CS514: Design and Analysis of Algorithms

Arijit Mondal

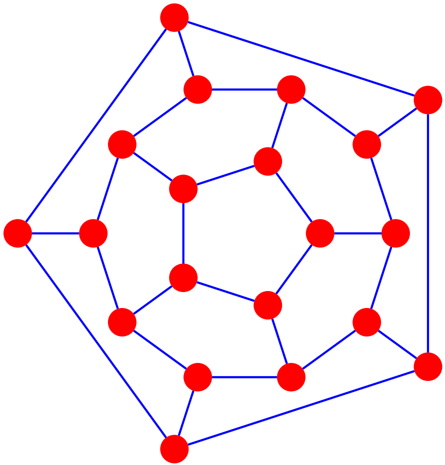
Dept of CSE

arijit@iitp.ac.in

https://www.iitp.ac.in/~arijit/

Matching

- Given an undirected graph $G = (V, E)$, a subset of edges $M \subseteq E$ is a matching if each node of the graph appears *at most* one edge of M.



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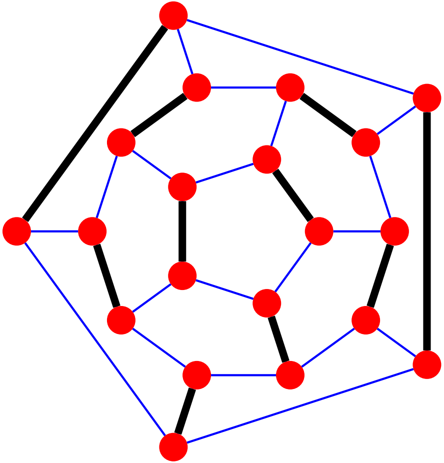
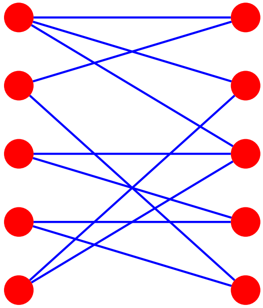


Bipartite matching

- A graph is bipartite if the nodes can be partitioned into two subsets X and Y such that every edge connects a node in X to a node in Y
- Given a bipartite graph $G = (X \cup Y, E)$, find a matching (M) that has the maximum cardinality ie., $|M|$ is maximum.



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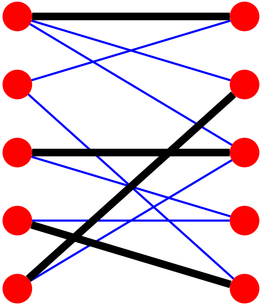
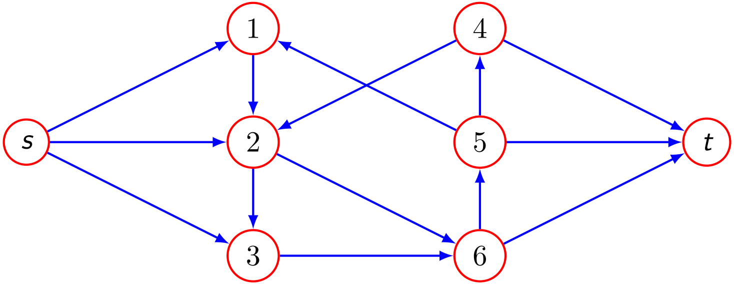


Edge-disjoint paths

- Two paths are edge-disjoint if they have no common edge. Given a directed graph $G = (V, E)$ and two nodes s and t, find the maximum number of edge-disjoint $s \rightsquigarrow t$ paths.



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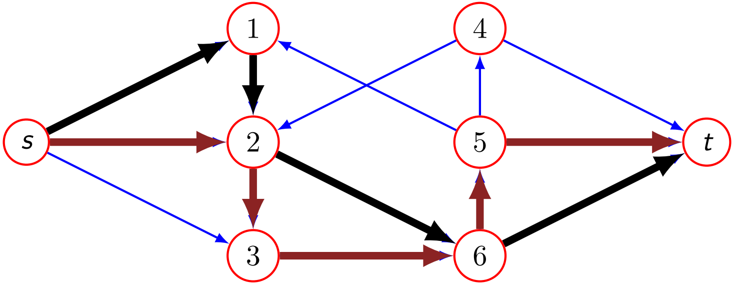
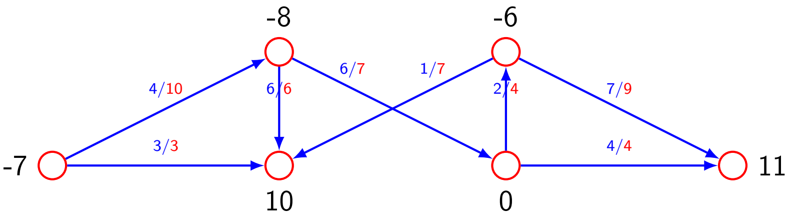


Circulation with demands

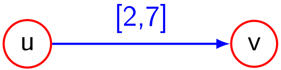
- Given a directed graph $V = (G, E)$ with non-negative edge capacities $c(e)$ and node supply and demands $d(v)$, a circulation is a function that satisfies
  - For each $e \in E$: $0 \leq f(e) \leq c(e)$ ($f(.)$ – flow along edge e)
  - For each $v \in V$: $\sum_{e \text{ in to } v} f(e) - \sum_{e \text{ out of } v} f(e) = d(v)$

  Does a circulation exist?
- $d(v) > 0$ - demand, $d(v) < 0$ - supply, $d(v) = 0$ - transshipment node



Circulation with lower bounds

- The problem is the same as previous one except that each edge has some lower bound on the flow. Hence, capacity along an edge will be specified as $[c_{lb}(u,v), c_{ub}(u,v)]$. What modifications are to be made in the graph to apply previous strategy?



Survey design

- Design a survey asking n_1 consumers about n_2 products that meets the following requirements, if possible.
  - Consumer i can survey about product j if they own it
  - Consumer i can be asked between c_i and c_i' questions
  - Ask between p_j and p_j' consumers about product j

Airline scheduling

- A set of lucrative flight segments (m say) are provided. A flight segment is specified by (a) origin airport, (b) destination airport, (c) departure time, (d) arrival time.
- It is possible to use a single plane for a flight segment i, and then later for a flight segment j, provided that
  - the destination of i is the same as the origin of j, and there's enough time to perform maintenance on the plane in between; or
  - you can add a flight segment in between that gets the plane from the destination of i to the origin of j with adequate time in between.
- Example:
  - B (depart 6am) – W (arrive 7am)
  - Ph (depart 7am) – Pt (arrive 8am)
  - W (depart 8am) – LA (arrive 11am)
  - Ph (depart 11am) – SF (arrive 2pm)
  - SF (depart 2:15pm) – S (arrive 3:15pm)
  - LV (depart 5pm) – S (arrive 6pm)
- Can k aircrafts serve all m flight segments?

Image segmentation

- Given the following
  - An image with V number of pixels and E number of pairs of neighboring pixels
  - $a_i \geq 0$ is likelihood pixel i in foreground
  - $b_i \geq 0$ is likelihood pixel i in background
  - $p_{ij} \geq 0$ is separation penalty for labeling one of i and j as foreground, and the other as background
- Goals:
  - Accuracy: if $a_i > b_i$ in isolation, prefer to label i in foreground
  - Smoothness: if many neighbors of i are labeled foreground, we should be inclined to label i as foreground
  - Find partition (A, B) that maximizes: $\sum_{i \in A} a_i + \sum_{j \in B} b_j - \sum_{\substack{(i,j) \in E \\ |A \cap \{i,j\}| = 1}} p_{ij}$