

Embedded Systems



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Modeling: Hybrid systems

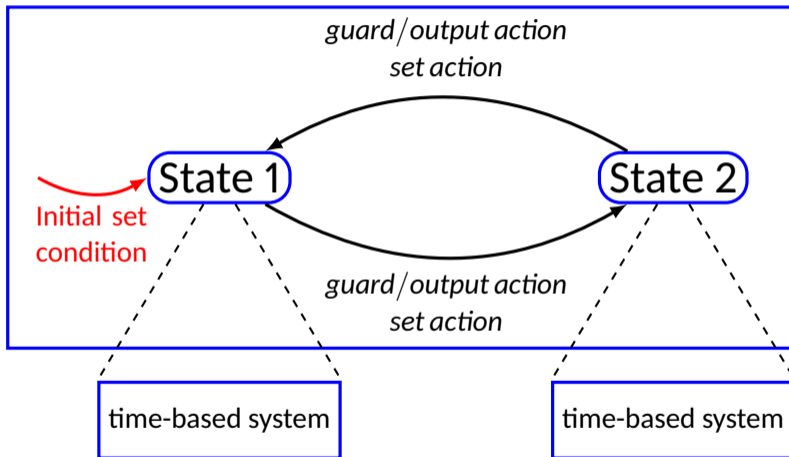
Introduction

- Most of the systems have both continuous and discrete behavior
- Continuous behavior can be modeled by ODE and discrete behavior by FSM
- Need to have separate modeling scheme to describe both the behavior
- Usually, states in discrete modelling are enhanced with time based behavior

Hybrid systems: example

- **Digital controller**
 - Thermostat
 - Automatic cruise control
 - Aircraft autopilot
- **Phased operation**
 - Bouncing ball
 - Biological cell growth
- **Multiagent systems**
 - Interaction of robots
 - Ground and air transportation systems

Timed automata

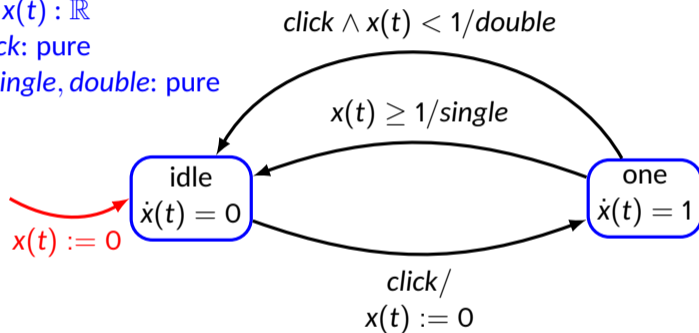


Double click detector

variable: $x(t) : \mathbb{R}$

input: $click$: pure

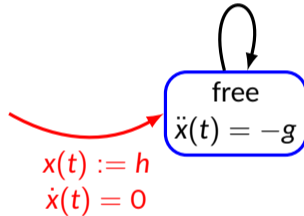
output: $single, double$: pure



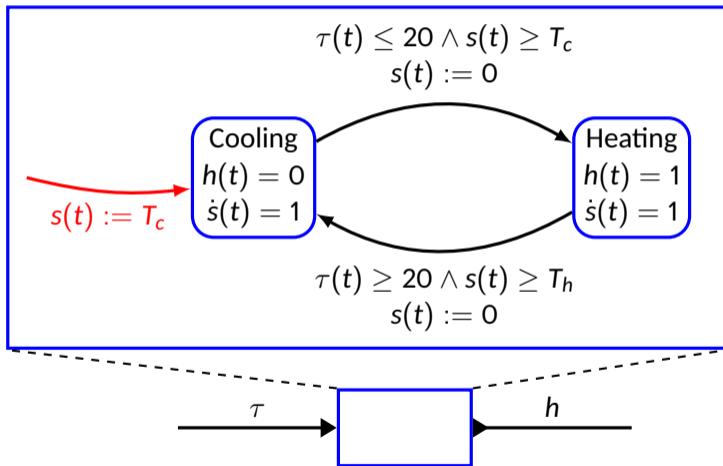
Bouncing ball

$$x(t) = 0 / \text{bump}$$

$$\dot{x}(t) := -a\dot{x}(t)$$



Thermostat

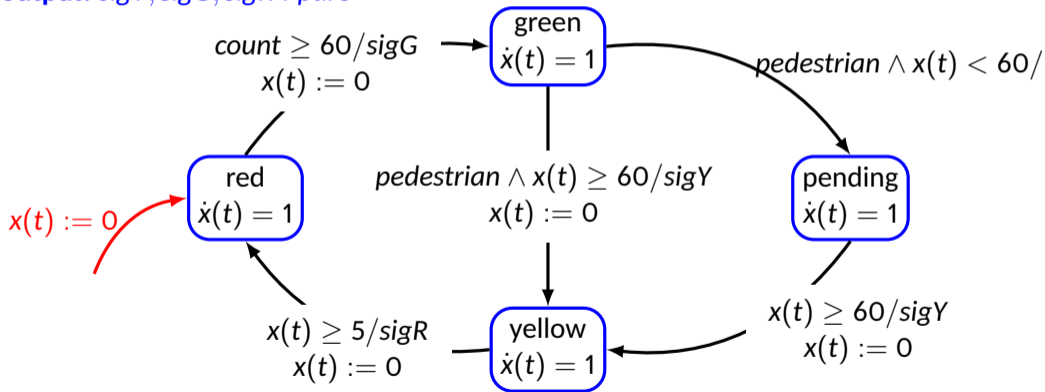


Example: pedestrian crosswalk

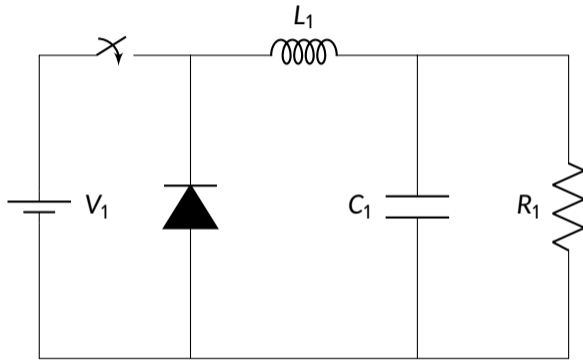
variable: $count : \{0, 1, \dots, 60\}$

input: $pedestrian : pure$

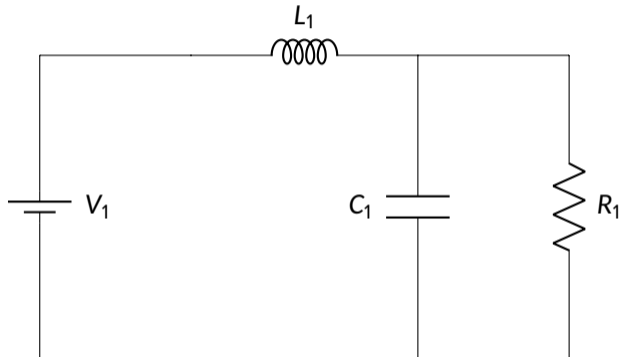
output: $sigY, sigG, sigR : pure$



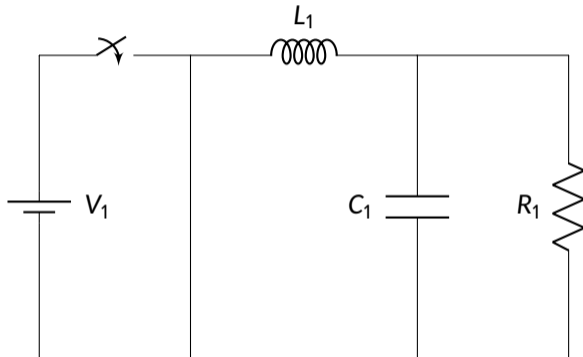
Buck converter



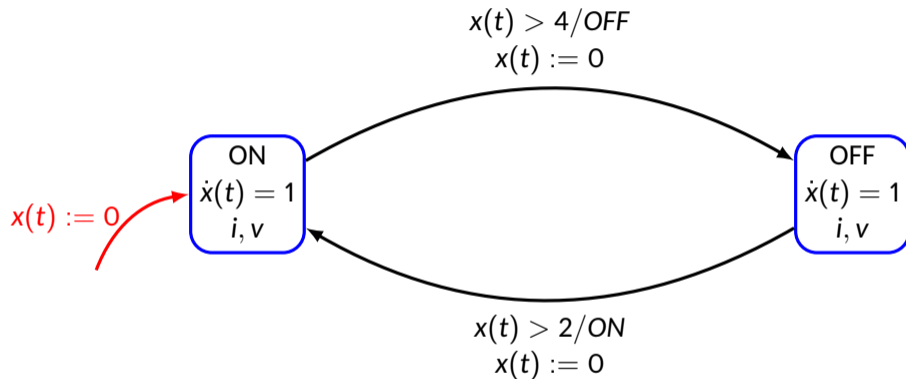
Buck converter: Mode 1



Buck converter: Mode 2



Buck converter: hybrid automata



Automatic cruise control

right

$$\dot{x}, \dot{y}, \dot{\theta}$$

$$e = f(x, y)$$

straight

$$\dot{x}, \dot{y}, \dot{\theta}$$

$$e = f(x, y)$$

stop

$$\dot{x}, \dot{y}, \dot{\theta}$$

$$e = f(x, y)$$

left

$$\dot{x}, \dot{y}, \dot{\theta}$$

$$e = f(x, y)$$

Automatic cruise control

right
 $\dot{x}, \dot{y}, \dot{\theta}$
 $e = f(x, y)$

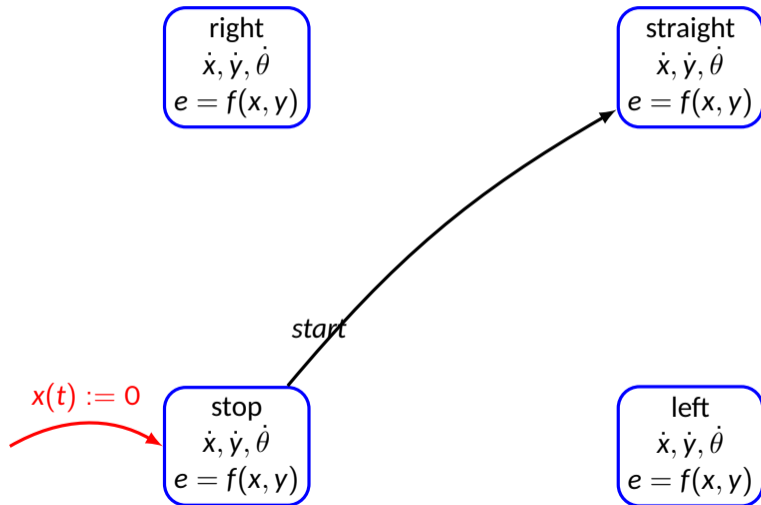
straight
 $\dot{x}, \dot{y}, \dot{\theta}$
 $e = f(x, y)$

$x(t) := 0$

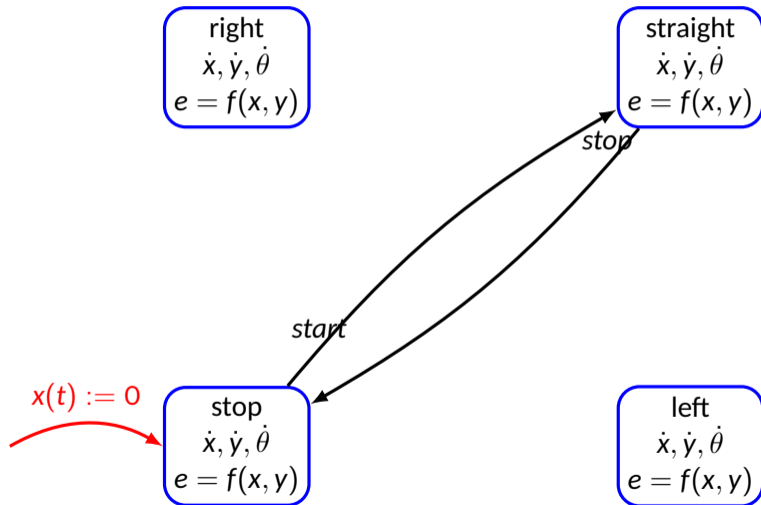
stop
 $\dot{x}, \dot{y}, \dot{\theta}$
 $e = f(x, y)$

left
 $\dot{x}, \dot{y}, \dot{\theta}$
 $e = f(x, y)$

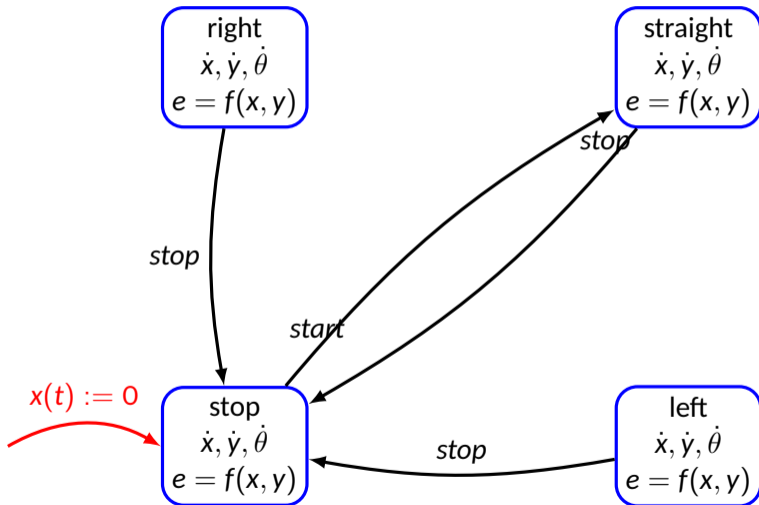
Automatic cruise control



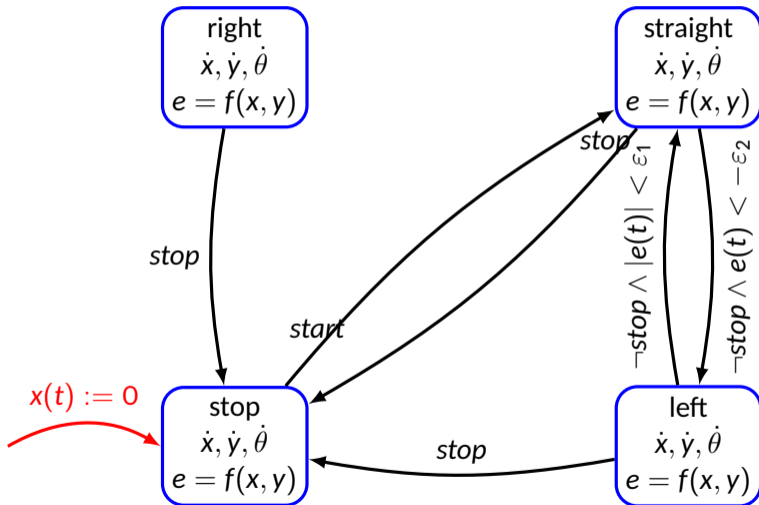
Automatic cruise control



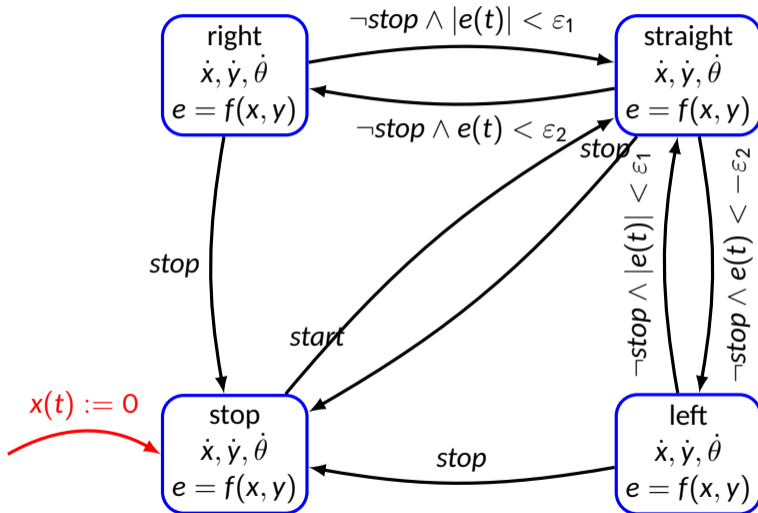
Automatic cruise control



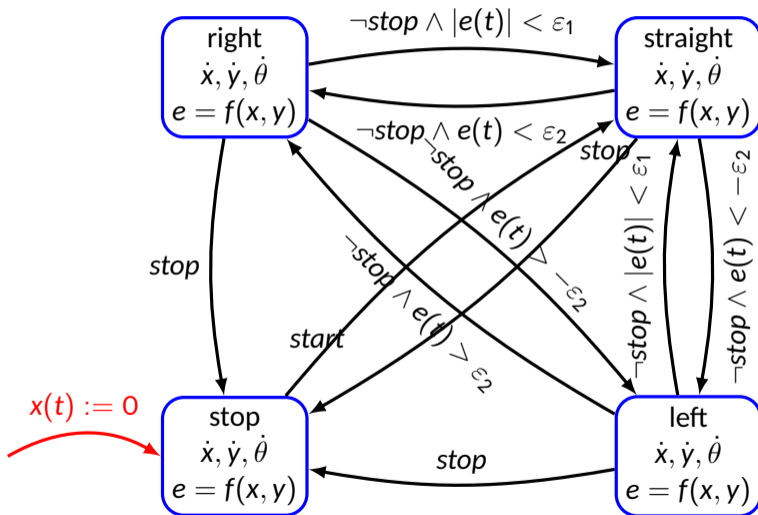
Automatic cruise control



Automatic cruise control



Automatic cruise control



Summary

- Hybrid systems provide a bridge between continuous and discrete dynamics
- Hierarchical description of the system
- Each mode is time-based operation, that is, refinement of state
- State machine to describe mode transition, refinement of state provides continuous dynamics