Embedded Systems

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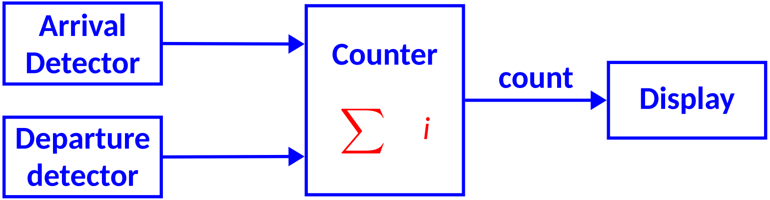
Modeling: Discrete systems

Introduction

- Embedded systems can include both discrete and continuous dynamics
- Continuous dynamics can be modeled by ordinary differential equation
- State machines are used to model discrete behavior of the systems
- A system operates in a sequence of discrete steps
- Example
    - Number of cars in a parking area

Car parking

- Arrival detector, departure detector



- Similar to integrator
- Input is not continuous $u : R \rightarrow \{absent, present\}$
  - Also known as pure signal

Event

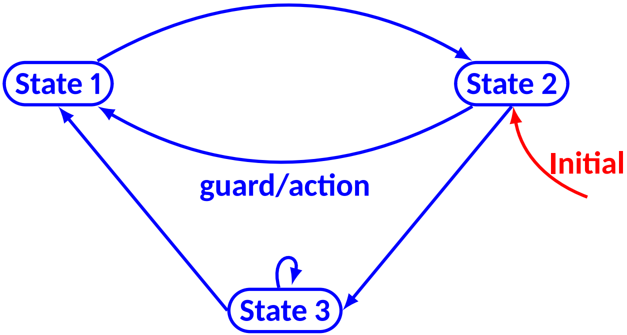
- **Systems are event triggered**
  - **Sequence of steps known as reaction**
- **A particular reaction will observe the values of the inputs at a particular time and calculate output values for the same time**
  - **An actor has input ports** $P = \{p_1, p_2, \ldots, p_N\}$
  - V_p **denotes the type of** p **(values may be received)**
  - **At each reaction a variable can take** $p \in V_p \cup \{absent\}$

Notion of state

- **State of a system is its condition at a particular point of time**
- **State affects how the system reacts to inputs**
- **Integrator : discrete vs continuous**
- **Discrete modes with finite state space are called finite state machine**

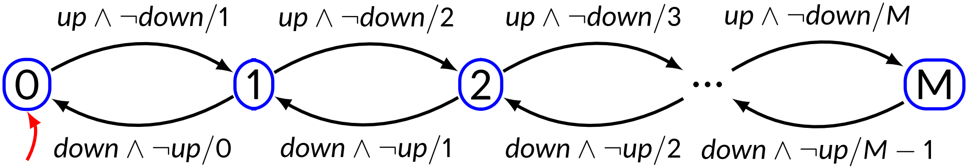
Finite State Machine

- A state machine is a model with discrete dynamics that maps valuations of the inputs to outputs where the map may depend on its current state



Finite State Machine: example

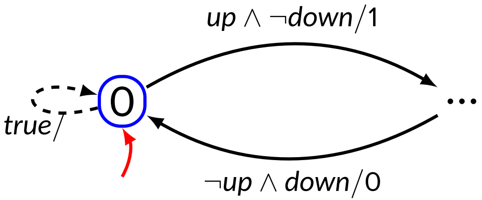
inputs: *up*, *down*: pure
outputs: *count*: $\{0, 1, \ldots, M\}$



Transition

- It governs the discrete dynamics of FSM
- Guard/Action
  - Guard determines whether the transition may take on a reaction
  - Action specifies the output for each reaction
- If p_1 and p_2 are inputs to FSM
  - *true* — transition is always enabled
  - p_1 — transition is enabled if p_1 is present
  - $\neg p_1$ — transition is enabled if p_1 is absent
  - $p_1 \wedge p_2$ — transition is enabled if both p_1 and p_2 are present
  - $p_1 \vee p_2$ — transition is enabled if either p_1 or p_2 are present

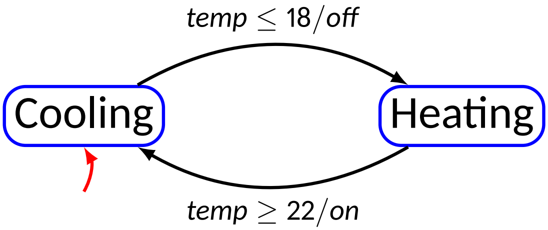
Default transition



Finite State Machine: example

inputs: *temp*: $\mathbb{R}$

outputs: *on*, *off*: pure

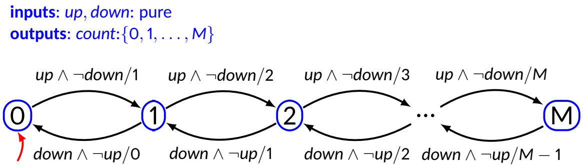


FSM Definition

- **It is a tuple** $\langle \mathit{States}, \mathit{Inputs}, \mathit{Outputs}, \mathit{Update}, \mathit{InitialState} \rangle$
- **States — finite number of states**
- **Inputs — set of input valuations**
- **Outputs — set of output valuations**
- **Update —** $\mathit{States} \times \mathit{Inputs} \rightarrow \mathit{States} \times \mathit{Outputs}$**, mapping a state and input valuation to a next state and a output valuation**
- **InitialState — start state**

FSM example

- **States =** $\{0, 1, 2, \ldots, M\}$
- **Inputs =** $\{up, down\} \to \{present, absent\}$
- **Outputs =** $\{count\} \to \{0, 1, 2, \ldots, M\}$
- **InitialState =** 0
- $$update(s, i) = \begin{cases} (s+1, s+1) \ \textbf{if}\ s < M \wedge up = present \wedge down = absent \\ (s-1, s-1) \ \textbf{if}\ s > 0 \wedge up = absent \wedge down = present \\ (s, absent) \ \textbf{otherwise} \end{cases}$$



A few terminologies

- **Determinacy — If for each state there is at most one transition enabled by each input value**
  - Update function is not one to many mapping
  - Same input will produce same output always
- **Receptiveness — If for each state there is at least one transition possible on each input symbol**
  - FSM is receptive as ’update’ is a function, not a partial function
- **Chattering — A system oscillates between two states rapidly**
- **Stuttering — A system remains in the state due to absence of inputs and outputs**
- **Hysteresis — Dependence of the state of a system on its history.**

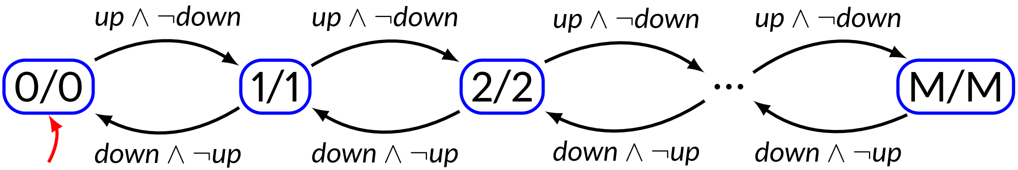
Mealy vs Moore machine

- Mealy machine
  - Named after George Mealy
  - Characterized by producing outputs when a transition is taken
- Moore machine
  - Named after Edward Moore
  - Produces the output when the machine is in a state
  - Output is function of state only
  - Strictly causal

- A Mealy machine can be converted into Moore machine
- A Moore machine can be converted into Mealy machine
- Mealy machine is preferred because of compactness and output is instantaneous with respect to inputs

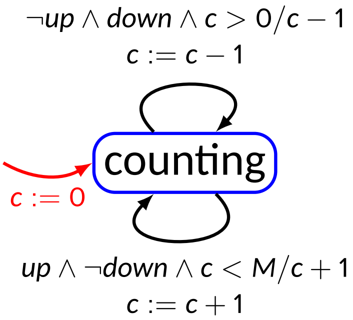
Moore machine: example

inputs: *up*, *down*: pure
outputs: *count*: $\{0, 1, \ldots, M\}$



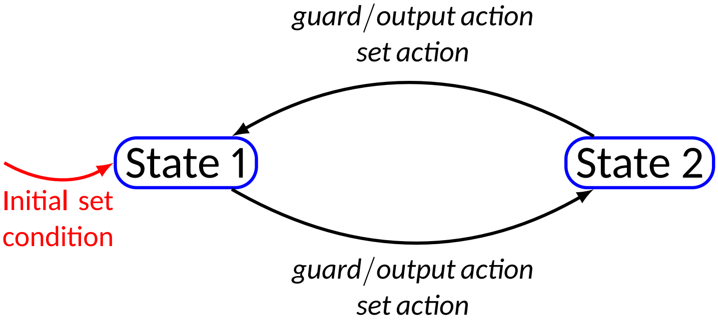
Extended FSM

variable: $c : \{0, 1, \ldots, M\}$
inputs: *up*, *down*: pure
outputs: *count*: $\{0, 1, \ldots, M\}$



Extended FSM

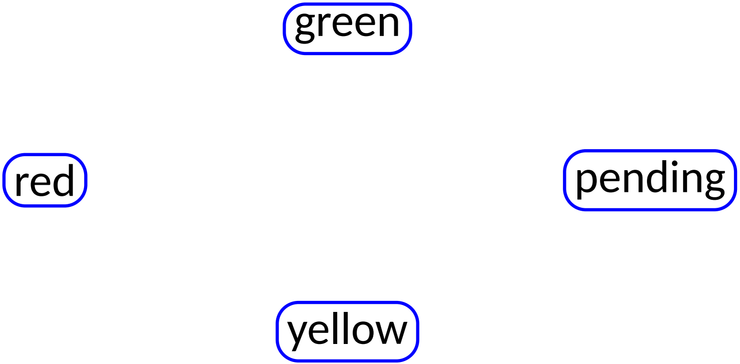
variable declaration
input declaration
output declaration



Example: pedestrian crosswalk

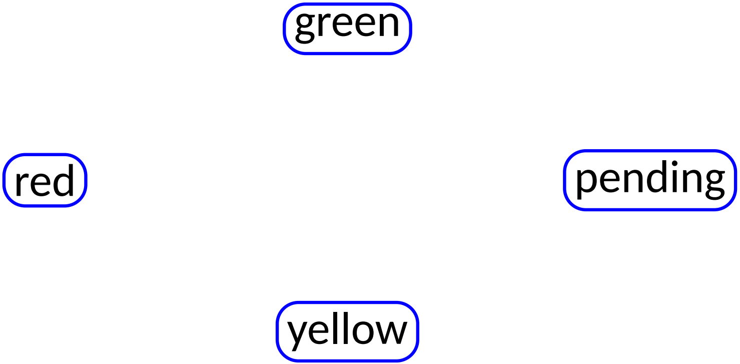
- It starts with red
- It moves to green after 60 seconds
- It will remain in green if there is no pedestrian
- If the light goes to green, then it remains there at least for 60 seconds
- If there is a pedestrian, light becomes yellow if it has been green for more than 60 seconds
- The yellow light will remain for 5 seconds before it turns to red

Example: pedestrian crosswalk



Example: pedestrian crosswalk

variable: $count : \{0, 1, \ldots, 60\}$
input: $pedestrian : pure$
output: $sigY, sigG, sigR : pure$



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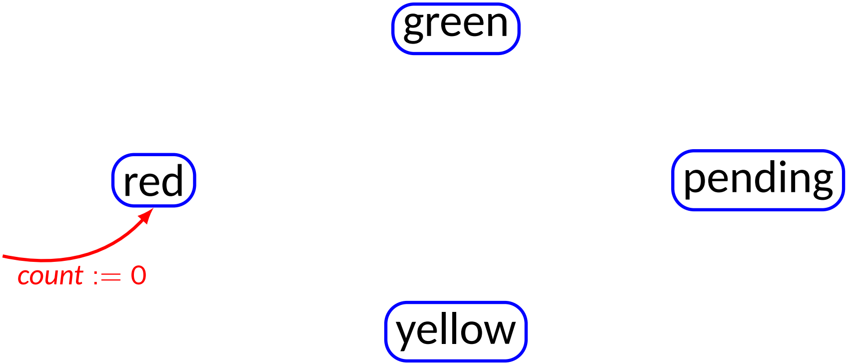

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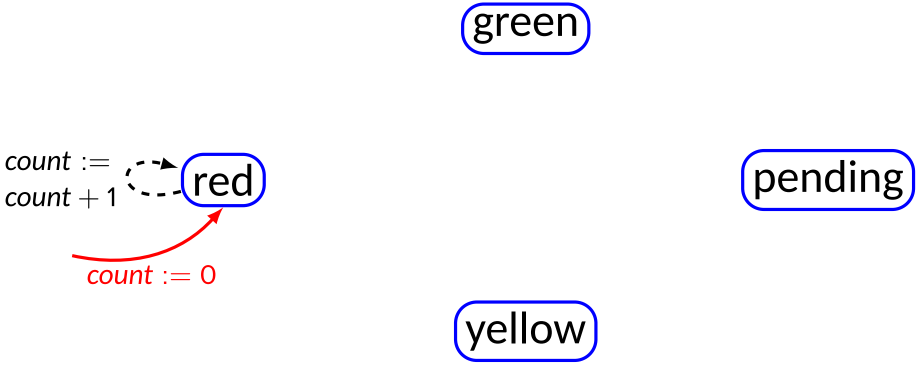

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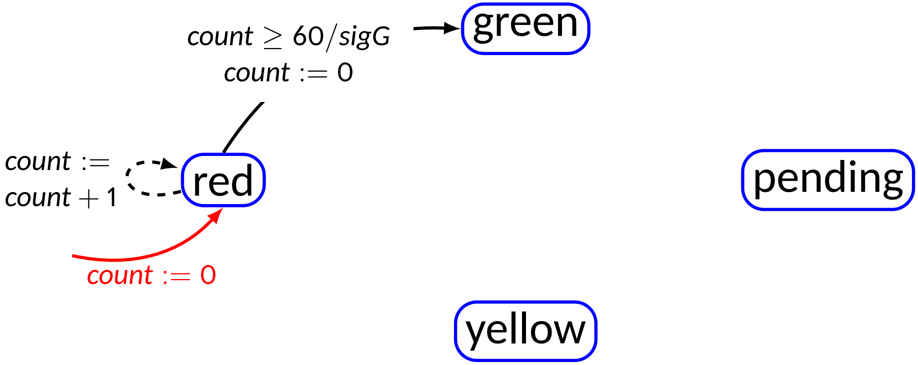

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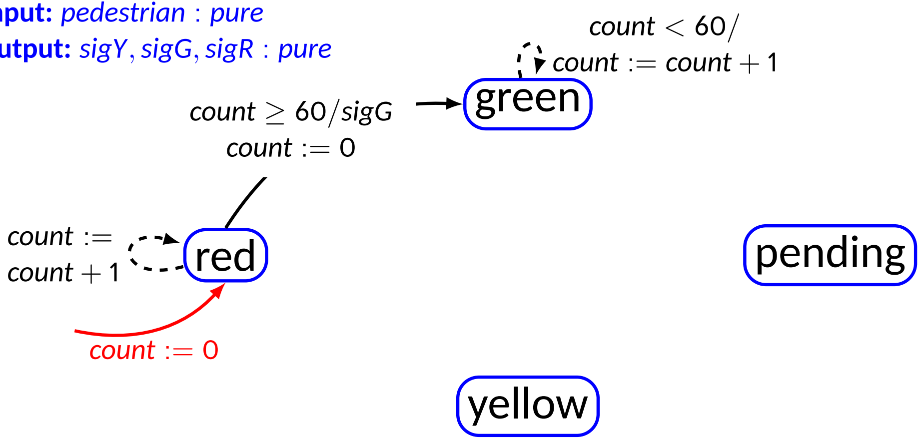

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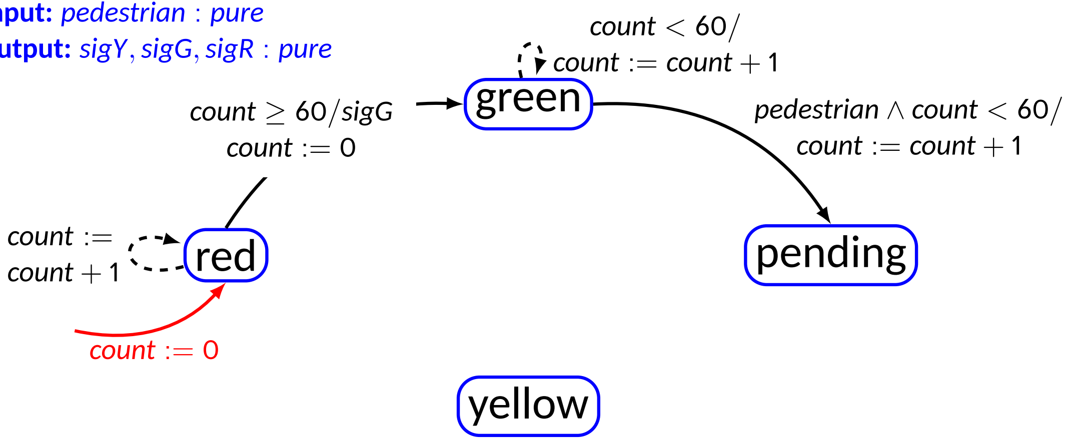

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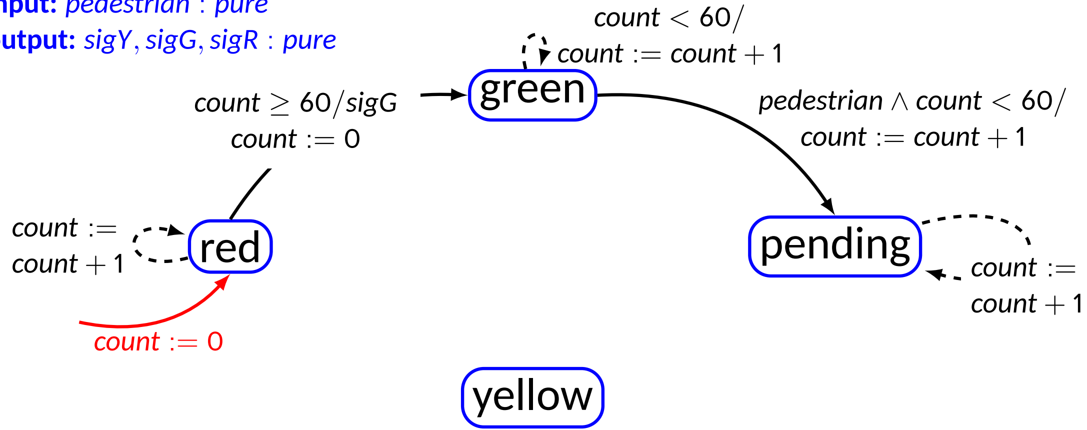

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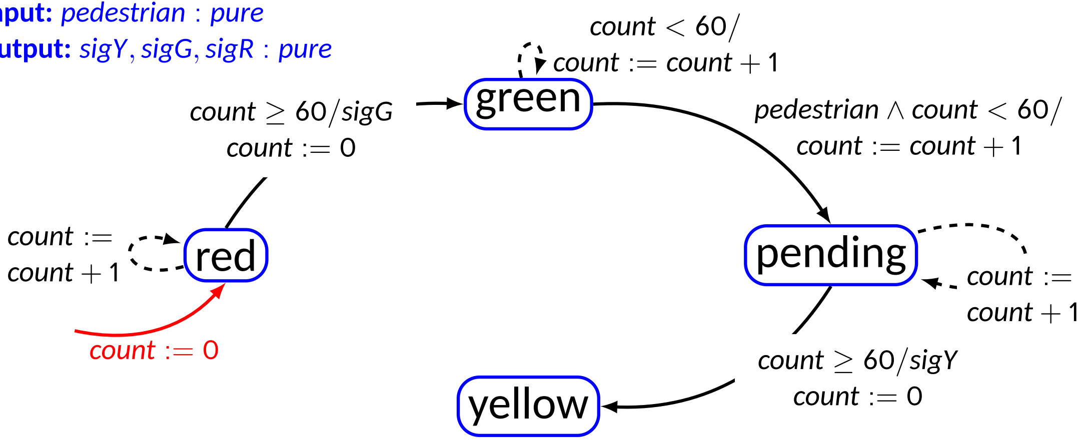

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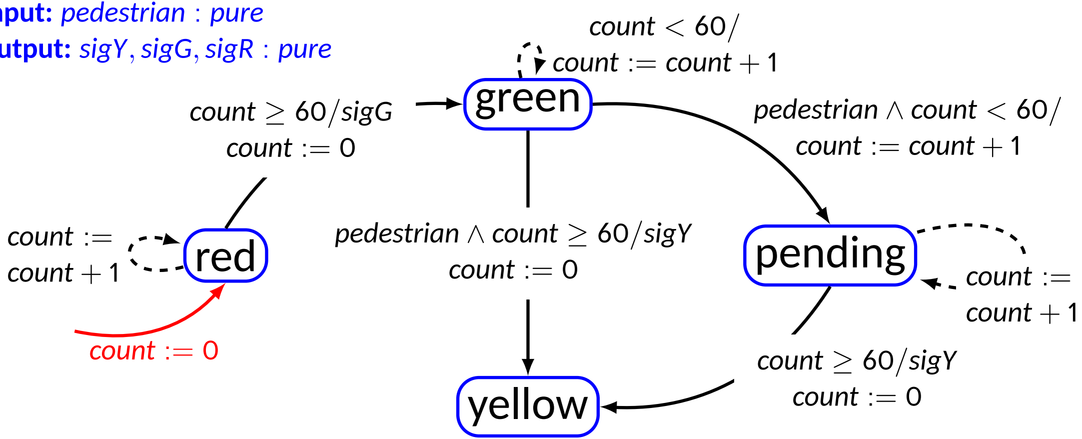
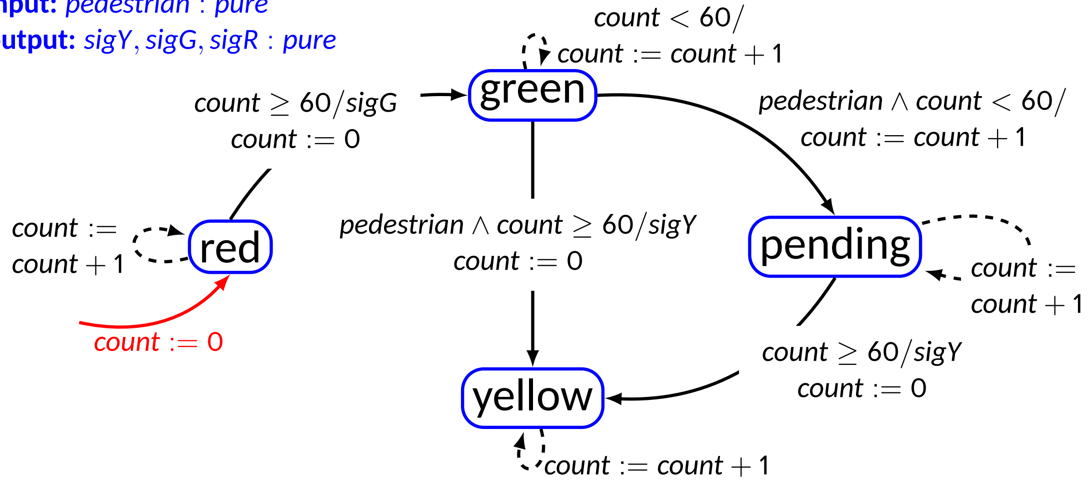


Example: pedestrian crosswalk

variable: *count* : $\{0, 1, \ldots, 60\}$
input: *pedestrian* : *pure*
output: *sigY*, *sigG*, *sigR* : *pure*



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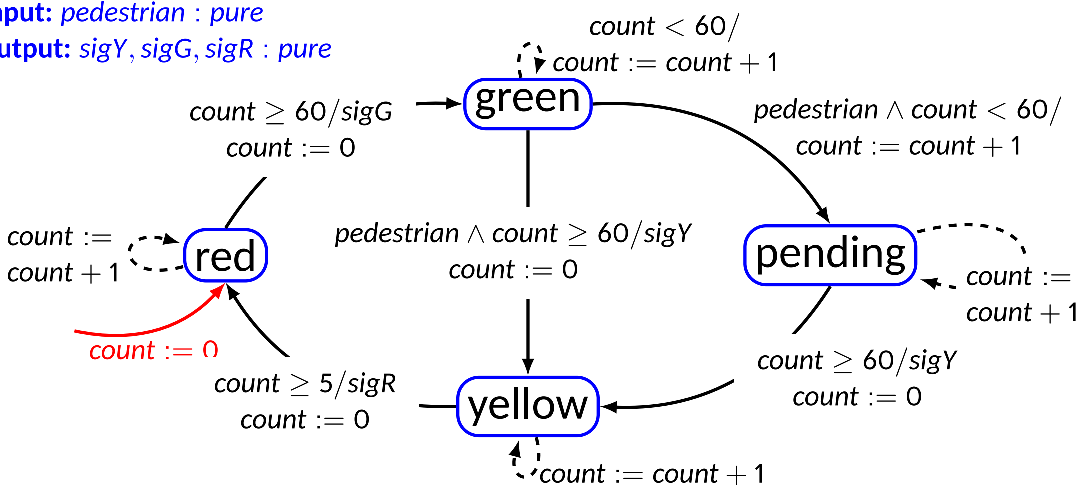
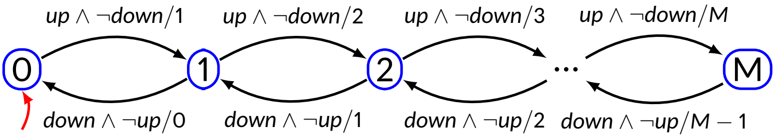


Extended FSM

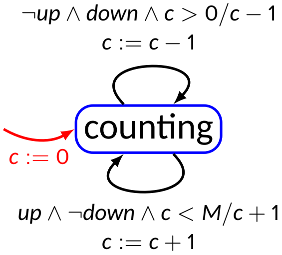
- **The state of an extended machine includes not only the information about which discrete state the machine is in, but also what values any variables have.**
  - **Suppose there is, n discrete states, m variables each of which can take one of p possible values**
  - **Size of the state space will be $|States| = np^m$**

Example

inputs: *up*, *down*: pure
outputs: *count*: $\{0, 1, \ldots, M\}$

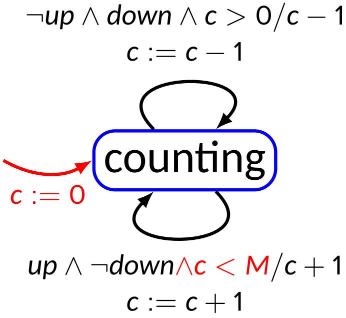


variable: $c : \{0, 1, \ldots, M\}$
inputs: *up*, *down*: pure
outputs: *count*: $\{0, 1, \ldots, M\}$



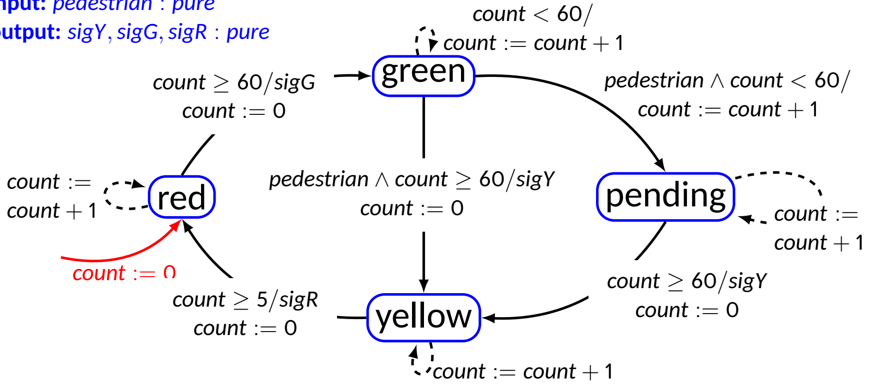
Example: infinite states

variable: $c : \{0, 1, \dots, M\}$
inputs: *up*, *down*: pure
outputs: *count*: $\{0, 1, \dots, M\}$



Pedestrian crosswalk: state count

variable: $count : \{0, 1, \ldots, M\}$
input: $pedestrian : pure$
output: $sigY, sigG, sigR : pure$

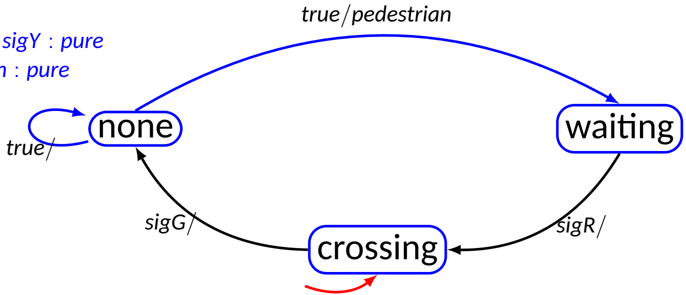


Nondeterminism

- **A state machine interacts with the environment**
- **Modeling of pedestrian**

- **If for any state, two distinct transitions with guards that can evaluate to true in the same reaction, then the machine is nondeterministic**

inputs: *sigR*, *sigG*, *sigY* : *pure*
output: *pedestrian* : *pure*

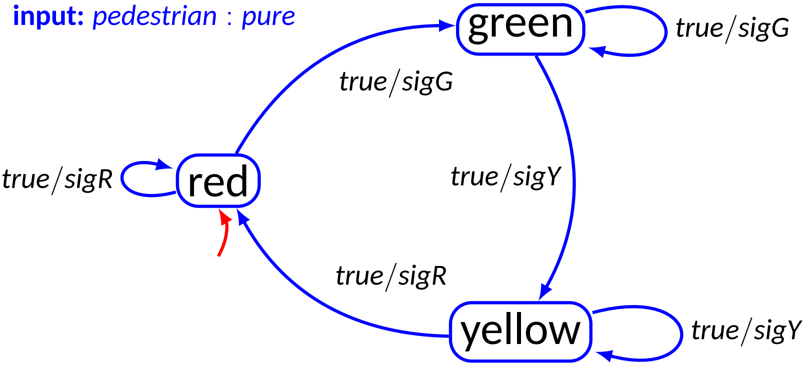


Nondeterministic FSM

- **It is a tuple** $\langle States, Inputs, Outputs, possibleUpdates, InitialStates\rangle$
- **States — finite number of states**
- **Inputs — set of input valuations**
- **Outputs — set of output valuations**
- **possibleUpdates —** $States \times Inputs \to 2^{States \times Outputs}$**, mapping a state and input valuation to a next state and a set of possible (next state, output) pairs. Also known as Transition Relation**
- **InitialStates — start states**

Nondeterministic FSM

output: *sigR*, *sigG*, *sigY* : *pure*
input: *pedestrian* : *pure*



Uses of nondeterminism

- **Environment modeling — to hide irrelevant details**
- **Specifications — system requirements imposes constraints on some features while the others are unconstrained**

- **Probabilistic FSM is different from Non-deterministic FSM**
  - **In probabilistic FSM, every transition is associated with some probability**

Behavior & Traces

- **Behavior of state machine is an assignment of such signals to each port such that the signal on any output port is the output sequence produced by the input signals**
- **Example: garage counter**

$$s_{up} = \{absent, absent, present, absent, present, present, \ldots\}$$
$$s_{down} = \{absent, absent, absent, present, absent, absent, \ldots\}$$
$$s_{count} = \{absent, absent, 1, 0, 1, 2, \ldots\}$$

- $s_{up}, s_{down}, s_{count}$ **together form the behavior**
- **For deterministic FSM if input sequence is known the output sequence can be determined**
- **Set of all behaviors of a state machine** M **is called its language** $L(M)$

Behavior & Traces (contd.)

- A behavior may be more conveniently represented as a sequence of valuations called observable trace
  - If x_i is input and Y_i is output then following is an observable sequence $((x_0, y_0), (x_1, y_1), \ldots)$

- An execution trace may be defined as

$$((x_0, s_0, y_0), (x_1, s_1, y_1), \ldots)$$

$$s_0 \xrightarrow{x_0/y_0} s_1 \xrightarrow{x_1/y_1} s_2 \ldots$$

Computation trees

- **For nondeterministic machine, it may be useful to represent all possible traces**