Discrete Mathematics

Predicate Logic: Introduction

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Insufficiency of Propositional Logic

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  - New connectors: $\exists$ (there exists), $\forall$ (for all)

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Example - 3

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Thank you!