Discrete Mathematics

Sets

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- Example:
  - The set of all vowels in the English alphabet, $V = \{a, e, i, o, u\}$
  - The set of odd positive integers less than 10, $O = \{1, 3, 5, 7, 9\}$
  - Alternative notation, $O = \{x | x \text{ is an odd positive integer less than 10}\}$
  - Set of positve integers, $O = \{x \in \mathbf{Z}^+ | x \text{ is odd and } x < 10\}$

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- Subset

Well known sets

- $\mathbf{N} = \{0, 1, 2, \ldots\}$, the set of all natural numbers
- $\mathbf{Z} = \{\ldots, -2, -2, 0, 1, 2, \ldots\}$, the set of all integers
- $\mathbf{Z}^+ = \{1, 2, \ldots\}$, the set of all positive integers
- $\mathbf{Q} = \{p/q | p \in \mathbf{Z}, q \in \mathbf{Z} \text{ and } q \neq 0\}$, the set of all rational numbers
- $\mathbf{R}$, the set of all real numbers
- $\mathbf{R}^+$, the set of all positive real numbers
- $\mathbf{C}$, the set of all complex numbers

Intervals

- **Closed interval –** $[a, b]$
- **Open interval –** (a, b)

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- For every set S, $\emptyset \subseteq S$

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- **A set is said to be *infinite* if it is not finite**

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  - $S = \emptyset$
  - $S = \{\emptyset\}$

Cartesian product

- **Let A and B be sets. The *Cartesian product* of A and B, denoted by $A \times B$, is the set of all ordered pairs (a, b), where $a \in A$ and $b \in B$. Hence, $A \times B = \{(a, b) | a \in A \land b \in B\}$**

Set operations & terminologies

- Union
- Intersection
- Disjoint set
- Set difference
- Complement set
- De Morgan's law

Problems

- **Let** $A_i = \{i, i+1, \ldots\}$ **for** $i = 1, 2, \ldots$. **Find** $\bigcup_{i=1}^{n} A_i, \quad \bigcap_{i=1}^{n} A_i$

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Functions

- **Let A and B be nonempty sets. A function f from A to B is an assignment of exactly one element of B to each element of A. We write $f(a) = b$ if b is the unique element of B assigned by the function f to the element a of A. If f is a function from A to B, we write $f : A \to B$.**

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- **If f is a function from A to B, we say that A is the domain of f and B is the codomain of f. If $f(a) = b$, we say that b is the image of a and a is a preimage of b. The range, or image, of f is the set of all images of elements of A. Also, if f is a function from A to B, we say that f maps A to B.**

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- **Let f_1 and f_2 be functions from A to R. Then $f_1 + f_2$ and $f_1 f_2$ are also functions from A to R defined for all $x \in A$ by $(f_1 + f_2)(x) = f_1(x) + f_2(x)$, $(f_1 f_2)(x) = f_1(x) f_2(x)$.**

One-to-One and Onto functions

- **A function f is said to be *one-to-one*, or an *injection*, if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f. A function is said to be *injective* if it is *one-to-one*.**

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- **The function f is a *one-to-one correspondence*, or a *bijection*, if it is both one-to-one and onto. We also say that such a function is *bijective*.**

Inverse function

- **Let f be a one-to-one correspondence from the set A to the set B. The *inverse function* of f is the function that assigns to an element b belonging to B the unique element a in A such that $f(a) = b$. The inverse function of f is denoted by f^{-1}. Hence, $f^{-1}(b) = a$ when $f(a) = b$.**

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  - Set of real numbers

Thank you!