Discrete Mathematics

Predicate Logic: Fundamentals

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Formulating Predicate Logic Statement

- **Wherever Mary goes, so does the lamb. Mary goes to school. So the lamb goes to school.**
  - **Predicate** $goes(x, y)$ **to denote** x **goes to** y
  - $F_1 : \forall x(goes(Mary, x) \rightarrow goes(Lamb, x))$
  - $F_2 : goes(Mary, School)$
  - $G : goes(Lamb, School)$
- **To prove:** $(F_1 \wedge F_2) \rightarrow G$ **is always true**

Use of quantifiers

- Someone likes everyone
- Everyone likes someone
- There is someone whom everyone likes
- Everyone likes everyone
- If everyone likes everyone then someone likes everyone
- If there is a person whom everyone likes then that person likes himself
- Laws of negation

Use of function symbols

- If x is greater than y and y is greater than z then x is greater than z
  - g(x,y) – x is greater than y
  - $\forall x \forall y \forall z((g(x,y) \wedge g(y,z)) \rightarrow g(x,z))$
- The age of a person is greater than the age of his child
  - Age(x) – Function symbol; child(x,y) – x is child of y
  - $\forall x \forall y(child(x,y) \rightarrow g(Age(y), Age(x)))$
  - Age(x) returns value
  - child(x,y) returns TRUE or FALSE
- Therefore the age of a person is greater than the age of his grandchild
- The sum of ages of two children are never more than the sum of ages of their parents

Variables and Predicate / Function Symbols

- Variables, Free variables, Bound variables
  - $\forall x(p(x, y))$
  - $\forall x\{p(x, y) \wedge \exists z\ q(x, y, z, w)\}$
  - $\forall x\{p(x, y) \wedge \exists z \exists y\ q(x, y, z, w)\}$
- Symbols – proposition symbols, constant symbols, function symbols, predicate symbols
- Variables can be quantified in first order predicate logic
- Symbols cannot be quantified in first order predicate logic
- Interpretations are mapping of symbols to relevant aspects of a domain

Terminology for Predicate Logic

- Domain: D
- Constant symbols: M, N, O, P, ...
- Variable symbols: x, y, z, ...
- Function symbols: F(x), G(x,y), ...
- Predicate symbols: p(x), q(x,y), ...
- Connectors: $\sim, \wedge, \vee, \rightarrow, \exists, \forall$
- Terms:
- Well-formed formula:
- Free and bound variables
- Interpretation, valid, non-valid, satisfiable, unsatisfiable

Validity, Satisfiability, Structure

- $F_1 : \forall x(goes(Mary, x) \to goes(Lamb, x))$
- $F_2 : goes(Mary, School)$
- $G : goes(Lamb, School)$
- **To prove:** $(F_1 \wedge F_2) \to G$ **is always true**
- **The above is the same as follows**
- $F_1 : \forall x(www(M, x) \to www(L, x))$
- $F_2 : www(M, S)$
- $G : www(L, S)$
- **To prove:** $(F_1 \wedge F_2) \to G$ **is always true**

Interpretations

- **What is an interpretation? Assign a domain set D, map constants, functions, predicates suitably**
- **The formula will now have a truth value**
- **Example:** $F_1 : \forall x(g(M, x) \rightarrow g(L, x))$, $F_2 : g(M, S)$, $G : g(L, S)$
- **Interpretation 1: D={Akash, Baby, Home Play, Ratan, Swim}, etc.**
- **Interpretation 2: D = set of integers**
- **How many interpretations can there be?**
- **To prove Validity, means $((F_1 \wedge F_2) \rightarrow G)$ is true under all interpretations**
- **To prove Satisfiability, means $((F_1 \wedge F_2) \rightarrow G)$ is true under at least one interpretation**

Russell’s Paradox

- There is a single barber in town.
- Those and only those who do not shave themselves are shaved by the barber
- Who shaves the barber?

Thank you!