Introduction to Deep Learning

Neural Networks

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Human brain vs von Neumann computer

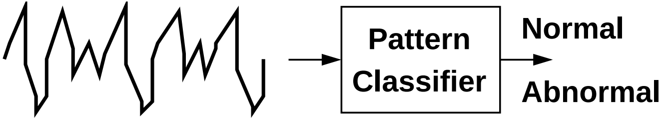
- Massive parallelism
- Distributed representation and computation
- Learning ability
- Generalization ability
- Adaptability
- Inherent contextual information processing
- Fault tolerance
- Low energy consumption

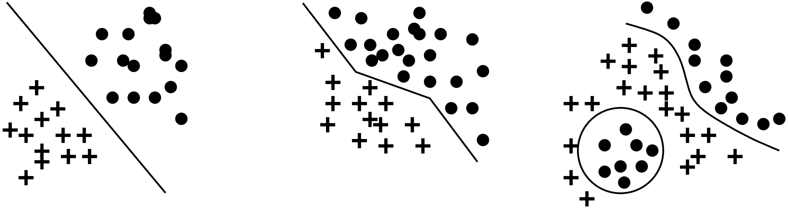
Computer vs Brain

	von Neumann	Neural system
Processor	Complex, high speed, one or a few	Simple, low speed, a large number
Memory	Separate from processor, Localized, Noncontent addressable	Integrated into processor, Distributed, Content addressable
Computing	Centralized, sequential, stored program	Distributed, parallel, self-learning
Reliability	Very vulnerable	Robust
Expertise	Numeric and symbolic manipulations	Perceptual problems
Operating environment	Well defined, well constrained	Poorly defined, unconstrained

Artificial Neuron: Applications

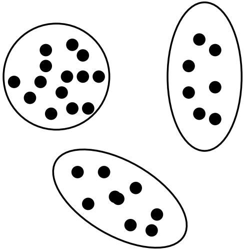
- **Pattern classification**





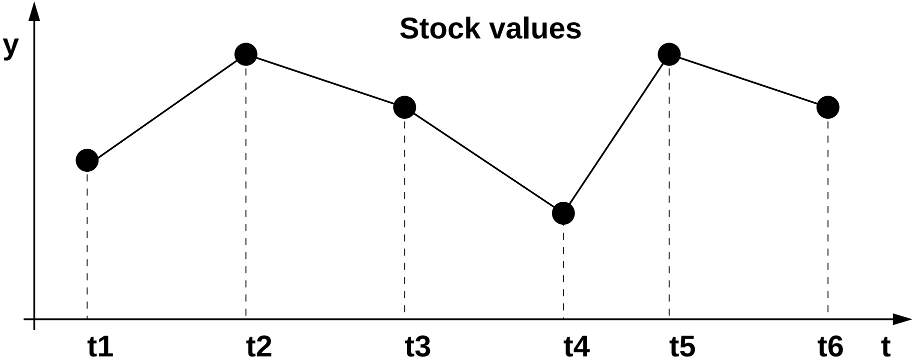
Artificial Neuron: Applications

- **Clustering/categorization**



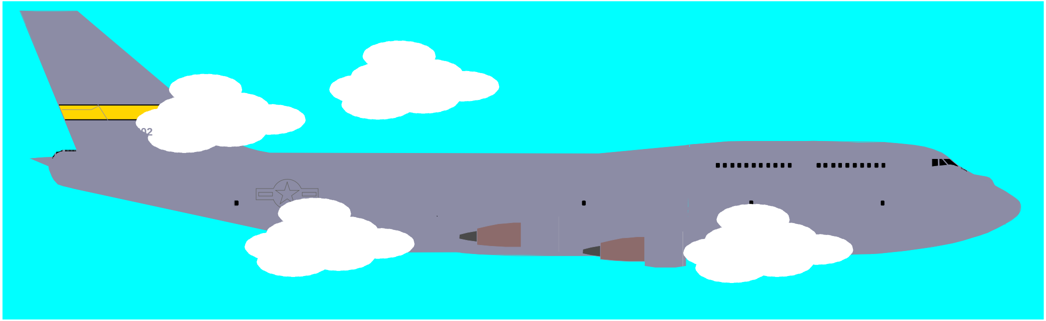
Artificial Neuron: Applications

- **Prediction**



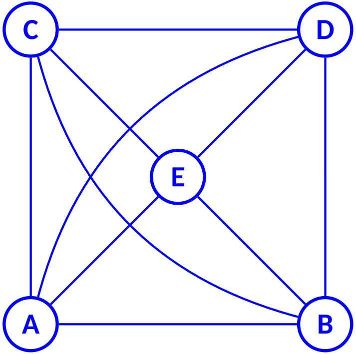
Artificial Neuron: Applications

- **Retrieval**



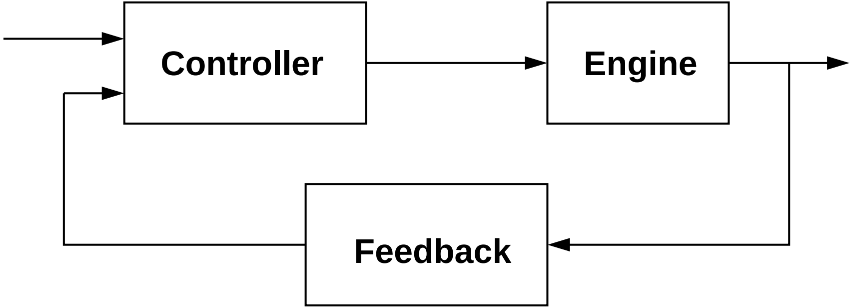
Artificial Neuron: Applications

- **Optimization**



Artificial Neuron

- Control



History

- Started in 1940s by McCulloch and Pitt
- Rosenblatt perceptron convergence theorem (1960)
- In 1980s ANN started gaining popularity
- Again became popular after 2006

Biological Neuron

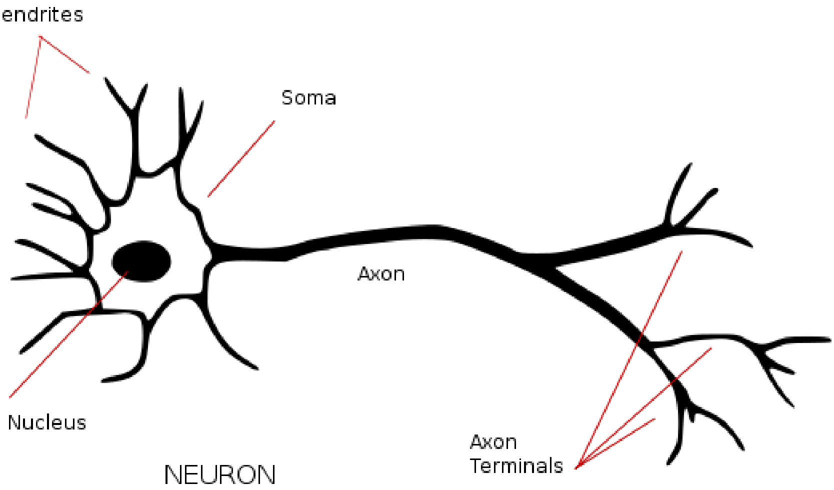
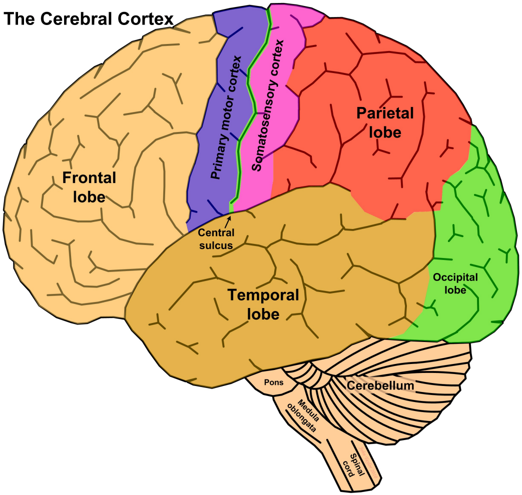


Image source: Internet

Cerebral cortex

- It is a flat sheet of neurons about 2-3 millimeter thick with surface area is 2200 cm^2
  - Twice the area of computer keyboard
- It contains around 10^{11} neurons
  - Number of stars in the Milky-way
- Each neuron is connected to 10^3-10^4 other neurons
- Total connections is around 10^{14}-10^{15}
- Connectionist model

Human brain



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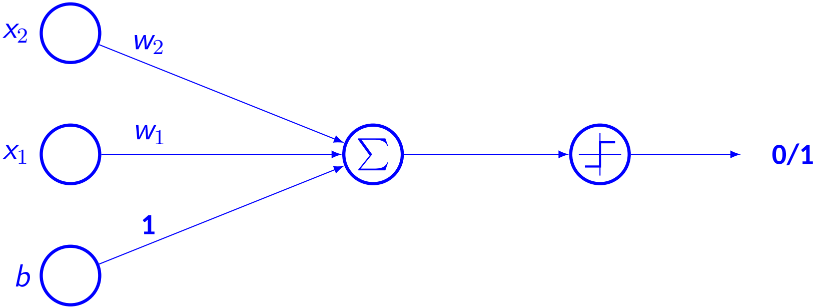
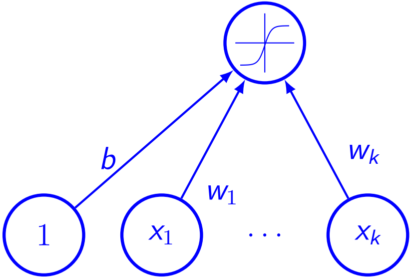
Neuron

- **One of the primitive models**

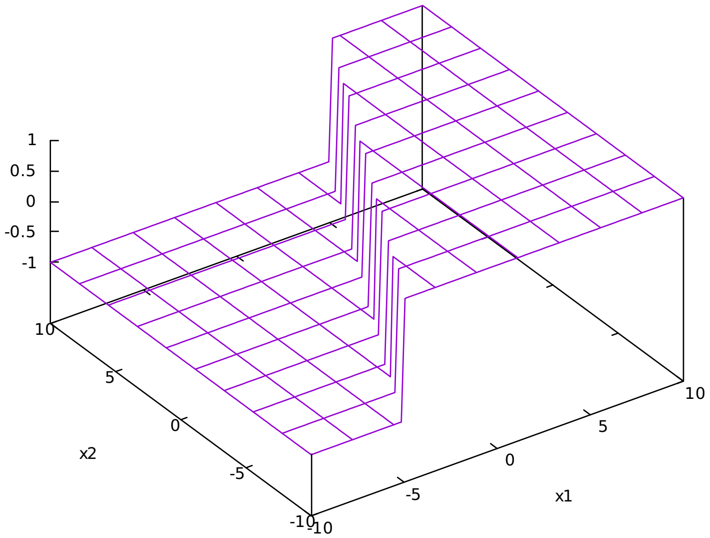


Artificial Neuron

- **Neuron pre-activation function**
  - $a(\mathrm{x}) = \sum_i w_i x_i + b = b + \mathrm{w}^T \mathrm{x}$
- **Neuron output activation function**
  - $h(\mathrm{x}) = g(a(\mathrm{x})) = g\left(\sum_i w_i x_i + b\right)$
- **Notations**
  - w — **Weight vector**
  - b — **Neuron bias**
  - $g(.)$ — **Activation function**

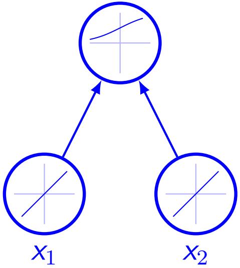


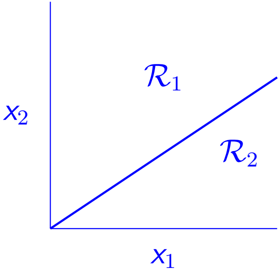
Physical interpretation



Classification using single neuron

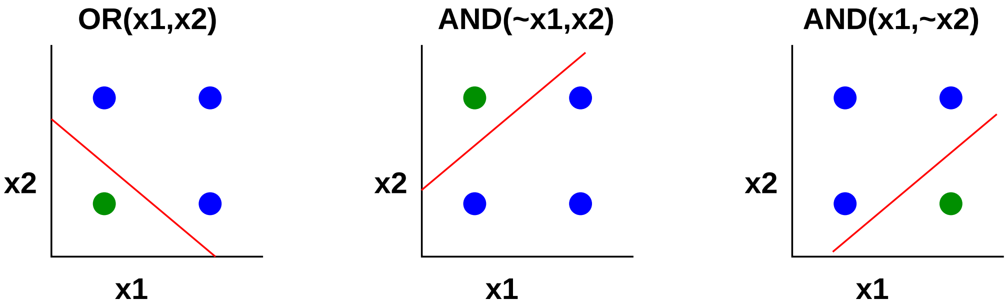
- Single neuron can do binary classification
  - Also known as logistic regression classifier





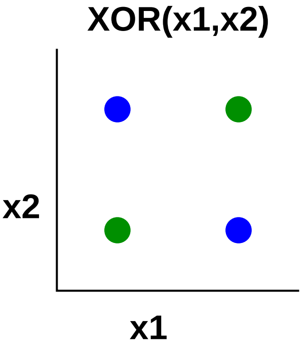
Artificial neuron

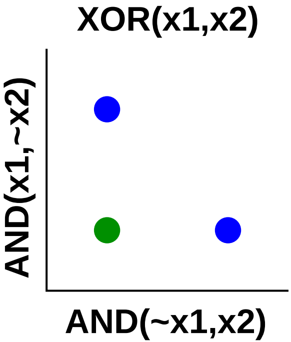
- Can solve linearly separable problems



Artificial neuron: XOR problem

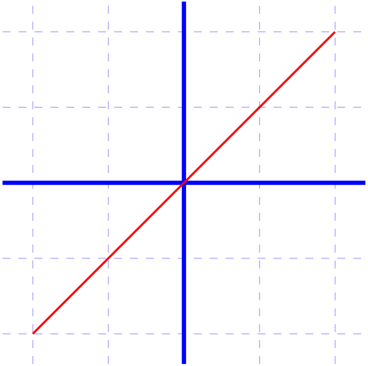
- There are issues for linear separation





Activation function

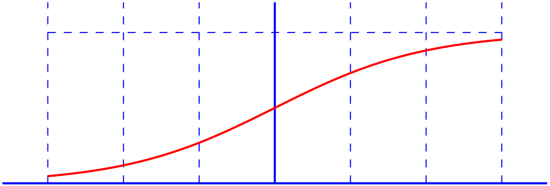
- Linear activation function
  - Not very interesting
  - No change in values
  - Huge range



$g(x) = x$

Activation function

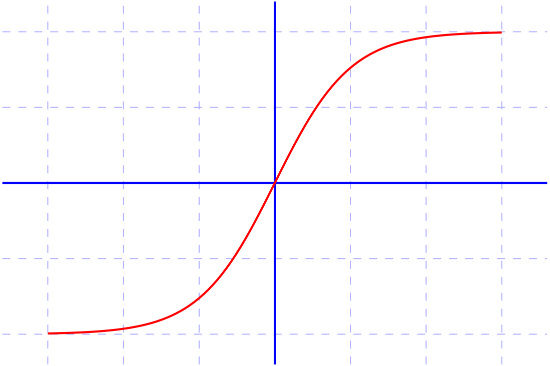
- Sigmoid function
  - Values lie between 0 and 1
  - Strictly increasing function
  - Bounded



$$g(x) = \mathbf{sigm}(x) = \frac{1}{1+\exp(-x)}$$

Activation function

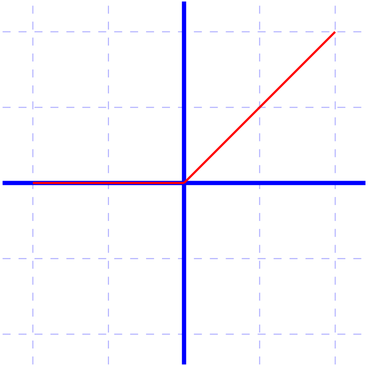
- **Hyperbolic Tangent (Tanh) function**
  - **Can be positive or negative**
  - **Values lie between -1 and 1**
  - **Strictly increasing function**
  - **Bounded**



$$g(x) = \mathbf{tanh}(x) = \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)}$$

Activation function

- Rectified linear activation function
  - Bounded below by 0
  - Strictly increasing function
  - Not upper bounded



$$g(x) = \textbf{reclin}(x) = \max(0, x)$$

Single hidden layer neural network

- **Hidden layer pre-activation**

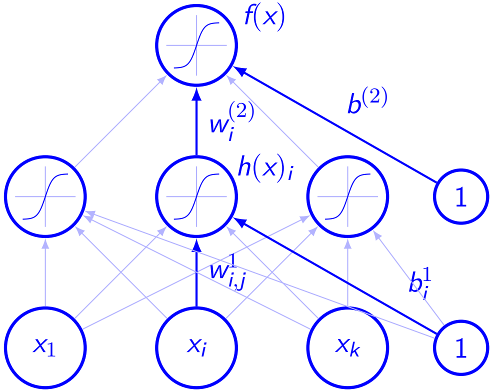
$$a(\mathbf{x}) = \mathbf{b}^1 + \mathbf{w}^1\mathbf{x}$$

- **Hidden layer activation**

$$h(\mathbf{x}) = g(a(\mathbf{x}))$$

- **Output layer activation**

$$f(\mathbf{x}) = o\left(b^{(2)} + \mathbf{w}^{(2)\,T} h^1(\mathbf{x})\right)$$



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Multi layer neural network

- **Pre-activation in layer**

  $k > 0\ (\mathrm{h}^{(0)}(\mathrm{x}) = x)$

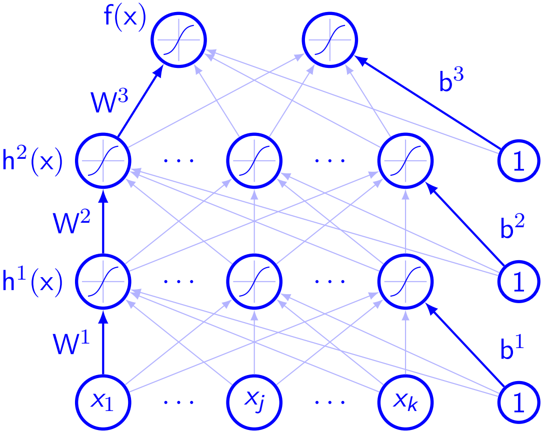
$$\mathrm{a}^{(k)}(\mathrm{x}) = \mathrm{b}^{(k)} + \mathrm{W}^{(k)}\mathrm{h}^{(k-1)}\mathrm{x}$$

- **Hidden layer activation**

$$\mathrm{h}^{(k)}(\mathrm{x}) = \mathrm{g}(\mathrm{a}^{(k)}(\mathrm{x}))$$

- **Output layer activation**

$$\mathrm{h}^{(L+1)}(\mathrm{x}) = o(\mathrm{a}^{(L+1)}(\mathrm{x})) = \mathrm{f}(\mathrm{x})$$



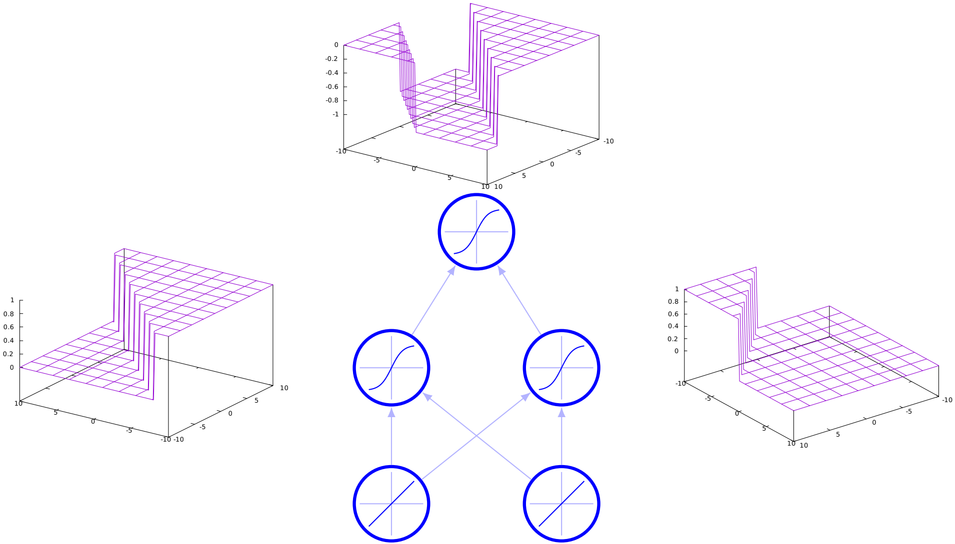
Multiclass classification

- Need multiple outputs that is one neuron for each class
- Need to determine probability of $p(y = c|\mathrm{x})$
- Softmax activation function is used at the output

$$\mathrm{o}(\mathrm{a}) = \textbf{softmax}(a) = \left[\frac{\exp(a_1)}{\sum_c \exp(a_c)} \quad \frac{\exp(a_2)}{\sum_c \exp(a_c)} \quad \cdots \quad \frac{\exp(a_c)}{\sum_c \exp(a_c)} \right]^T$$

- Strictly positive
- Sum to 1
- Class having the highest probability will be the predicted output

Capacity of neural network



Capacity of neural network

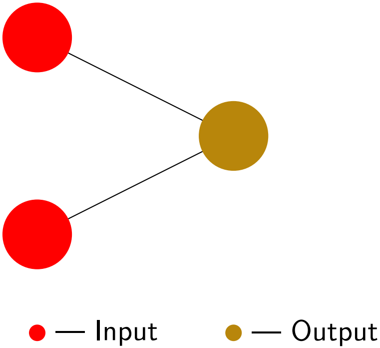
- **Universal approximation theorem (Hornik,1991)**
  - **A single hidden layer neural network with a linear output unit can approximate any continuous function arbitrarily well, given enough hidden units.**
- **The result is applicable for other hidden layer activation functions such as sigmoid, tanh, etc.**
- **This is a promising result, but it does not say that there is a learning algorithm to find the necessary parameter values!**

Types of Neural Network

- Feed forward neural network
- Radial basis function network
- Recurrent neural network
- Boltzmann machine
- Long short term memory network
- and many more

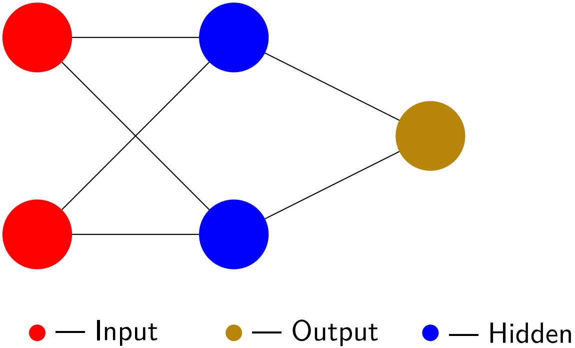
Perceptron

- **Simplest form of neural network**



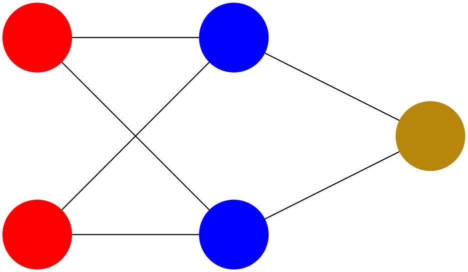
Feed Forward

- **With single hidden layer only**



Radial Basis Function

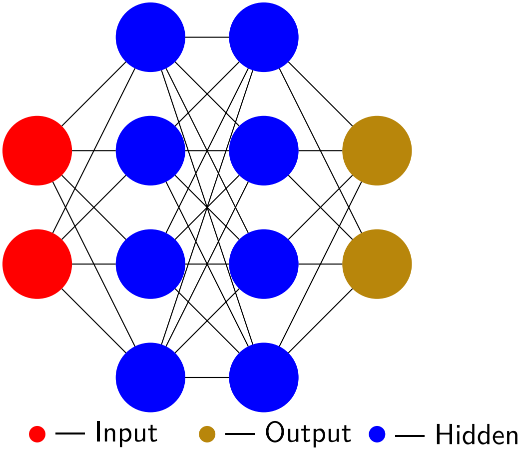
- Typically it will have 3 layers
- Distance from a center vector is computed
- Radial basis function as activation $o = \sum_i a_i \exp(\beta(\mathrm{x} - \mathrm{c})^2)$
- Usage - function approximation, time series prediction, classification, system control



● — Input ● — Output ● — Hidden

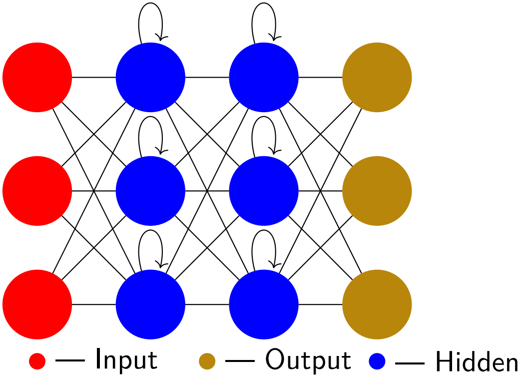
Deep Feed Forward

- Can have multiple hidden layers
- More complicated functions can be represented



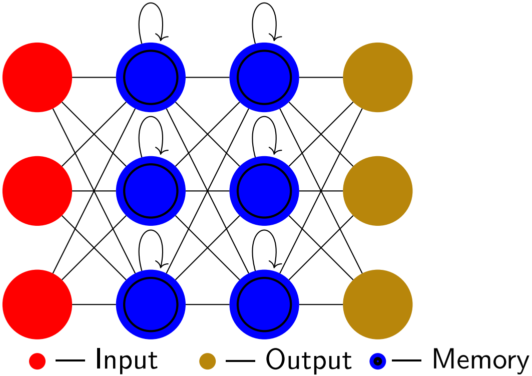
Recurrent Neural Network

- It has feedback loop
- Used for modelling dependencies such as temporal



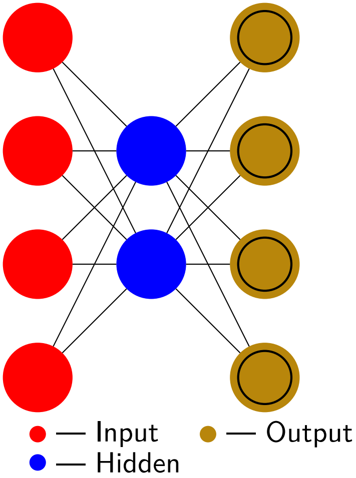
Long Short Term Memory

- Feedback loop with memory
- Application - NLP, time series modeling

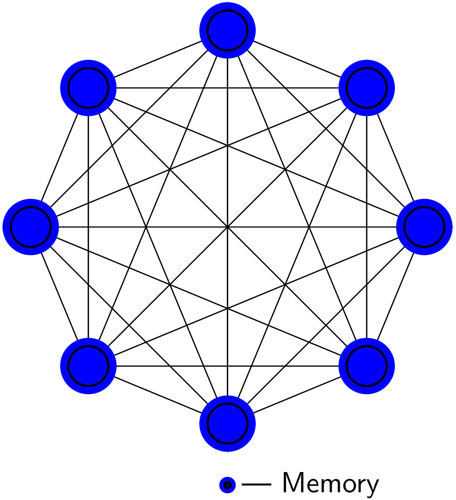


Auto Encoder

- **Learning the data in unsupervised mode**
- **Dimensionality reduction**

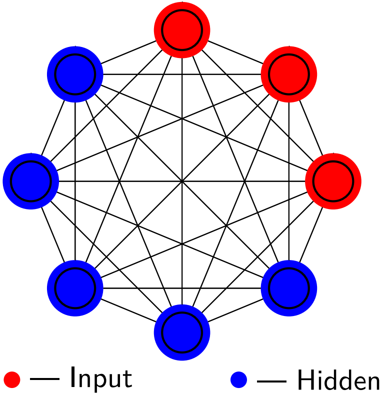


Markov chain



Boltzmann Machine

- Stochastic network
- Each neuron can have value either 0 or 1
- Some are hidden neurons
- Total energy (computed using states and the edge weights) is minimized



Learning the parameters

- The network must learn the connection weights from available training examples
- Learning can be
  - Supervised
  - Unsupervised
  - Hybrid
- Four basic types of learning rule
  - Error correction rule
  - Boltzmann learning
  - Hebbian
  - Competitive learning

Error correction rule

- Output is generated based on the weight values but this may vary from desired value
- The error information is used to update the weight value
- Perceptron learning algorithm
  - Initialize the weights and threshold to small random numbers
  - Present a pattern vector and evaluate the output of neuron
  - Update the weight according to $w_j(t+1) = w_j(t) + \eta(d - y)x_j$
- Back propagation algorithm

Boltzmann learning

- Usually symmetric recurrent network consisting of binary units
- A subset of neurons interact with environment
- Generally it has two modes
  - Clamped — Visible neurons are clamped to specific states
  - Free-running - Visible and hidden unit operate freely
- Stochastic learning rule derived from information theoretic and thermodynamic principles
- Learning rule is given by $\Delta w_{ij} = \eta(\bar{\rho}_{ij} - \rho_{ij})$

Hebbian rule

- **One of the oldest learning rules**
- **If neuron on both sides of a synapse are activated synchronously and repeatedly, the synapse's strength is selectively increased**
- **Mathematically, it can be described as** $w_{ij}(t+1) = w_{ij}(t) + \eta y_j(t) x_i(t)$

Competitive learning rule

- Output units compete among themselves for activation
- Only one output is active at time
- Also known as winner-take-all
- Mathematically, it can be represented as $w_{i^*}x \geq w_i x$
- Competitive learning rule can be stated as

$$\Delta w_{ij} = \begin{cases} \eta(x_j^u - w_{i^*j}) & i = i^* \\ 0 & i \neq i^* \end{cases}$$

Summary

- Error correction rule — Single or multilayer perceptron
  - Pattern classification, function approximation, prediction, control
- Boltzmann — Recurrent
  - Pattern classification
- Hebbian — Multilayer feed forward
  - Pattern classification, data analysis
- Competitive
  - Within class categorization, data compression