_r}{dt} = \frac{d\theta}{dt}\hat{e}_\theta$$

$$\vec{V} = \frac{d(r\hat{e}_r)}{dt} = \frac{dr}{dt}\hat{e}_r + r\frac{d\theta}{dt}\hat{e}_\theta$$

$\frac{d\hat{e}_r}{d\theta}$ Through a geometrical consideration



$$\frac{d\hat{e}_r}{d\theta} \approx \lim_{\Delta\theta \to 0} \frac{\Delta\hat{e}_r}{\Delta\theta}$$

$$\Delta\hat{e}_r = \Delta\theta\hat{e}_\theta$$

$$\frac{d\hat{e}_r}{d\theta} = \hat{e}_\theta$$