CS514: Design and Analysis of Algorithms

Arijit Mondal

Dept of CSE

arijit@iitp.ac.in

https://www.iitp.ac.in/~arijit/

General Information

- Class timings
  - Monday — 1600-1700
  - Tuesday — 1700-1800
  - Friday — 1500-1600
- Room - 301 (Block 9)

Books

- Thomas H Cormen, Charles E Lieserson, Ronald L Rivest and Clifford Stein, *Introduction to Algorithms*, Third Edition, MIT Press/McGraw-Hill
- Sanjoy Dasgupta, Christos H. Papadimitriou and Umesh V. Vazirani, *Algorithms*, Tata McGraw-Hill, 2008.
- Steven Skiena, *The Algorithm Design Manual*, Springer
- Jon Kleinberg and Éva Tardos, *Algorithm Design*, Pearson, 2005.
- Robert Sedgewick and Kevin Wayne, *Algorithms*, fourth edition, Addison Wesley, 2011.
- Udi Manber, *Algorithms – A Creative Approach*, Addison-Wesley, 1989
- Jeff Erickson, *Algorithms*

Evaluation

- Two quizes - 20%
- Midsem - 30%
- Endsem - 50%

CS514: Design and Analysis of Algorithms

Introduction

Arijit Mondal

Dept of CSE

arijit@iitp.ac.in

https://www.iitp.ac.in/~arijit/

Algorithm

- Is it a jumbled form of *logarithm*?

Algorithm

- Is it a jumbled form of *logarithm*?
- The word algorithm came into existence sometime after 1957

Algorithm

- Is it a jumbled form of *logarithm*?
- The word algorithm came into existence sometime after 1957
- Closest word that existed was algorism – it means the process of doing arithmetic using Arabic numerals

Algorithm

- Is it a jumbled form of *logarithm*?
- The word algorithm came into existence sometime after 1957
- Closest word that existed was algorism – it means the process of doing arithmetic using Arabic numerals
- It came from the name of a famous Persian textbook author, Abu Abd Allah Muhammad ibn Musa al-Khwarizmi

Algorithm

- Is it a jumbled form of *logarithm*?
- The word algorithm came into existence sometime after 1957
- Closest word that existed was algorism – it means the process of doing arithmetic using Arabic numerals
- It came from the name of a famous Persian textbook author, Abu Abd Allah Muhammad ibn Musa al-Khwarizmi
- He wrote the celebrated Arabic text Kitab al-jabr wa'l-muqabala ("Rules of restoring and equating")

Algorithm

- Is it a jumbled form of *logarithm*?
- The word algorithm came into existence sometime after 1957
- Closest word that existed was algorism – it means the process of doing arithmetic using Arabic numerals
- It came from the name of a famous Persian textbook author, Abu Abd Allah Muhammad ibn Musa al-Khwarizmi
- He wrote the celebrated Arabic text Kitab al-jabr wa'l-muqabala ("Rules of restoring and equating")
- Gradually the form and meaning of algorism became corrupted and resulted into algorithm

Properties of algorithm

- Input — an algorithm has zero or more inputs

Properties of algorithm

- Input — an algorithm has zero or more inputs
- Output — an algorithm has one or more outputs

Properties of algorithm

- Input — an algorithm has zero or more inputs
- Output — an algorithm has one or more outputs
- Finiteness — an algorithm must terminates after finite number of steps

Properties of algorithm

- Input — an algorithm has zero or more inputs
- Output — an algorithm has one or more outputs
- Finiteness — an algorithm must terminates after finite number of steps
- Definiteness — each step of algorithm needs to be defined precisely and unabiguously

Properties of algorithm

- Input — an algorithm has zero or more inputs
- Output — an algorithm has one or more outputs
- Finiteness — an algorithm must terminates after finite number of steps
- Definiteness — each step of algorithm needs to be defined precisely and unabiguously
  - *add salt to taste*

Properties of algorithm

- Input — an algorithm has zero or more inputs
- Output — an algorithm has one or more outputs
- Finiteness — an algorithm must terminates after finite number of steps
- Definiteness — each step of algorithm needs to be defined precisely and unabiguously
  - *add salt to taste*
- Effectiveness — operations must all be sufficiently basic that they can in principle be done exactly and in a finite length of time by someone using pencil and paper

Properties of algorithm

- Input — an algorithm has zero or more inputs
- Output — an algorithm has one or more outputs
- Finiteness — an algorithm must terminates after finite number of steps
- Definiteness — each step of algorithm needs to be defined precisely and unabiguously
  - *add salt to taste*
- Effectiveness — operations must all be sufficiently basic that they can in principle be done exactly and in a finite length of time by someone using pencil and paper
  - *If* 4 *is the largest integer* n *for which there is a solution to the equation* $w^n + x^n + y^n = z^n$ *in positive integers* $w, x, y,$ *and* z*, then go to step 6*

Overview

- Algorithm and program
- Pseudo-code
- Algorithm + Data-Structures = Program
- Initial solution + Analysis + Solution Refinement + Data-Structures = Final Program
- Use of recursive definition for initial solution
- Use recurrence equation for proofs and analysis
- Solution refinement through recursion transformation and traversal
- Data structures for saving past results for future use

- Sample problems
  - Finding MAX
  - Finding MAX and MIN
  - Finding MAX and 2nd-MAX
  - Fibonacci numbers
  - Searching in ordered / unordered list
  - Sorting
  - Pattern matching
  - Permutation and combination
  - Shortest path

Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

1. max(L)
2. if $|L| = 1$ return x_1
3. $L' = L - \{x_1\}$
4. $x' = \text{max}(L')$
5. if $(x_1 > x')$ return x_1
6. else return x'

Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

$\{5, 6, 10, 3, 1, 12, 2\}$

1. max(L)
2. if $|L| = 1$ return x_1
3. $L' = L - \{x_1\}$
4. $x' = \max(L')$
5. if $(x_1 > x')$ return x_1
6. else return x'

Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

```
max(L)
    if |L| = 1 return x_1
    L' = L - {x_1}
    x' = max(L')
    if (x_1 > x') return x_1
    else return x'
```

$\{5, 6, 10, 3, 1, 12, 2\}$

$x_1 = 5, L' = \{6, 10, 3, 1, 12, 2\}$

Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

1. max(L)
2. if $|L| = 1$ return x_1
3. $L' = L - \{x_1\}$
4. $x' = \max(L')$
5. if $(x_1 > x')$ return x_1
6. else return x'

$\{5, 6, 10, 3, 1, 12, 2\}$

$\swarrow$

$x_1 = 5, L' = \{6, 10, 3, 1, 12, 2\}$

$\searrow$

$x_1 = 6, L' = \{10, 3, 1, 12, 2\}$

Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

1. max(L)
2. if $|L| = 1$ return x_1
3. $L' = L - \{x_1\}$
4. $x' = \max(L')$
5. if $(x_1 > x')$ return x_1
6. else return x'

$$\{5, 6, 10, 3, 1, 12, 2\}$$

↙

$$x_1 = 5, L' = \{6, 10, 3, 1, 12, 2\}$$

↘

$$x_1 = 6, L' = \{10, 3, 1, 12, 2\}$$

↘

$$x_1 = 10, L' = \{3, 1, 12, 2\}$$

Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

```
max(L)
    if |L| = 1 return x_1
    L' = L - {x_1}
    x' = max(L')
    if (x_1 > x') return x_1
    else return x'
```

$$\{5, 6, 10, 3, 1, 12, 2\}$$

$$\swarrow$$

$$x_1 = 5, L' = \{6, 10, 3, 1, 12, 2\}$$

$$\searrow$$

$$x_1 = 6, L' = \{10, 3, 1, 12, 2\}$$

$$\searrow$$

$$x_1 = 10, L' = \{3, 1, 12, 2\}$$

$$\searrow$$

$$x_1 = 3, L' = \{1, 12, 2\}$$

Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

max(L)
    if $|L| = 1$ return x_1
    $L' = L - \{x_1\}$
    $x' = \max(L')$
    if $(x_1 > x')$ return x_1
    else return x'

$\{5, 6, 10, 3, 1, 12, 2\}$

$\swarrow$

$x_1 = 5, L' = \{6, 10, 3, 1, 12, 2\}$

$\searrow$

$x_1 = 6, L' = \{10, 3, 1, 12, 2\}$

$\searrow$

$x_1 = 10, L' = \{3, 1, 12, 2\}$

$\searrow$

$x_1 = 3, L' = \{1, 12, 2\}$

$\searrow$

$x_1 = 1, L' = \{12, 2\}$

Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

```
max(L)
    if |L| = 1 return x1
    L' = L - {x1}
    x' = max(L')
    if (x1 > x') return x1
    else return x'
```

$$\{5, 6, 10, 3, 1, 12, 2\}$$

$$\swarrow$$

$$x_1 = 5, L' = \{6, 10, 3, 1, 12, 2\}$$

$$\searrow$$

$$x_1 = 6, L' = \{10, 3, 1, 12, 2\}$$

$$\searrow$$

$$x_1 = 10, L' = \{3, 1, 12, 2\}$$

$$\searrow$$

$$x_1 = 3, L' = \{1, 12, 2\}$$

$$\searrow$$

$$x_1 = 1, L' = \{12, 2\}$$

$$\searrow$$

$$x_1 = 12, L' = \{2\}$$

Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

```
max(L)
    if |L| = 1 return x_1
    L' = L − {x_1}
    x' = max(L')
    if (x_1 > x') return x_1
    else return x'
```

$$\{5, 6, 10, 3, 1, 12, 2\}$$

$$\swarrow$$

$$x_1 = 5, L' = \{6, 10, 3, 1, 12, 2\}$$

$$\searrow$$

$$x_1 = 6, L' = \{10, 3, 1, 12, 2\}$$

$$\searrow$$

$$x_1 = 10, L' = \{3, 1, 12, 2\}$$

$$\searrow$$

$$x_1 = 3, L' = \{1, 12, 2\}$$

$$\searrow$$

$$x_1 = 1, L' = \{12, 2\}$$

$$\searrow$$

$$x_1 = 12, L' = \{2\}$$

$$\searrow$$

$$L' = \{2\}$$

Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

max(L)
    if $|L| = 1$ return x_1
    $L' = L - \{x_1\}$
    $x' = \max(L')$
    if $(x_1 > x')$ return x_1
    else return x'

$\{5, 6, 10, 3, 1, 12, 2\}$

$x_1 = 5, L' = \{6, 10, 3, 1, 12, 2\}$

$x_1 = 6, L' = \{10, 3, 1, 12, 2\}$

$x_1 = 10, L' = \{3, 1, 12, 2\}$

$x_1 = 3, L' = \{1, 12, 2\}$

$x_1 = 1, L' = \{12, 2\}$

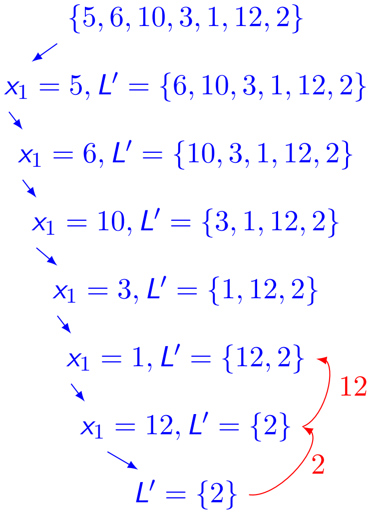
$x_1 = 12, L' = \{2\}$

$L' = \{2\}$ → 2

Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

```
max(L)
    if |L| = 1 return x1
    L' = L − {x1}
    x' = max(L')
    if (x1 > x') return x1
    else return x'
```



Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

1. max(L)
2. if $|L| = 1$ return x_1
3. $L' = L - \{x_1\}$
4. $x' = \max(L')$
5. if $(x_1 > x')$ return x_1
6. else return x'

$\{5, 6, 10, 3, 1, 12, 2\}$

$x_1 = 5, L' = \{6, 10, 3, 1, 12, 2\}$

$x_1 = 6, L' = \{10, 3, 1, 12, 2\}$

$x_1 = 10, L' = \{3, 1, 12, 2\}$

$x_1 = 3, L' = \{1, 12, 2\}$

$x_1 = 1, L' = \{12, 2\}$

$x_1 = 12, L' = \{2\}$

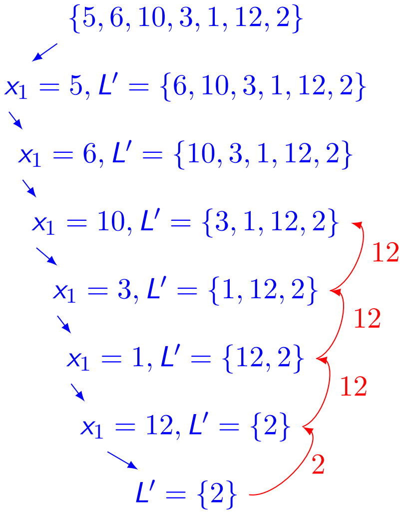
$L' = \{2\}$

Returned values: 2, 12, 12

Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

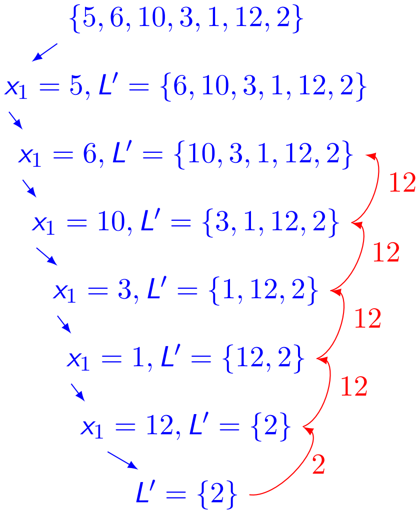
max(L)
    if $|L| = 1$ return x_1
    $L' = L - \{x_1\}$
    $x' = \max(L')$
    if $(x_1 > x')$ return x_1
    else return x'



Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

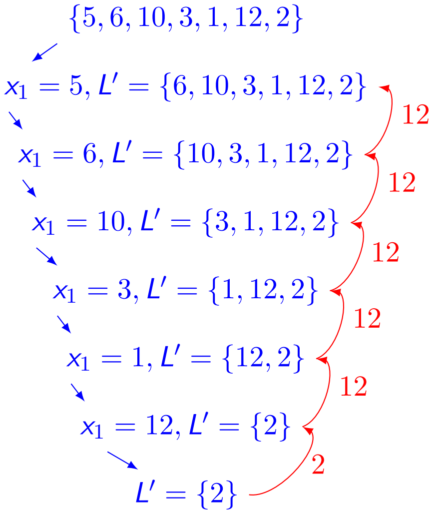
max(L)
    if $|L| = 1$ return x_1
    $L' = L - \{x_1\}$
    $x' = \max(L')$
    if $(x_1 > x')$ return x_1
    else return x'



Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

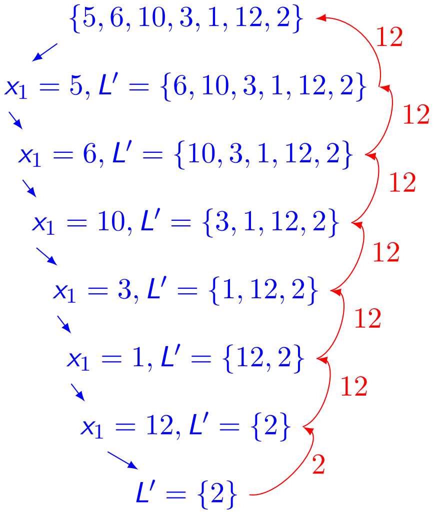
1. max(L)
2. if $|L| = 1$ return x_1
3. $L' = L - \{x_1\}$
4. $x' = \max(L')$
5. if $(x_1 > x')$ return x_1
6. else return x'



Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

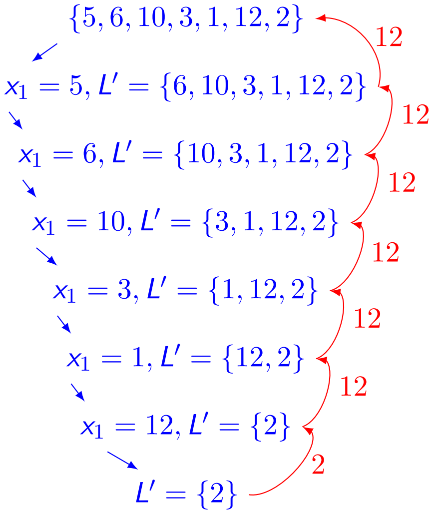
```
max(L)
    if |L| = 1 return x1
    L' = L − {x1}
    x' = max(L')
    if (x1 > x') return x1
    else return x'
```



Finding MAX of n elements (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Sequential comparison:

1. $\text{max}(L)$
2. if $|L| = 1$ return x_1
3. $L' = L - \{x_1\}$
4. $x' = \text{max}(L')$
5. if $(x_1 > x')$ return x_1
6. else return x'



Complexity analysis:

$$
\begin{aligned}
T(n) &= T(n-1) + 1, & n > 1 \\
&= 0, & n = 1
\end{aligned}
$$

Finding MAX of n elements (2)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Recursive formulation:

1. max2(L)
2. if $|L| = 1$ return x_1
3. Split L into 2 non-empty sets L_1, L_2
4. $x =$max2(L_1)
5. $y =$max2(L_2)
6. if $(x > y)$ return x
7. else return y

Finding MAX of n elements (2)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Recursive formulation:

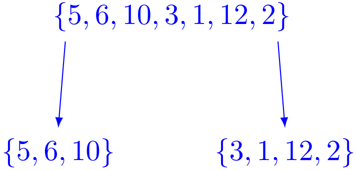
1. max2(L)
2. if $|L| = 1$ return x_1
3. Split L into 2 non-empty sets L_1, L_2
4. $x =$max2(L_1)
5. $y =$max2(L_2)
6. if $(x > y)$ return x
7. else return y

$\{5, 6, 10, 3, 1, 12, 2\}$

Finding MAX of n elements (2)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Recursive formulation:

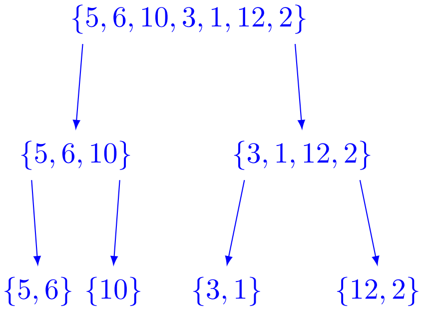
1. max2(L)
2. if $|L| = 1$ return x_1
3. Split L into 2 non-empty sets L_1, L_2
4. $x =$max2(L_1)
5. $y =$max2(L_2)
6. if $(x > y)$ return x
7. else return y



Finding MAX of n elements (2)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Recursive formulation:

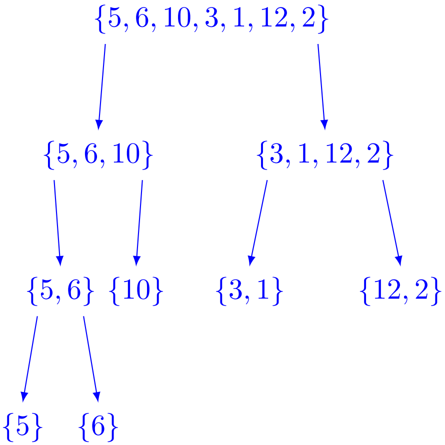
max2(L)
  if $|L| = 1$ return x_1
  Split L into 2 non-empty sets L_1, L_2
  $x =$max2(L_1)
  $y =$max2(L_2)
  if $(x > y)$ return x
  else return y



Finding MAX of n elements (2)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Recursive formulation:

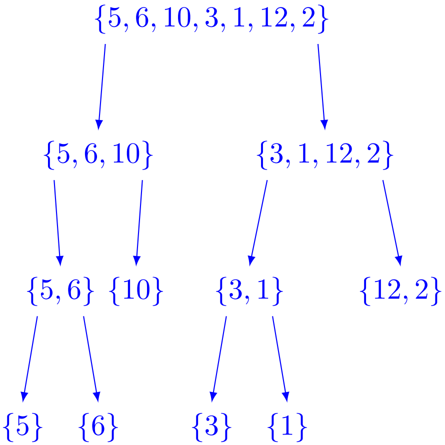
1. max2(L)
2. if $|L| = 1$ return x_1
3. Split L into 2 non-empty sets L_1, L_2
4. $x =$max2(L_1)
5. $y =$max2(L_2)
6. if $(x > y)$ return x
7. else return y



Finding MAX of n elements (2)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Recursive formulation:

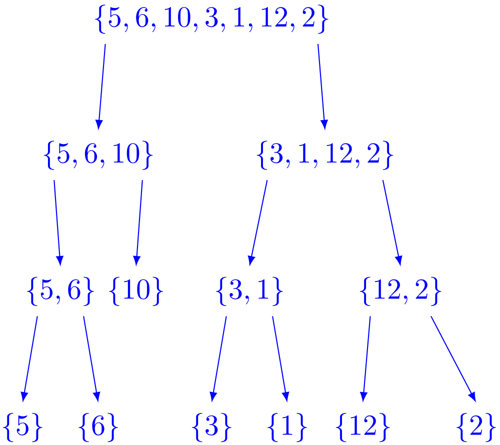
max2(L)
  if $|L| = 1$ return x_1
  Split L into 2 non-empty sets L_1, L_2
  $x =$max2(L_1)
  $y =$max2(L_2)
  if $(x > y)$ return x
  else return y



Finding MAX of n elements (2)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Recursive formulation:

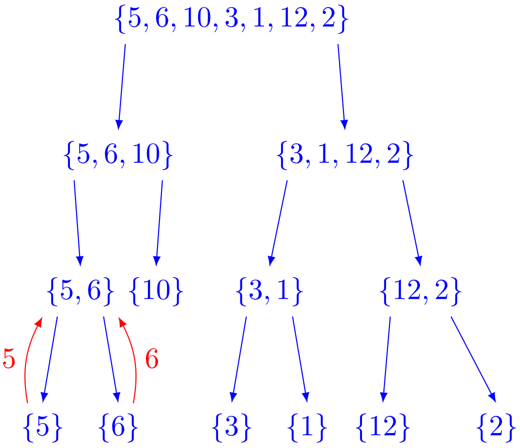
max2(L)
    if $|L| = 1$ return x_1
    Split L into 2 non-empty sets L_1, L_2
    $x =$max2(L_1)
    $y =$max2(L_2)
    if $(x > y)$ return x
    else return y



Finding MAX of n elements (2)

- Given $L = \{x_1, x_2, \dots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Recursive formulation:

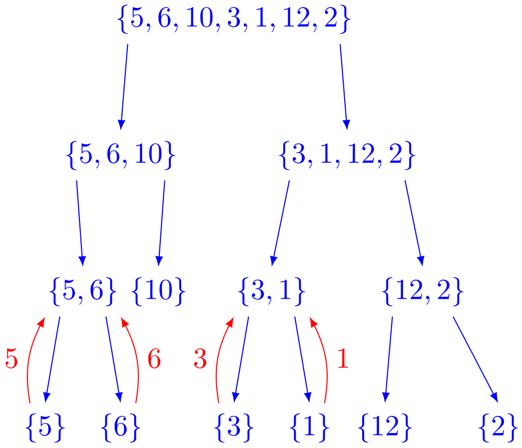
1. max2(L)
2.   if $|L| = 1$ return x_1
3.   Split L into 2 non-empty sets L_1, L_2
4. $x =$max2(L_1)
5. $y =$max2(L_2)
6.   if $(x > y)$ return x
7.   else return y



Finding MAX of n elements (2)

- Given $L = \{x_1, x_2, \dots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Recursive formulation:

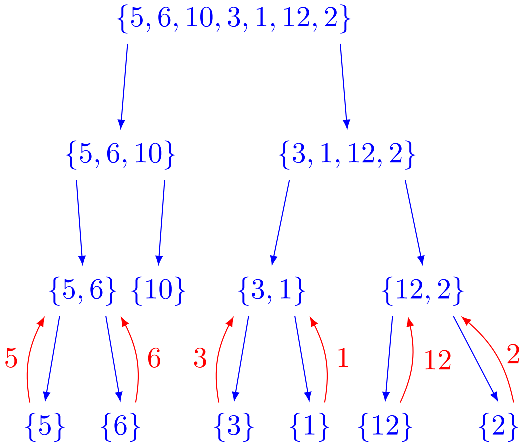
max2(L)
  if $|L| = 1$ return x_1
  Split L into 2 non-empty sets L_1, L_2
  $x =$max2(L_1)
  $y =$max2(L_2)
  if $(x > y)$ return x
  else return y



Finding MAX of n elements (2)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Recursive formulation:

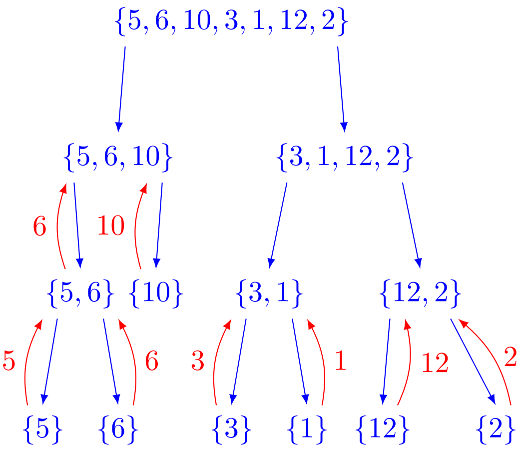
1. max2(L)
2. if $|L| = 1$ return x_1
3. Split L into 2 non-empty sets L_1, L_2
4. $x =$max2(L_1)
5. $y =$max2(L_2)
6. if $(x > y)$ return x
7. else return y



Finding MAX of n elements (2)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Recursive formulation:

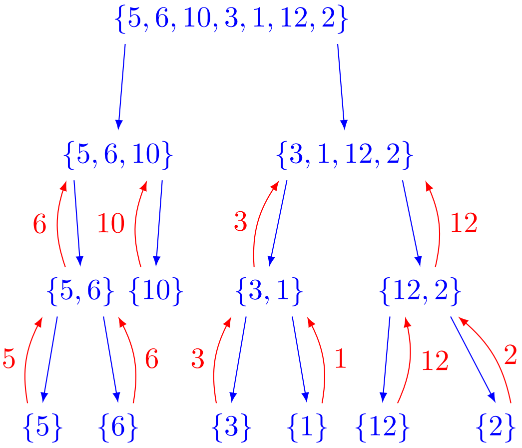
max2(L)
  if $|L| = 1$ return x_1
  Split L into 2 non-empty sets L_1, L_2
  $x =$max2(L_1)
  $y =$max2(L_2)
  if $(x > y)$ return x
  else return y



Finding MAX of n elements (2)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Recursive formulation:

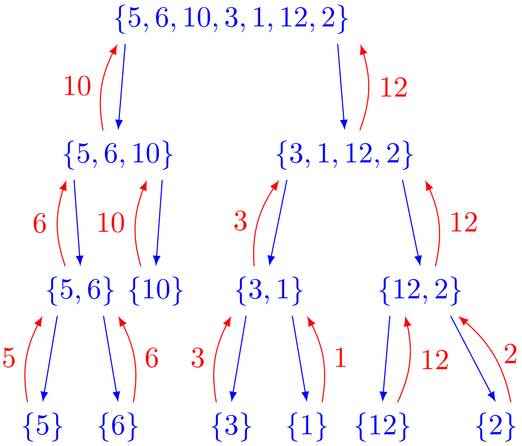
1. max2(L)
2. if $|L| = 1$ return x_1
3. Split L into 2 non-empty sets L_1, L_2
4. $x =$max2(L_1)
5. $y =$max2(L_2)
6. if $(x > y)$ return x
7. else return y



Finding MAX of n elements (2)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Recursive formulation:

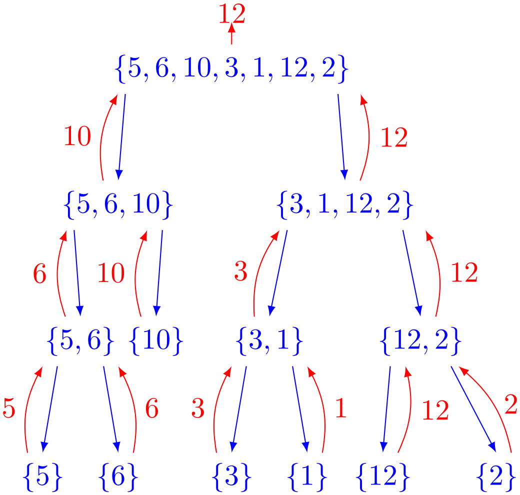
1. max2(L)
2. if $|L| = 1$ return x_1
3. Split L into 2 non-empty sets L_1, L_2
4. $x =$max2(L_1)
5. $y =$max2(L_2)
6. if $(x > y)$ return x
7. else return y



Finding MAX of n elements (2)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Recursive formulation:

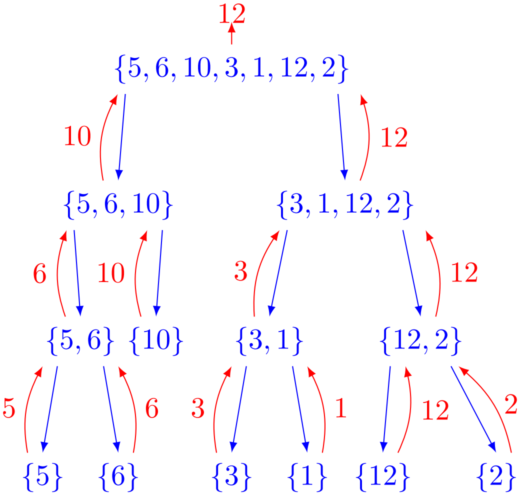
```
max2(L)
   if |L| = 1 return x_1
   Split L into 2 non-empty sets L_1, L_2
   x = max2(L_1)
   y = max2(L_2)
   if (x > y) return x
   else return y
```



Finding MAX of n elements (2)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$
- Recursive formulation:

1. max2(L)
2. if $|L| = 1$ return x_1
3. Split L into 2 non-empty sets L_1, L_2
4. $x =$max2(L_1)
5. $y =$max2(L_2)
6. if $(x > y)$ return x
7. else return y



Complexity analysis:

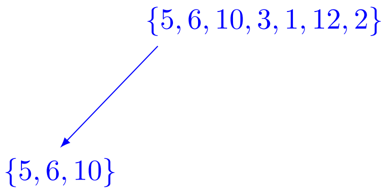
$$T(n) = T(n-k) + T(k) + 1, \quad n > 1$$
$$= 0, \quad n = 1$$

Recursion Flow: Finding MAX of n elements (2)

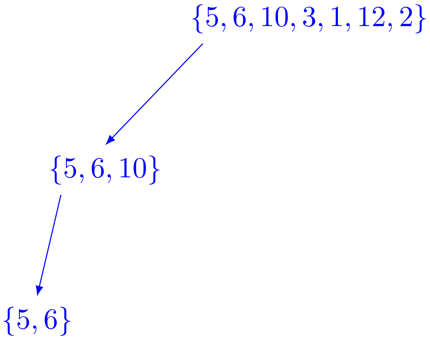
Recursion Flow: Finding MAX of n elements (2)

$$\{5, 6, 10, 3, 1, 12, 2\}$$

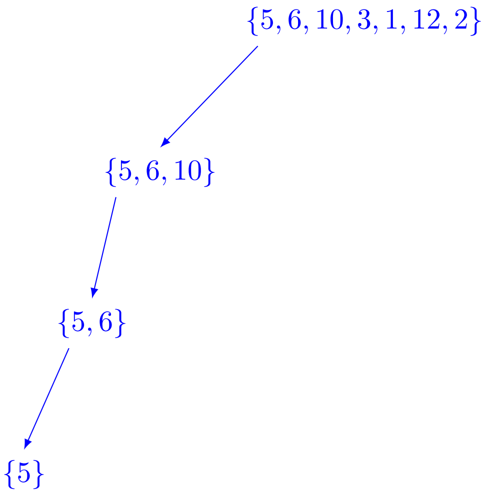
Recursion Flow: Finding MAX of n elements (2)



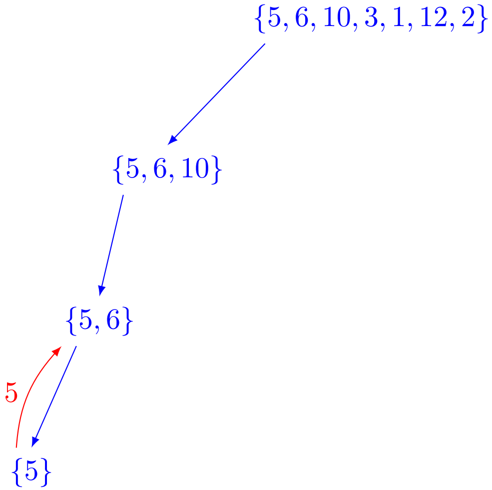
Recursion Flow: Finding MAX of n elements (2)



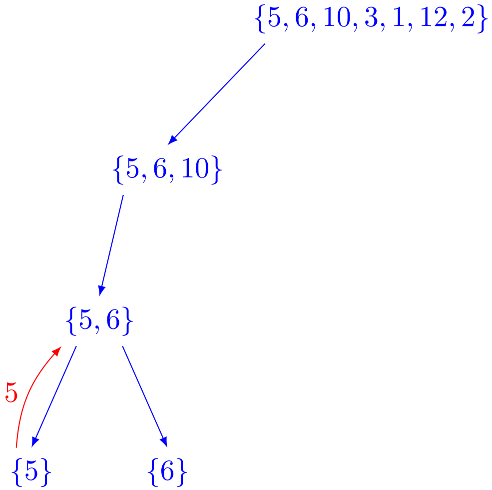
Recursion Flow: Finding MAX of n elements (2)



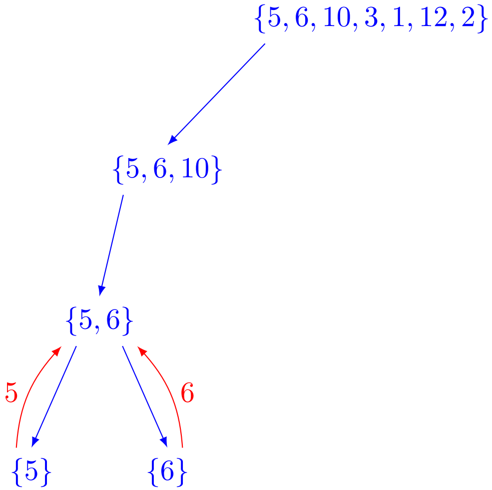
Recursion Flow: Finding MAX of n elements (2)



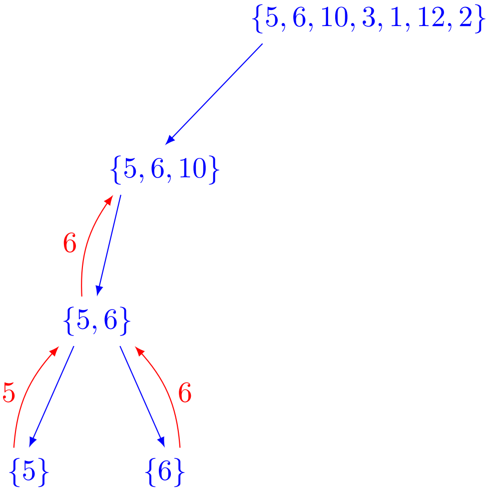
Recursion Flow: Finding MAX of n elements (2)



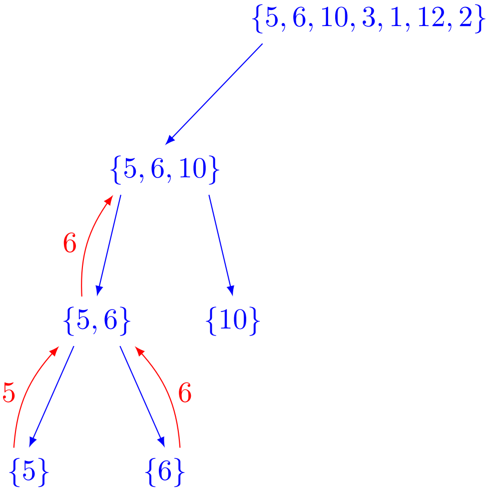
Recursion Flow: Finding MAX of n elements (2)



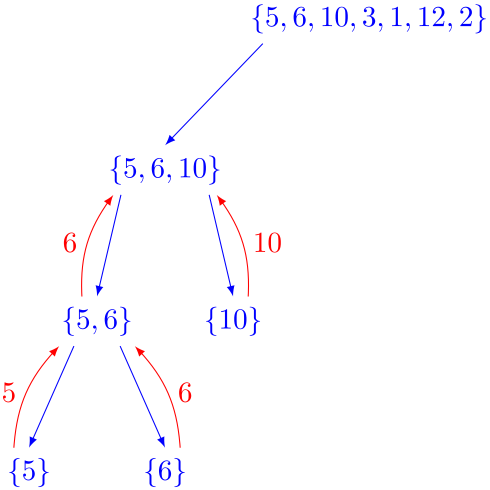
Recursion Flow: Finding MAX of n elements (2)



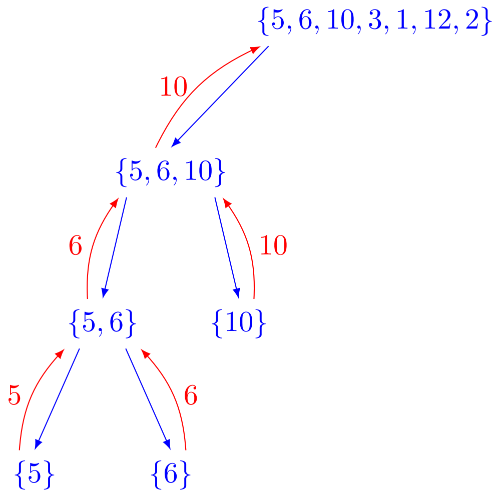
Recursion Flow: Finding MAX of n elements (2)



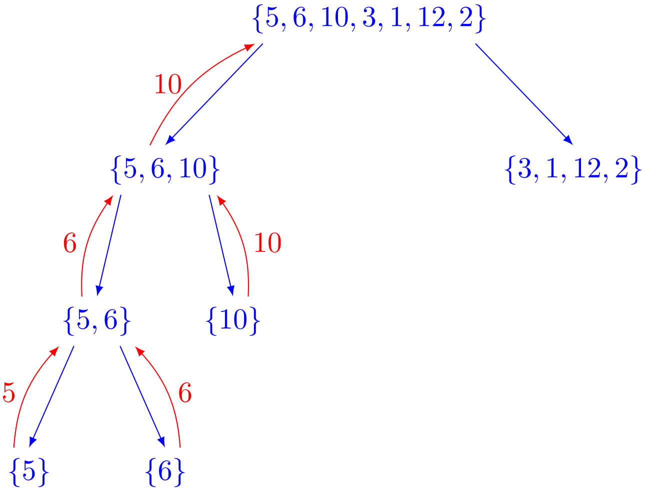
Recursion Flow: Finding MAX of n elements (2)



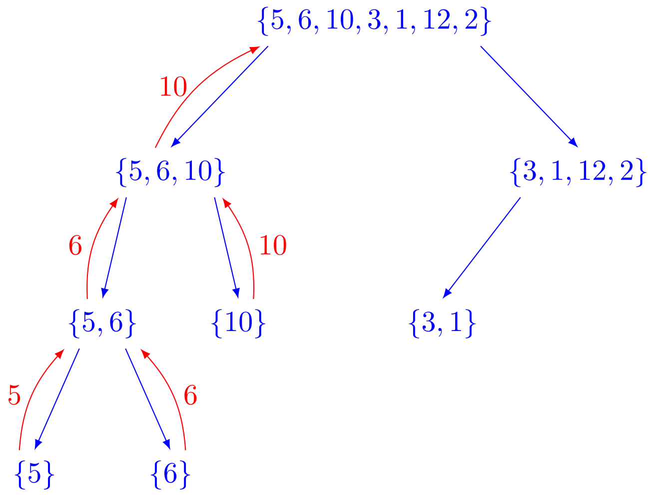
Recursion Flow: Finding MAX of n elements (2)



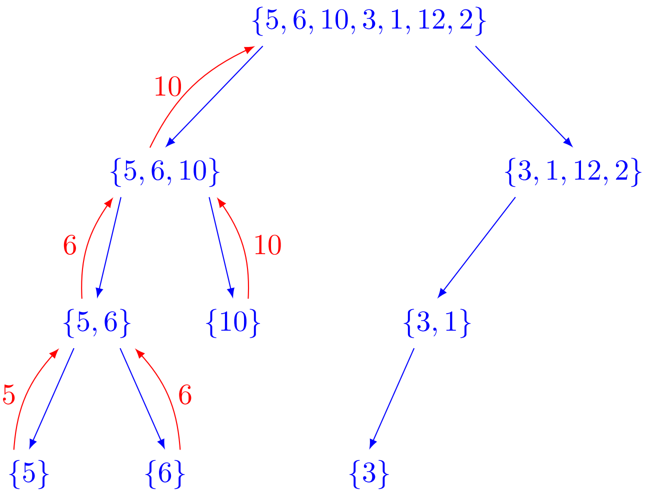
Recursion Flow: Finding MAX of n elements (2)



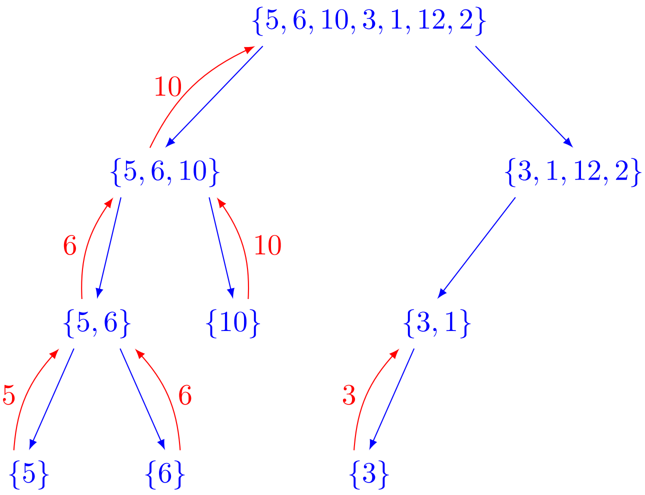
Recursion Flow: Finding MAX of n elements (2)



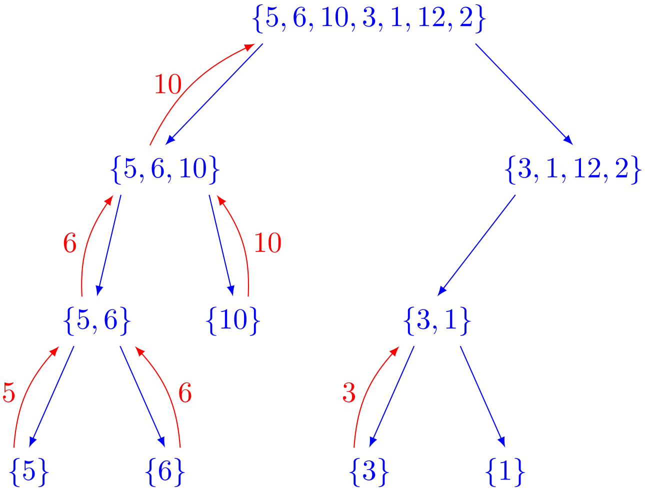
Recursion Flow: Finding MAX of n elements (2)



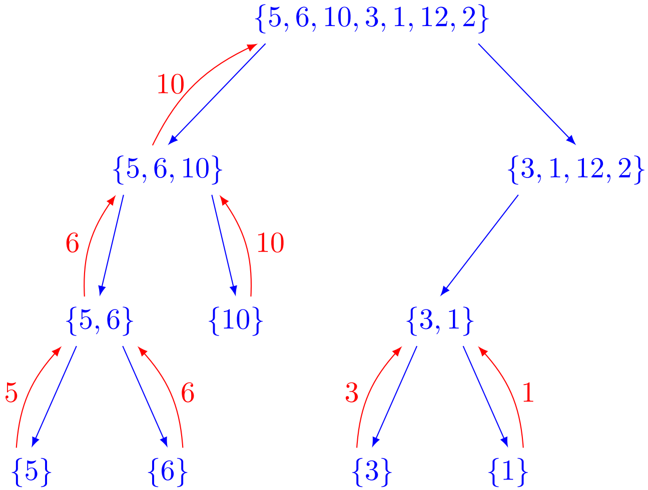
Recursion Flow: Finding MAX of n elements (2)



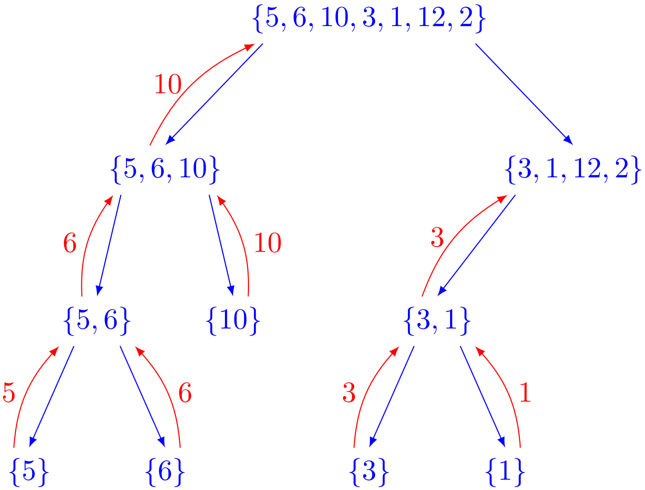
Recursion Flow: Finding MAX of n elements (2)



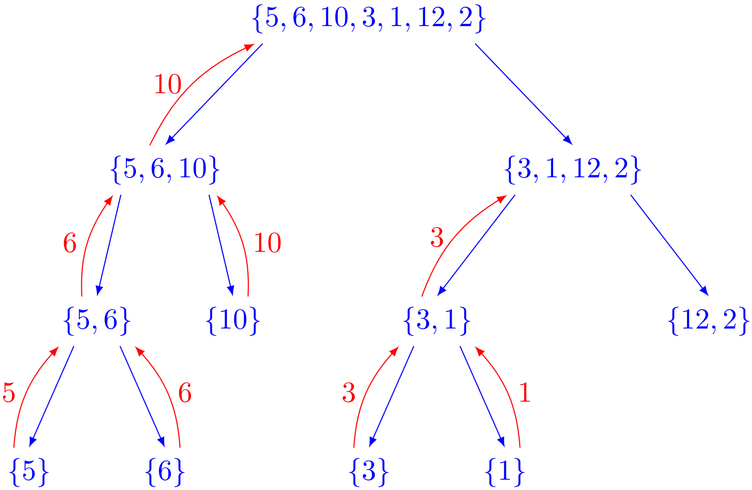
Recursion Flow: Finding MAX of n elements (2)



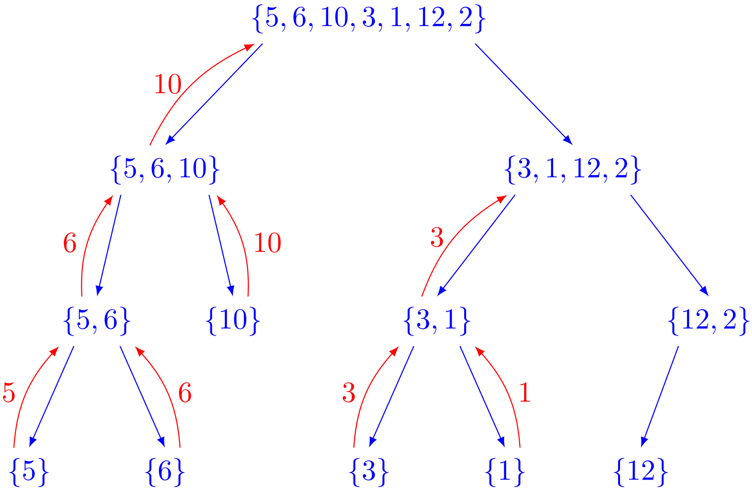
Recursion Flow: Finding MAX of n elements (2)



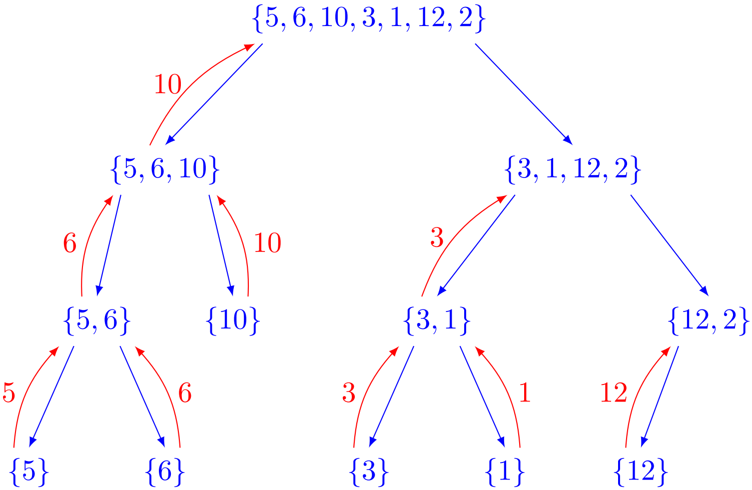
Recursion Flow: Finding MAX of n elements (2)



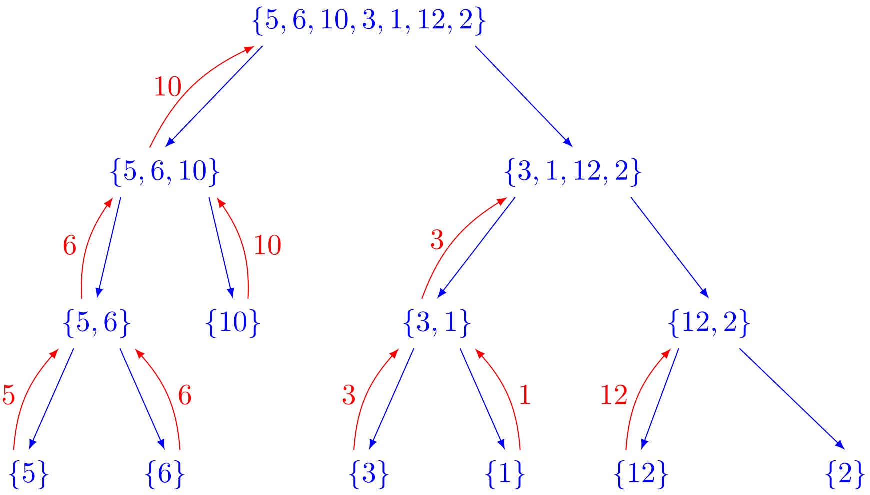
Recursion Flow: Finding MAX of n elements (2)



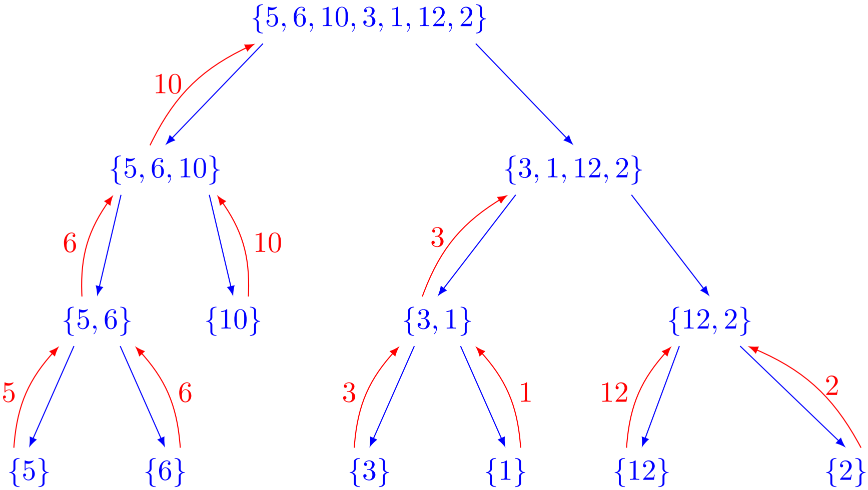
Recursion Flow: Finding MAX of n elements (2)



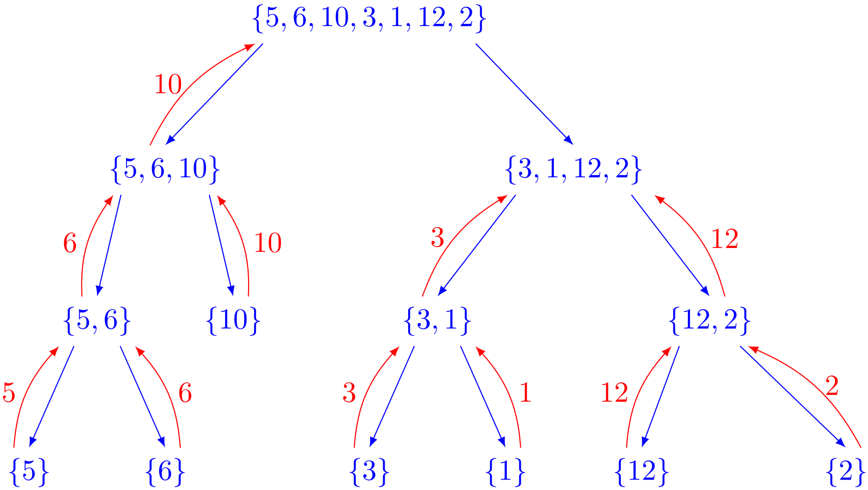
Recursion Flow: Finding MAX of n elements (2)



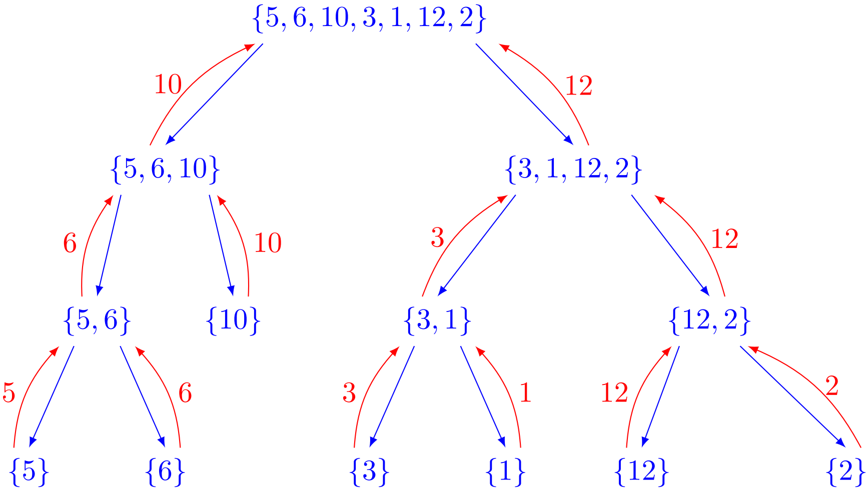
Recursion Flow: Finding MAX of n elements (2)



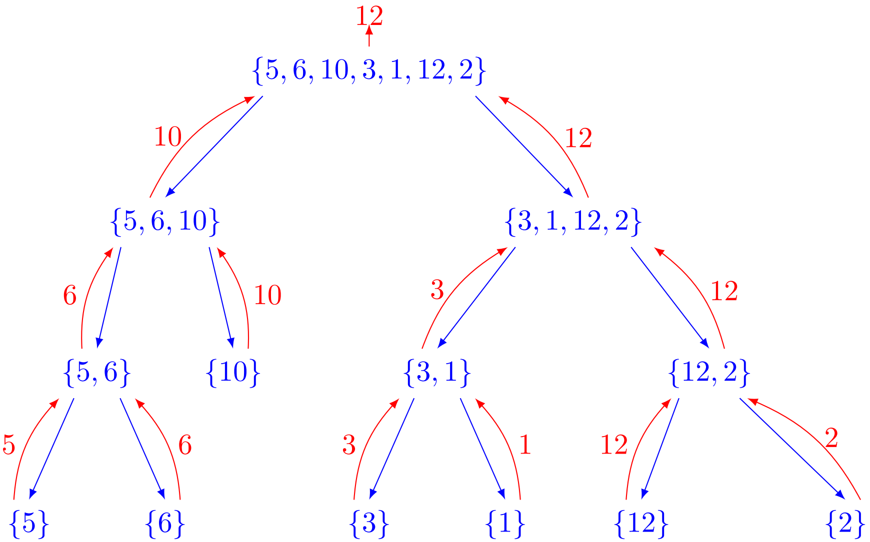
Recursion Flow: Finding MAX of n elements (2)



Recursion Flow: Finding MAX of n elements (2)



Recursion Flow: Finding MAX of n elements (2)



Comparison Tournament

- Finding of maximum can be viewed as a tournament of players taken two at a time

x_0

x_1

x_2

x_3

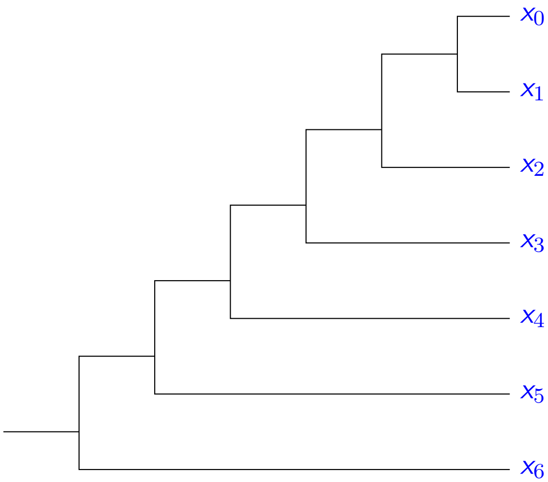
x_4

x_5

x_6

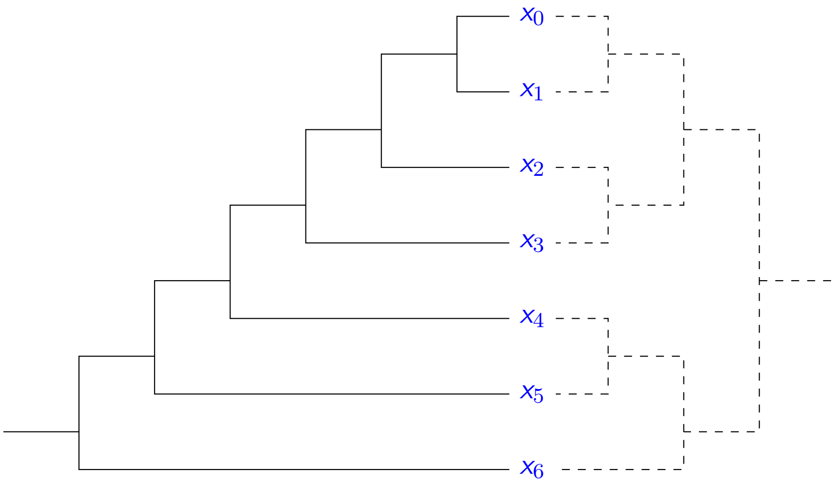
Comparison Tournament

- Finding of maximum can be viewed as a tournament of players taken two at a time



Comparison Tournament

- Finding of maximum can be viewed as a tournament of players taken two at a time



MAX & MIN (1)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$ and $\min\{L\}$
- Sequential comparison

1. maxmin(L)
2. if $|L|=1$ return $\langle x_1, x_1 \rangle$
3. $L' = L - \{x_1\}$
4. $\langle y_1, y_2 \rangle =$ maxmin(L')
5. if $x_1 > y_1$ then $m_1 = x_1$ else $m_1 = y_1$
6. if $x_1 < y_2$ then $m_2 = x_1$ else $m_2 = y_2$
7. return $\langle m_1, m_2 \rangle$

MAX & MIN (2)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find max$\{L\}$ and min$\{L\}$
- Recursive definition

maxmin2(L)
  if $|L|=1$ return $\langle x_1, x_1 \rangle$
  if $|L|=2$ if $x_1 > x_2$ return $\langle x_1, x_2 \rangle$ else return $\langle x_2, x_1 \rangle$
  Split L into 2 non-empty sets L_1, L_2
  $\langle y_1, y_2 \rangle$ = maxmin2(L_1)
  $\langle z_1, z_2 \rangle$ = maxmin2(L_2)
  if $y_1 > z_1$ then $m_1 = y_1$ else $m_1 = z_1$
  if $y_2 < z_2$ then $m_2 = y_2$ else $m_2 = z_2$
  return $\langle m_1, m_2 \rangle$

MAX & MIN (3)

- Given $L = \{x_1, x_2, \ldots, x_n\}$, all x_i are integers. We need to find $\max\{L\}$ and $\min\{L\}$
- Recursive definition - Choice of split
  - Recurrence relation:

$$\begin{aligned} T(n) &= 0, & n = 1 \\ &= 1, & n = 2 \\ &= T(k) + T(n-k) + 2, & n = 2 \end{aligned}$$

Thank you!