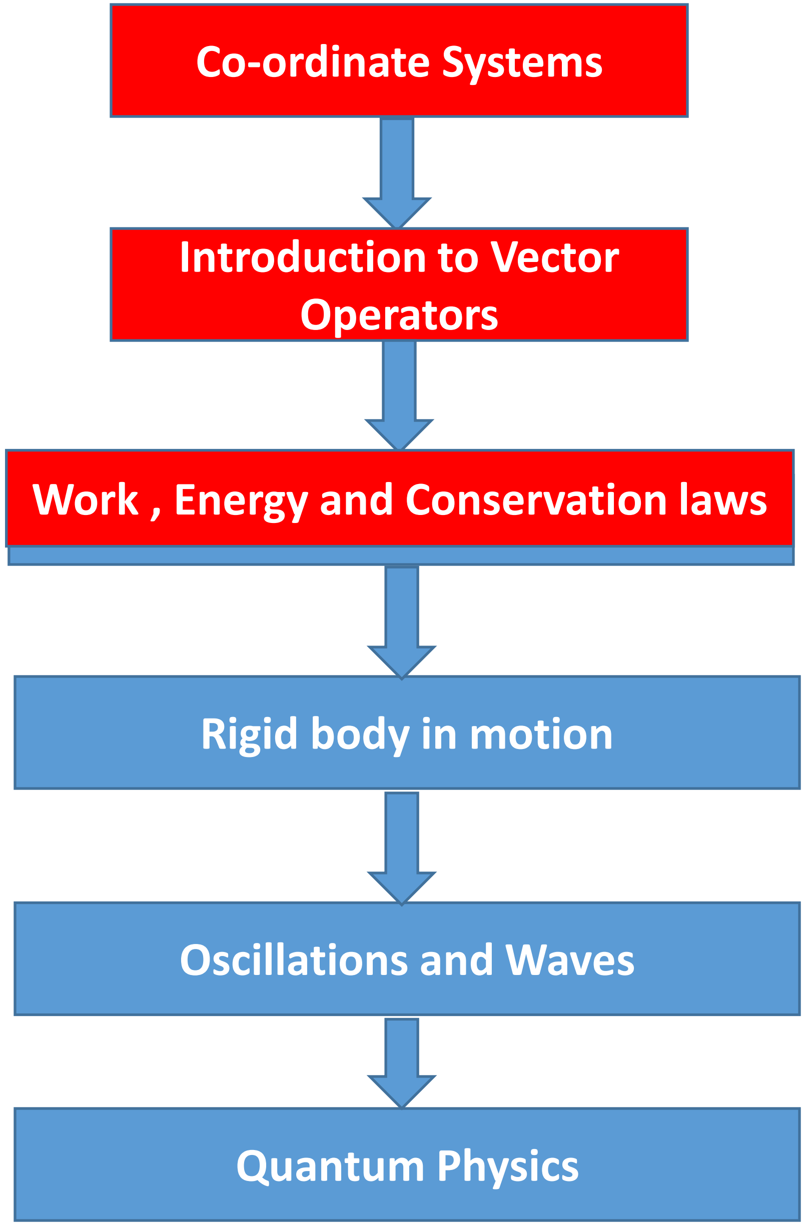
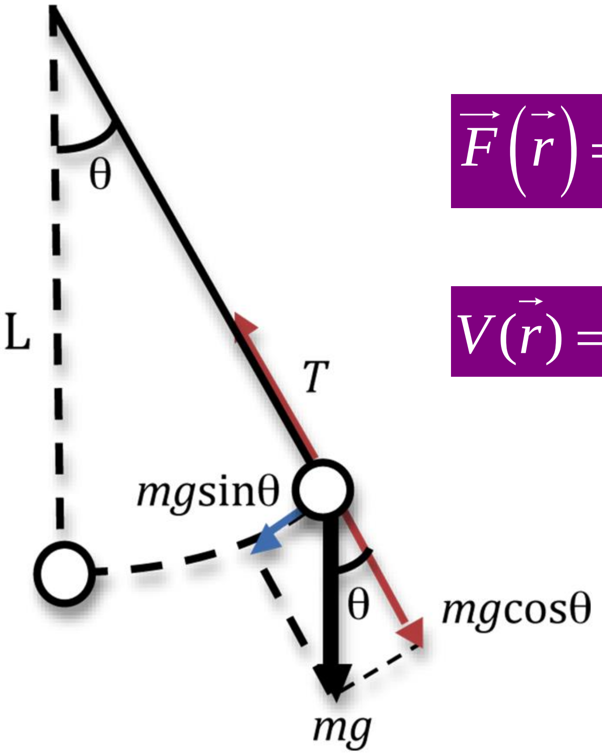
Highlights of the course



Chapter-3:Work , Energy and Conservation laws

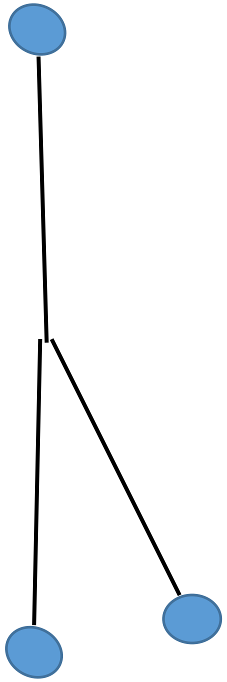
Concept of equilibrium

Simple Pendulum

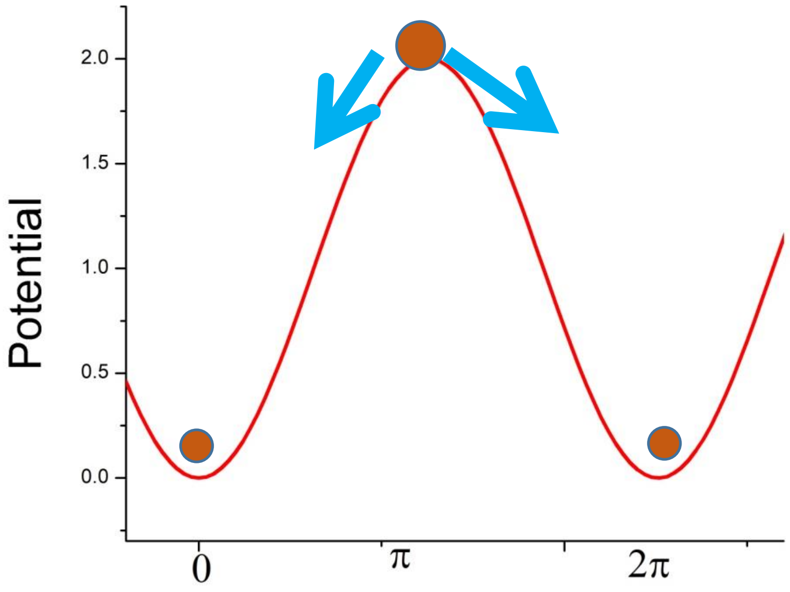


$$\vec{F}(\vec{r}) = -mg\sin\theta\hat{e}_\theta$$

$$V(\vec{r}) = mgl(1-\cos\theta)$$



Unstable equilibrium



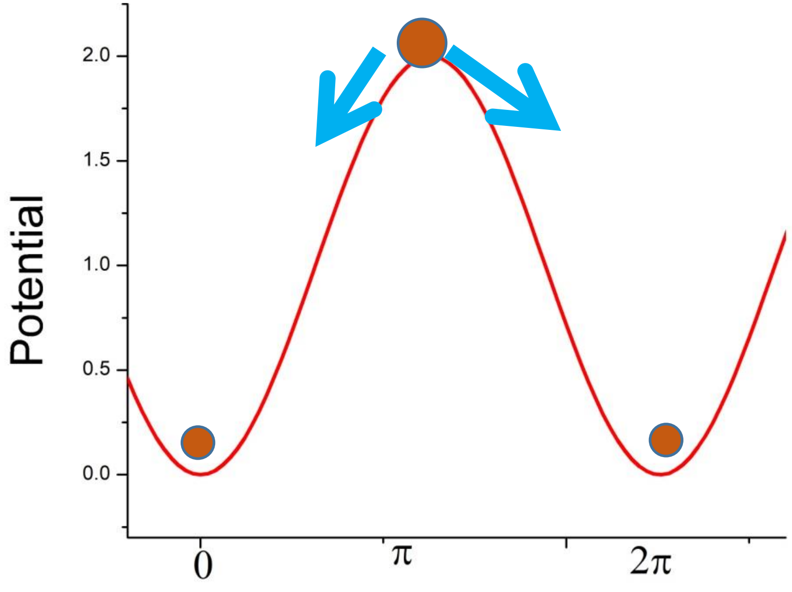
$\theta = \pi$

$\theta = 0$

$$V(\vec{r}) = mgl(1-\cos\theta)$$

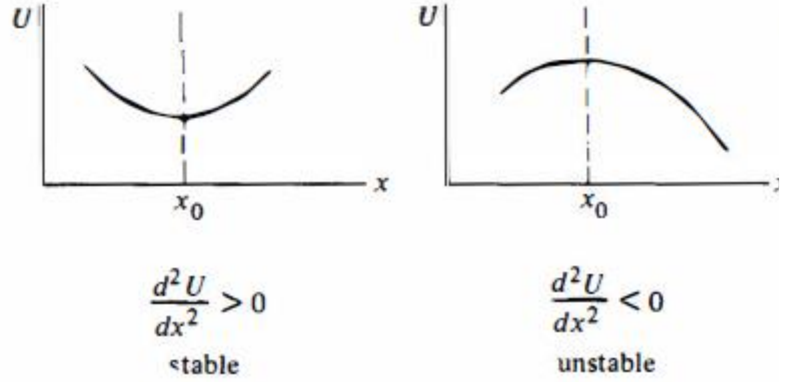
$$\frac{dV(\vec{r})}{d\theta} = mgl\sin\theta$$

$$\frac{d^2V(\vec{r})}{d\theta^2} = mgl\cos\theta$$



$$\frac{d^2V(\vec{r})}{d\theta^2} = mgl(\theta = 0)$$

$$\frac{d^2V(\vec{r})}{d\theta^2} = -mgl(\theta = \pi)$$



Plot $\dot{\theta}$ v/s θ ?

$$E = \frac{1}{2} m l^2 \dot{\theta}^2 + mgl(1 - \cos\theta)$$

$$E = \frac{1}{2} m l^2 \dot{\theta}^2 + mgl(1 - (1 - \frac{\theta^2}{2!} + ...)$$

$$E = \frac{1}{2} m l^2 \dot{\theta}^2 + mgl \frac{\theta^2}{2!}$$

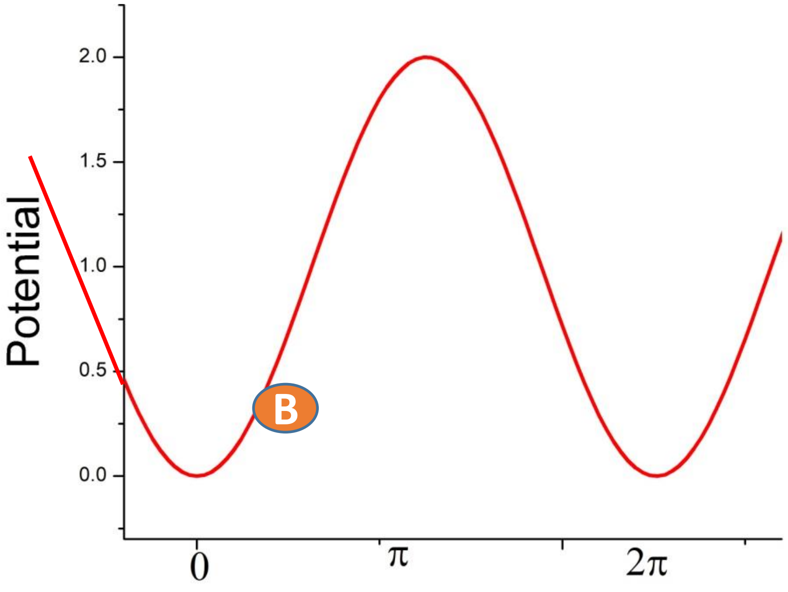
$$\frac{\dot{\theta}^2}{2E / ml^2} + \frac{\theta^2}{2E / mgl} = 1$$

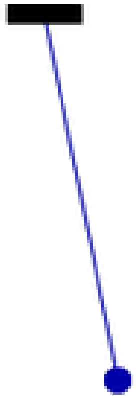
Equation of ellipse

For small values of ɵ the equation is of ellipse. But as ɵ becomes larger it Deviates from an ellipse

$$E = \frac{1}{2} m l^2 \dot{\theta}^2 + mgl \frac{\theta^2}{2!} - mgl \frac{\theta^4}{4!} + ...$$

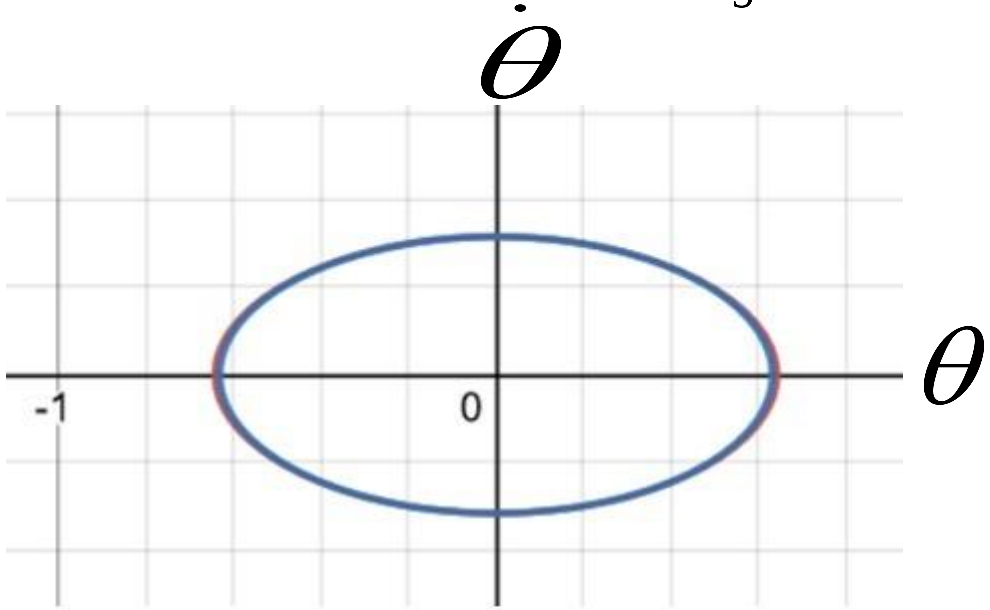
$$\frac{\dot{\theta}^2}{2E / ml^2} + \frac{\theta^2}{2E / mgl} - \frac{\theta^4}{24E / mgl} + ... = 1$$

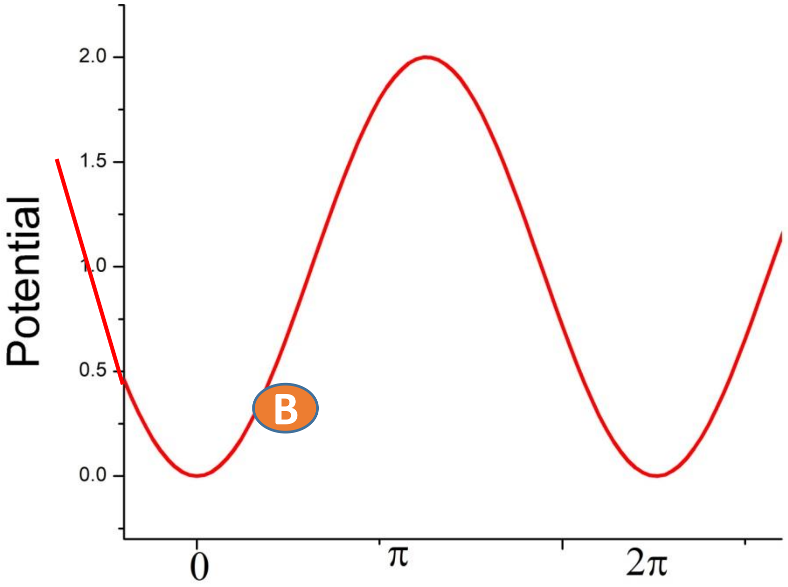


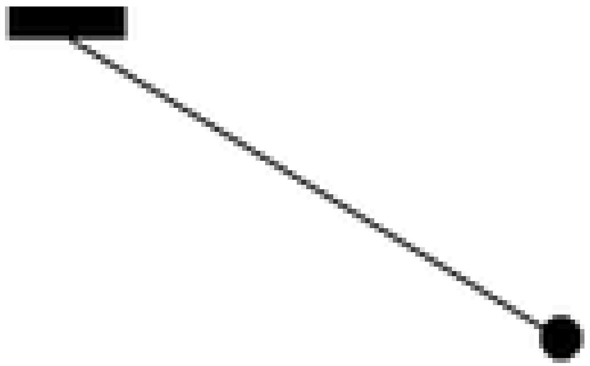


$$\frac{\dot{\theta}^2}{2E/ml^2}+\frac{\theta^2}{2E/mgl}=1$$

Small Oscillations

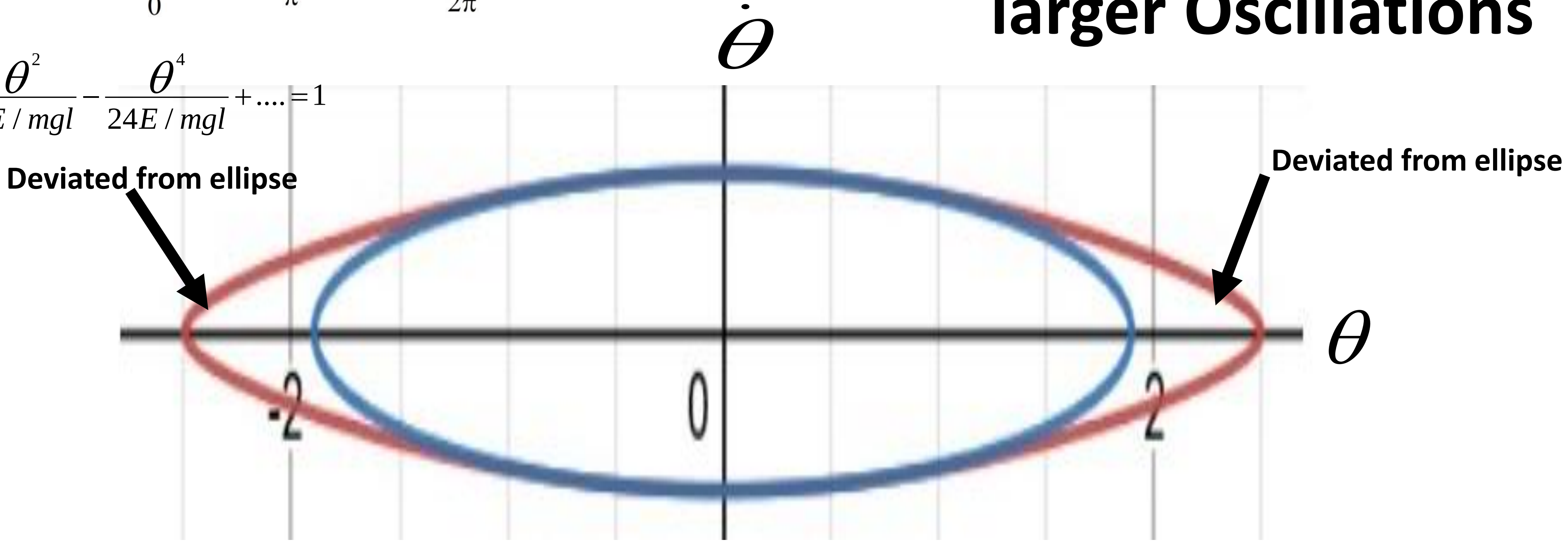




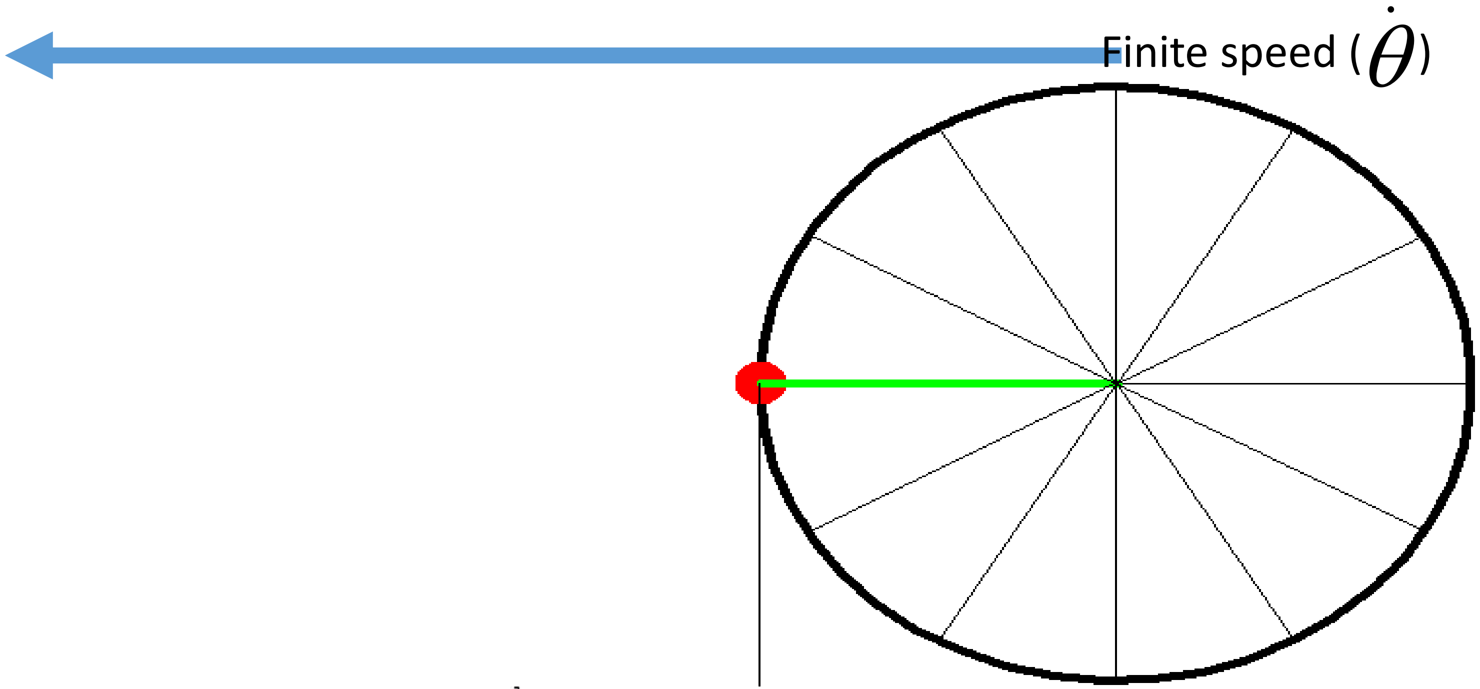


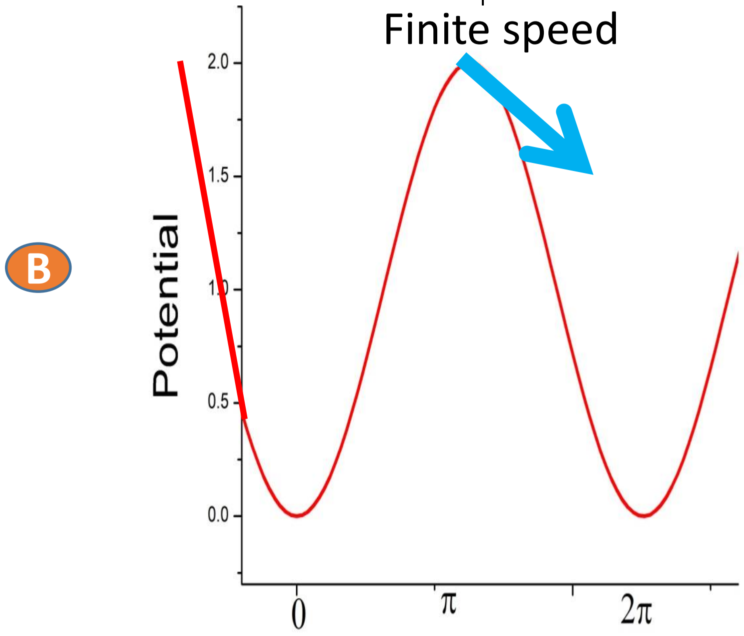
larger Oscillations

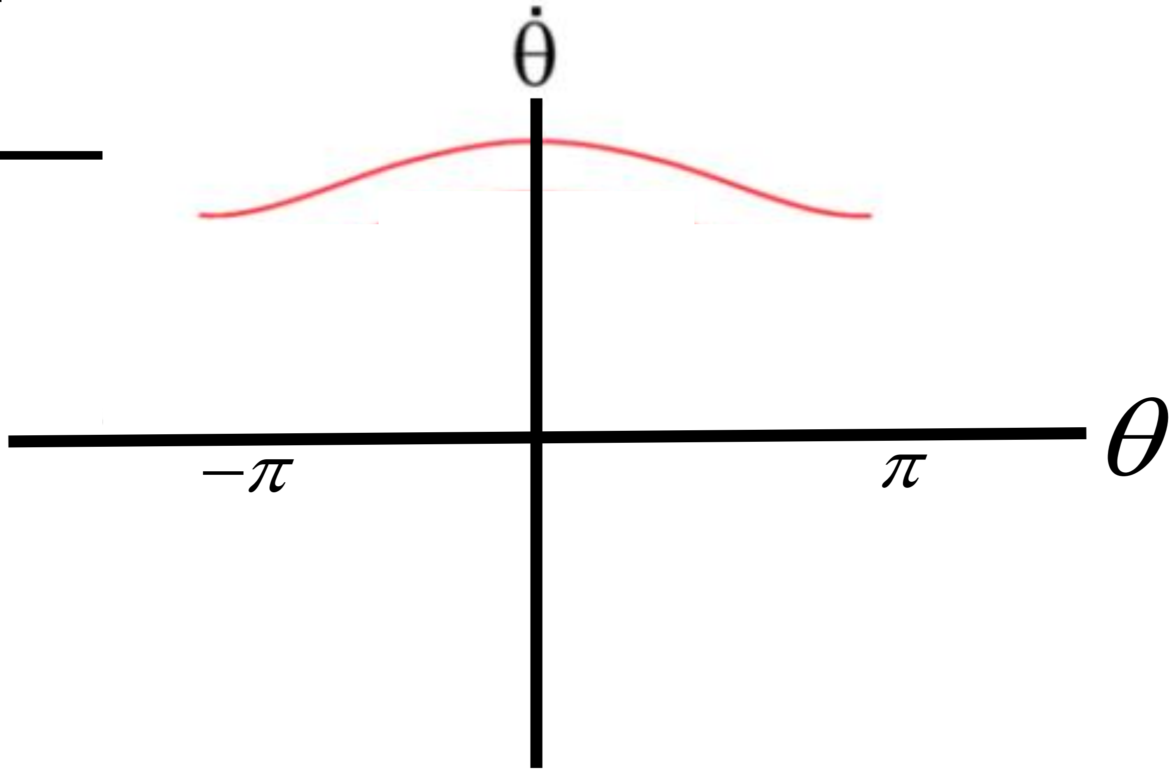
$$\frac{\dot{\theta}^2}{2E/ml^2}+\frac{\theta^2}{2E/mgl}-\frac{\theta^4}{24E/mgl}+....=1$$



Circular motion







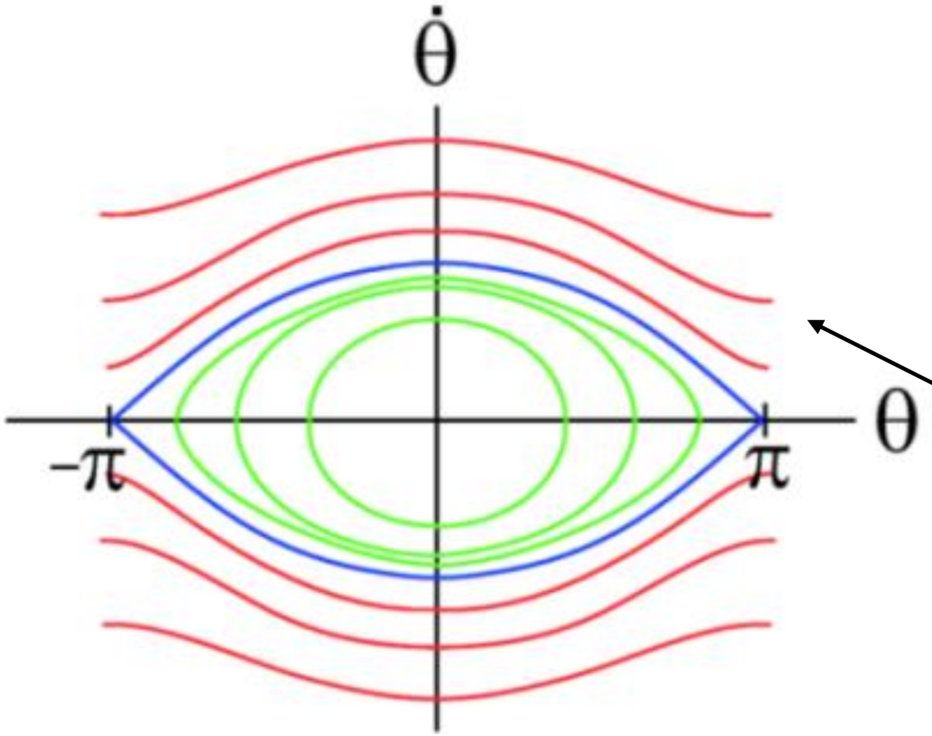
Plot $\dot{\theta}$ v/s θ

$$E = \frac{1}{2} m l^2 \dot{\theta}^2 + mgl \frac{\theta^2}{2!}$$

$$\frac{\dot{\theta}^2}{2E/ml^2} + \frac{\theta^2}{2E/mgl} = 1$$

$$E = \frac{1}{2} m l^2 \dot{\theta}^2 + mgl \frac{\theta^2}{2!} - mgl \frac{\theta^4}{4!} + \ldots\ldots.$$

$$\frac{\dot{\theta}^2}{2E/ml^2} + \frac{\theta^2}{2E/mgl} - \frac{\theta^4}{24E/mgl} + \ldots. = 1$$

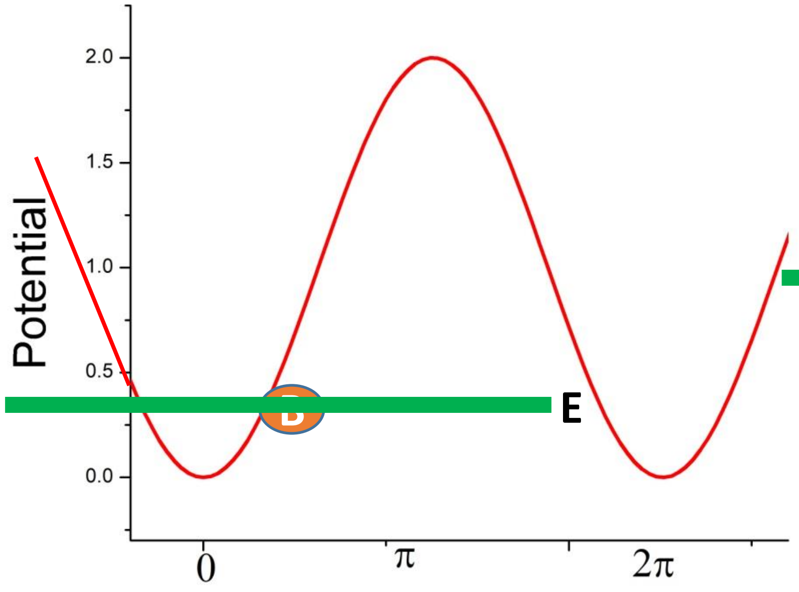


For larger E , motion Of the pendulum Becomes Circular motion

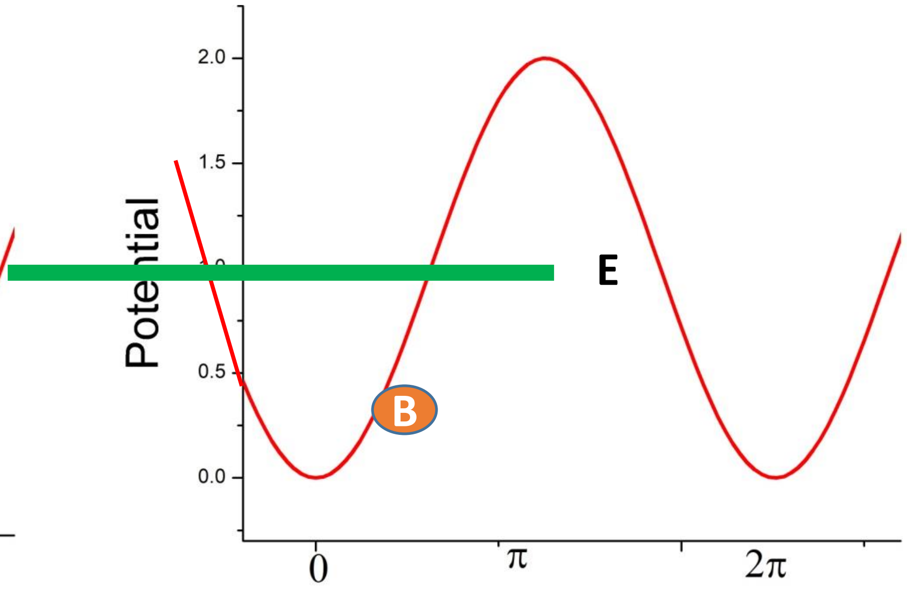
Simple Pendulum

Bounded motion

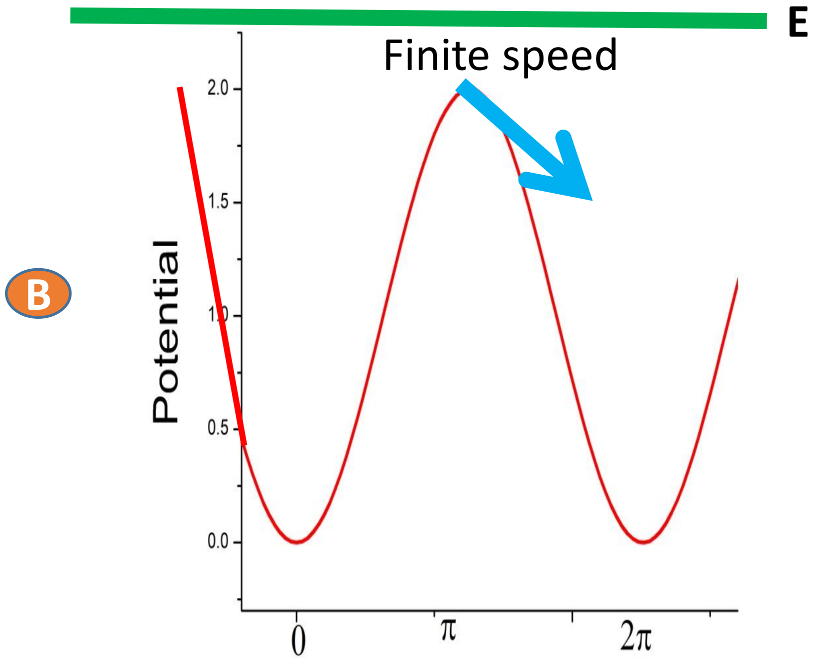
Unbounded motion



Small Oscillations



larger Oscillations



Circular motion

Concept of equilibrium

Duffing Oscillator (Georg Duffing, German engineer)

$$V(x) = \frac{1}{2}\beta x^2 + \frac{1}{4}\alpha x^4, \quad \beta < 0$$

Plot V(x) v/s x

How to Plot the graph?

1. Find Maxima and Minima
2. Find the zero crossing points
3. Imagine the function for smaller and larger values of x

1. Maxima and Minima

Condition for maxima and minima of a function

A function V(x) is maximum when $\frac{dV(x)}{dx} = 0$ and $\frac{d^2V(x)}{dx^2} < 0$

A function V(x) is minimum when $\frac{dV(x)}{dx} = 0$ and $\frac{d^2V(x)}{dx^2} > 0$

1. Maxima and Minima

$$V(x) = \frac{1}{2}\beta x^2 + \frac{1}{4}\alpha x^4$$

To find maxima and minima:

$$\frac{dV(x)}{dx} = 0$$

$$\beta x + \alpha x^3 = 0$$

$$\Rightarrow \quad x(\beta + \alpha x^2) = 0$$

$$\Rightarrow \quad x = 0 \text{ and } x = \pm\sqrt{\frac{-\beta}{\alpha}}$$

1. Maxima and Minima

$$\frac{d^2V(x)}{dx^2} = \beta + 3\alpha x^2$$

$$\text{When x=0, } \frac{d^2V(x)}{dx^2} = \beta < 0 \Rightarrow \text{Maxima}$$

$$\text{When x} = \pm\sqrt{\frac{-\beta}{\alpha}}, \ \frac{d^2V(x)}{dx^2} = -2\beta > 0$$

$$(\text{since}\beta < 0) \Rightarrow \text{Minima}$$

2. Zero crossing points

$$V(x) = 0$$

$$\frac{1}{2}\beta x^2 + \frac{1}{4}\alpha x^4 = 0$$

$$\Rightarrow \quad x = \pm\sqrt{\frac{-2\beta}{\alpha}} \text{ and } x=0$$

Plot for small and large values of x

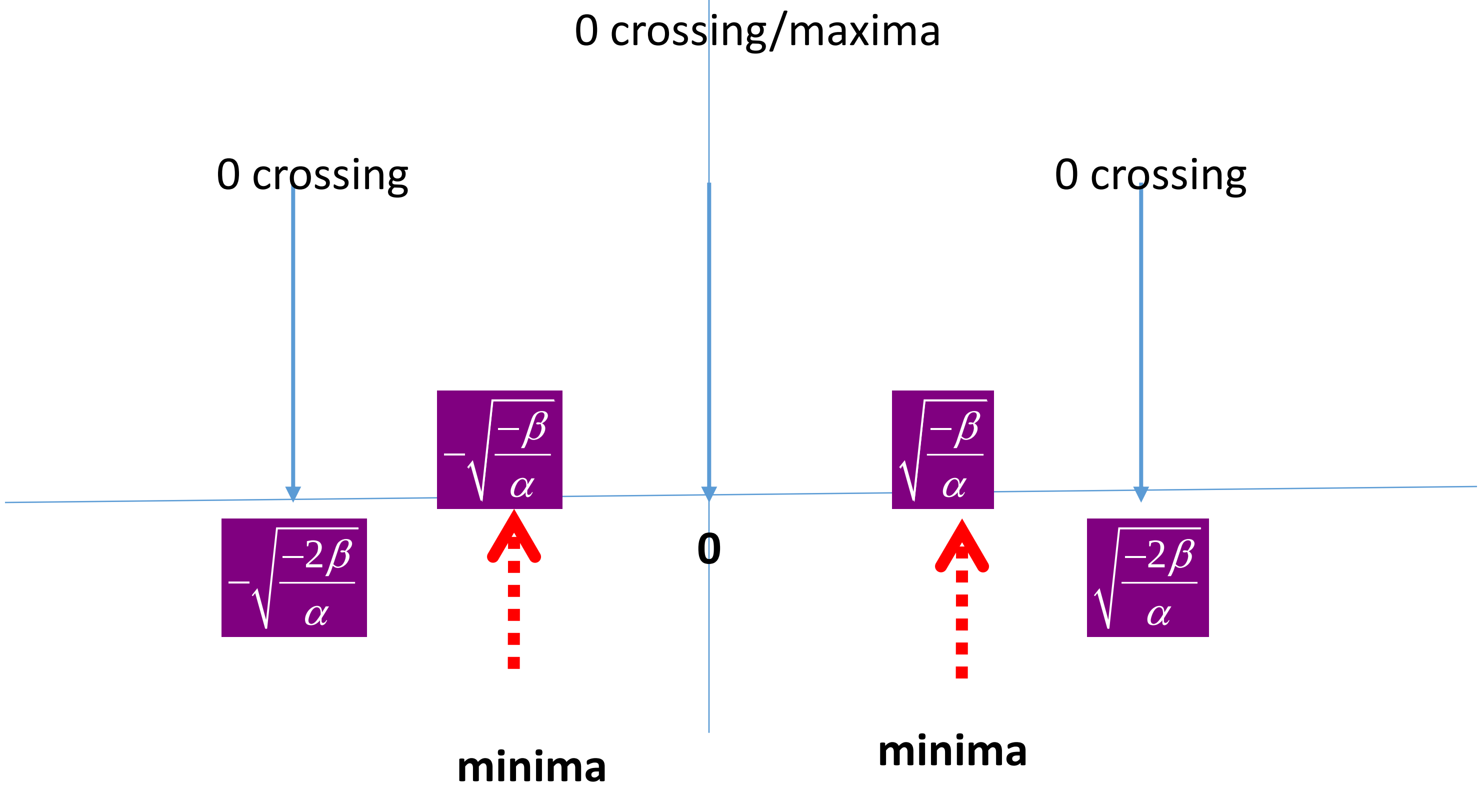
$$V(x) = \frac{1}{2}\beta x^2 + \frac{1}{4}\alpha x^4$$

1. Maxima at x =0

Minima at $-\sqrt{\frac{-\beta}{\alpha}}$

and $\sqrt{\frac{-\beta}{\alpha}}$

2. Zero crossing at 3 points, x = 0, $-\sqrt{\frac{-2\beta}{\alpha}}$ $\sqrt{\frac{-2\beta}{\alpha}}$

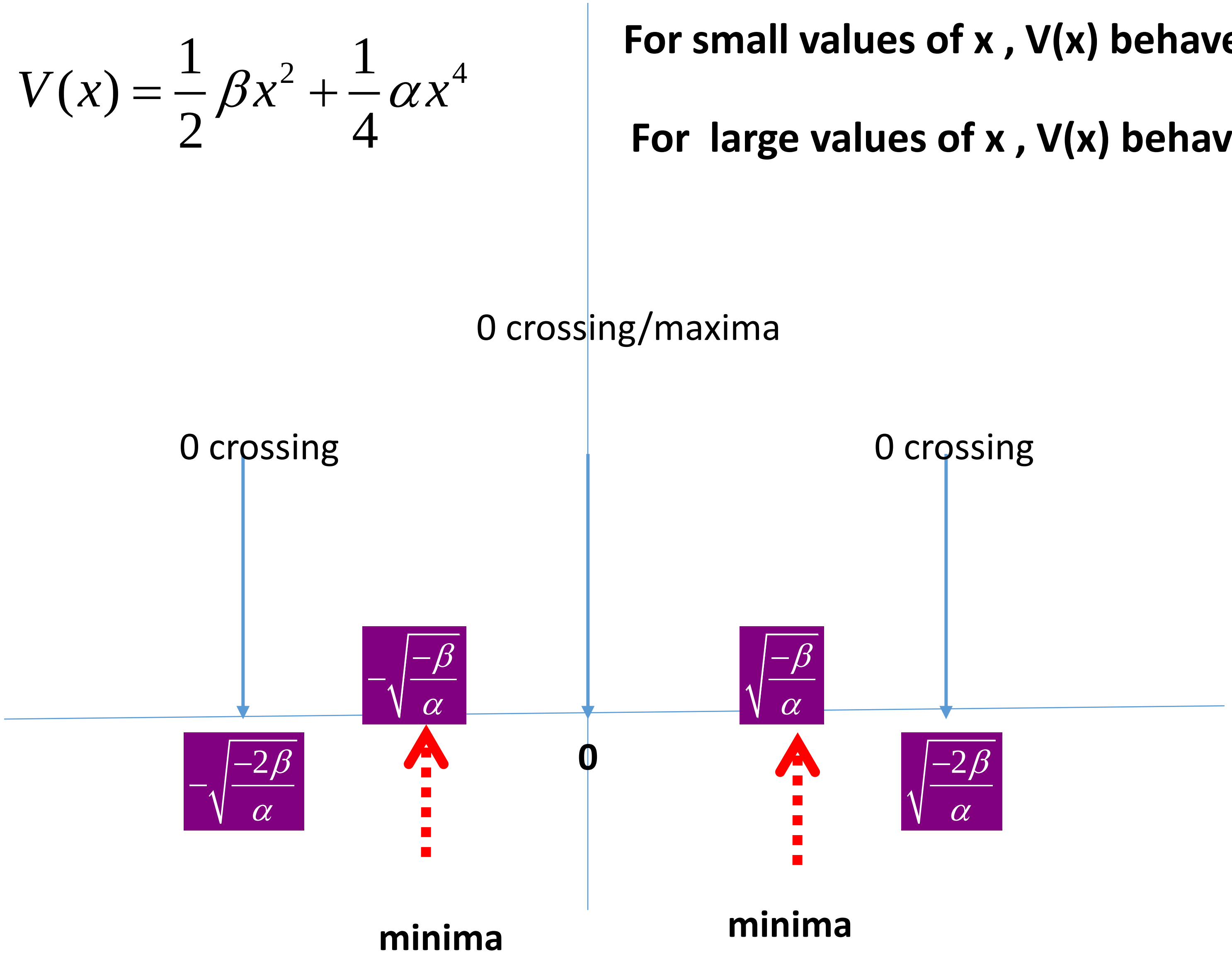


Plot for small and large values of x

$$V(x) = \frac{1}{2}\beta x^2 + \frac{1}{4}\alpha x^4$$

For small values of x , V(x) behaves as x^2

For large values of x , V(x) behaves as x^4

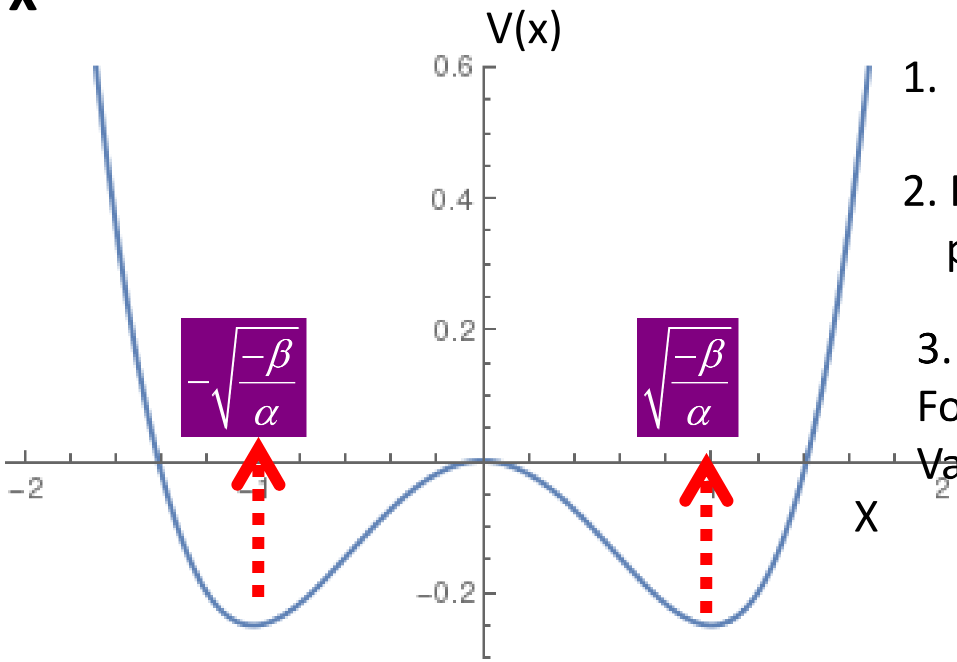


Concept of equilibrium

Duffing Oscillator

$$V(x) = \frac{1}{2}\beta x^2 + \frac{1}{4}\alpha x^4, \quad \beta < 0$$

Plot V(x) v/s x



1. Maxima at x =0

Minima at $-\sqrt{\frac{-\beta}{\alpha}}$

and $\sqrt{\frac{-\beta}{\alpha}}$

2. Zero crossing at 3 points, x = 0, $-\sqrt{\frac{-2\beta}{\alpha}}$ $\sqrt{\frac{-2\beta}{\alpha}}$

To Plot the graph

1. Find Maxima and Minima

2. Find the zero crossing points

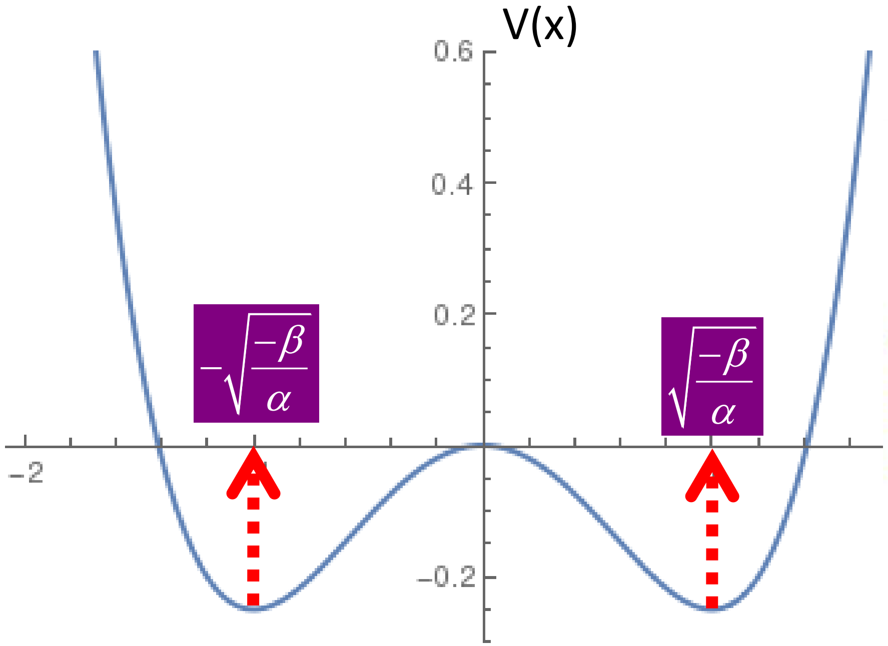
3. Imagine the function For smaller and larger Values of x

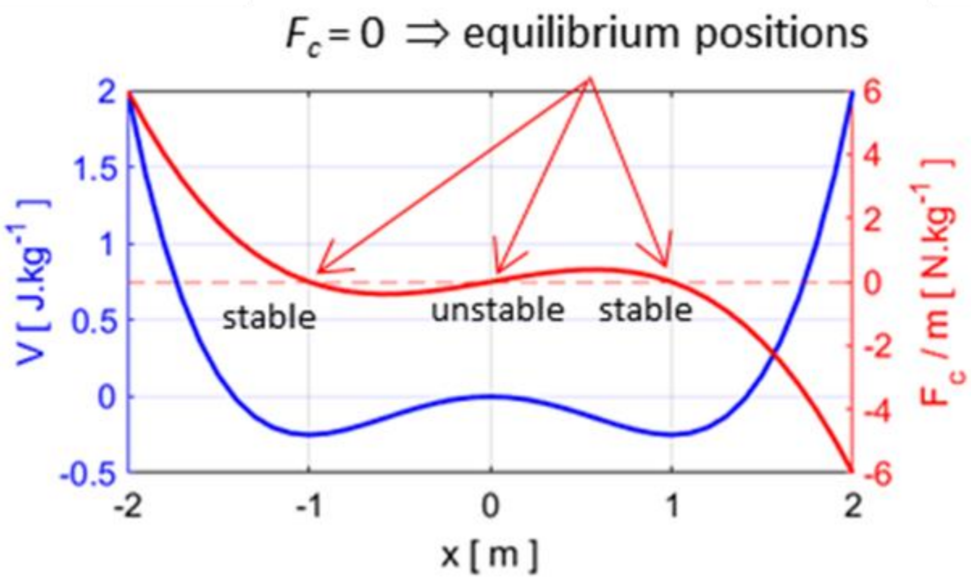
Concept of equilibrium

Duffing Oscillator

$$V(x) = \frac{1}{2}\beta x^2 + \frac{1}{4}\alpha x^4, \quad \beta < 0$$

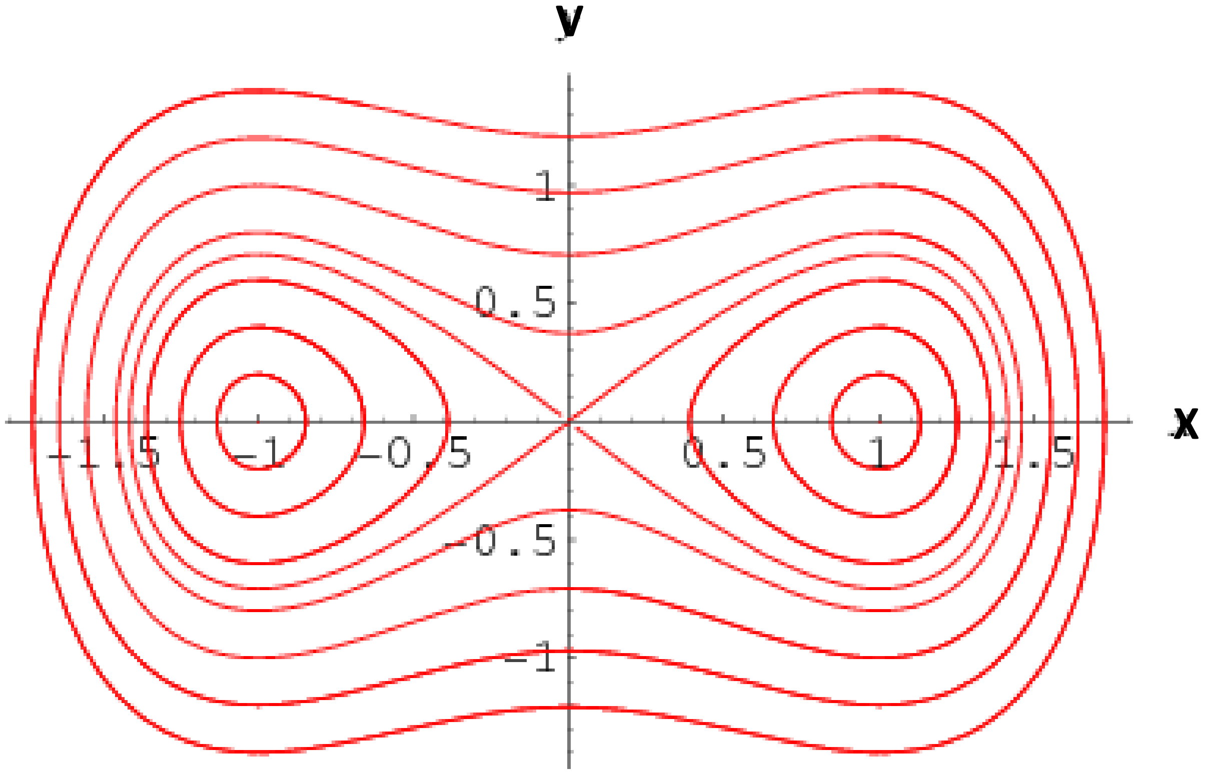
Component of Force $F(x) = -\frac{dV(x)}{dx} = -\beta x - \alpha x^3$



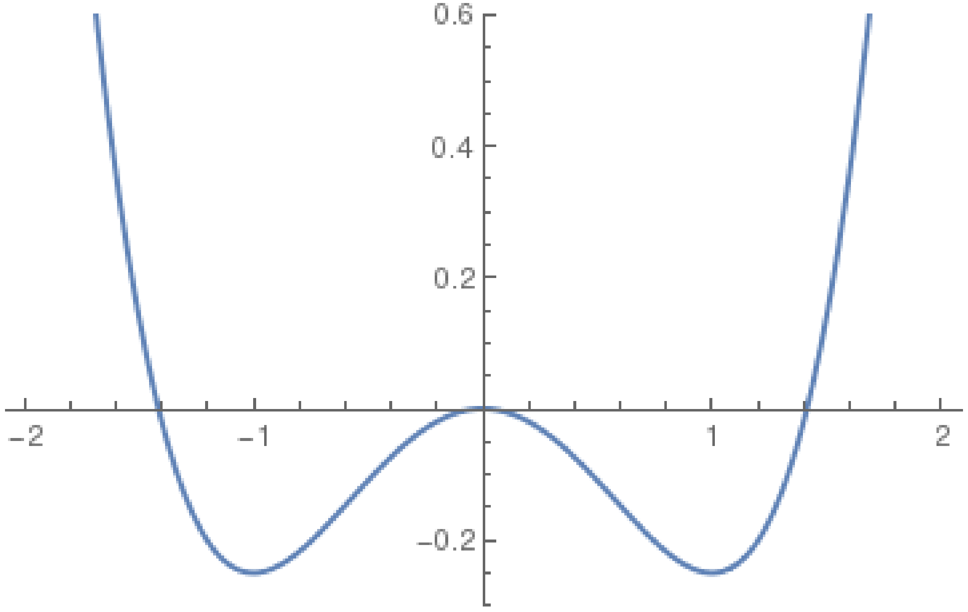


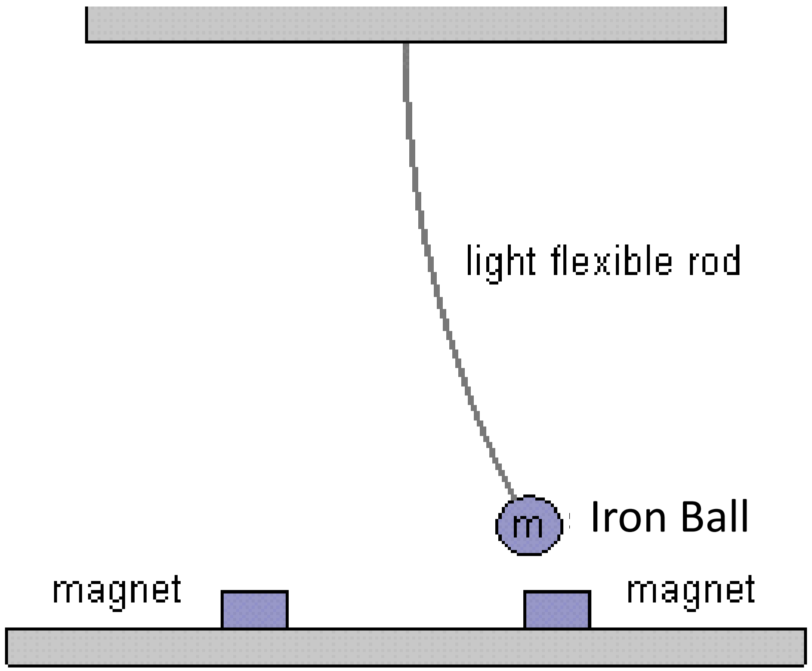
Velocity Vs. Position plot

$$E = \frac{1}{2}\beta x^2 + \frac{1}{4}\alpha x^4 + \frac{1}{2}mv^2$$

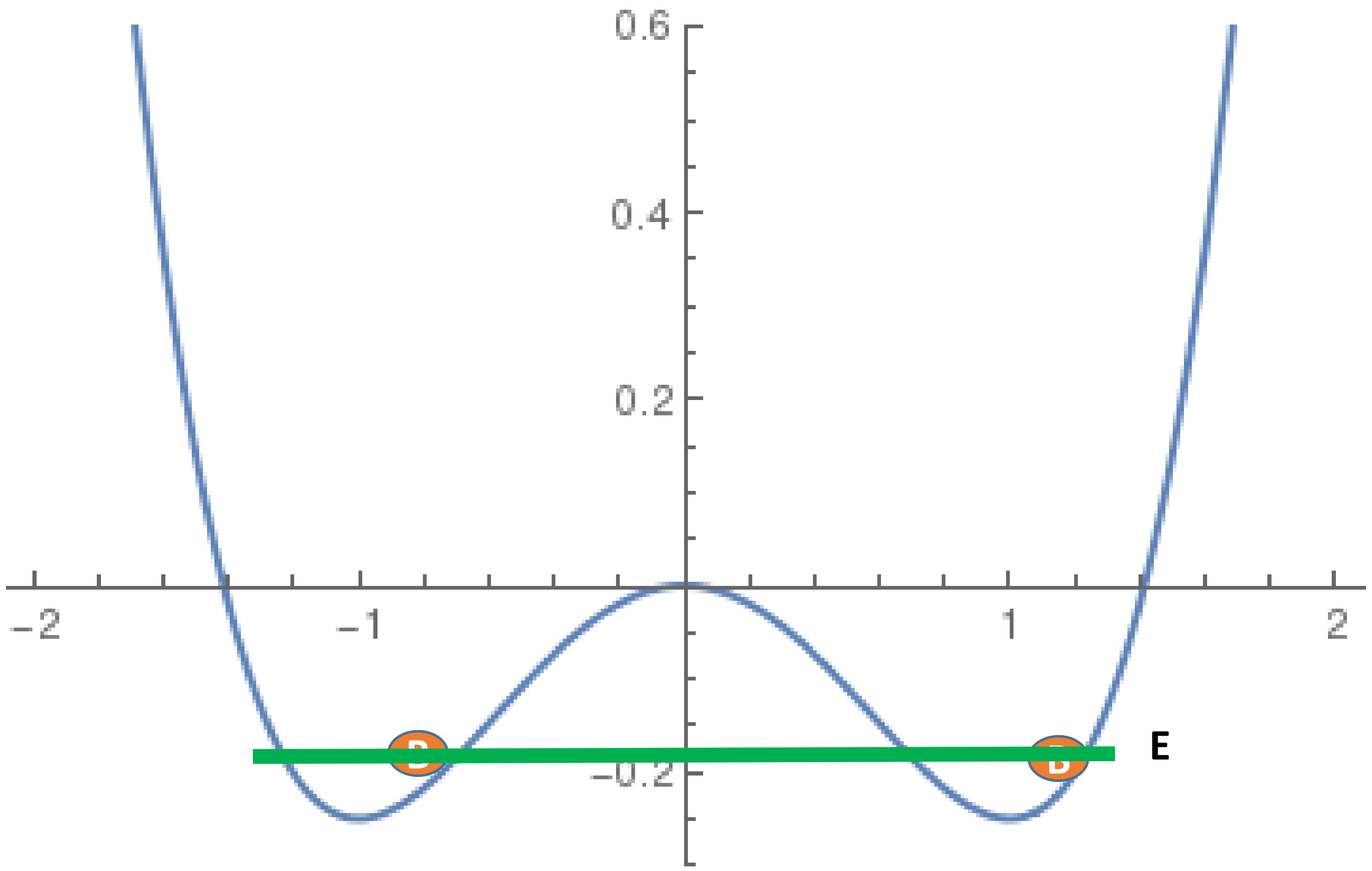


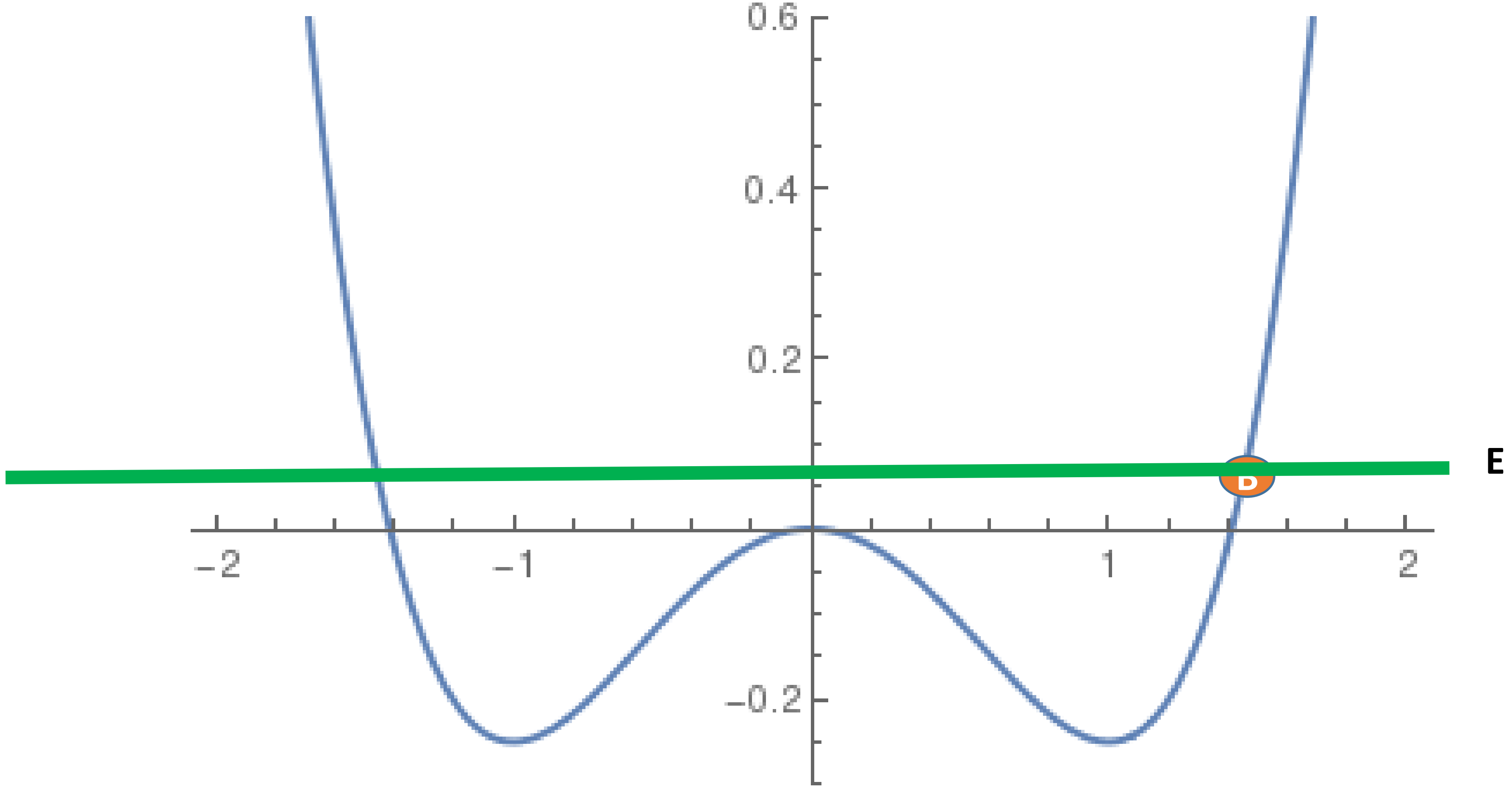
Physical Example of double well





Duffing Oscillator (Georg Duffing, German engineer)




0.6
0.4
0.2
−0.2
−2
−1
1
2
E
B

Velocity Vs. Position plot (Summary)