

# Discrete Mathematics

## Propositional Logic: Introduction



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# First few examples

- If I am the President then I am well-known. I am the President. So I am well-known.

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  - $F_1: a \rightarrow b$



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  - $F_1: a \rightarrow b$
  - $F_2: a$
  - $G: b$
- **The final formula for deduction**  $(F_1 \wedge F_2) \rightarrow G$  that is  $((a \rightarrow b) \wedge a) \rightarrow b$

# Deduction using Propositional Logic: Example 1

- If I am the President then I am well-known. I am the President. So I am well-known.
- Coding: Variables
  - $a$ : I am the President,  $b$ : I am well-known
- Coding sentences
  - $F_1: a \rightarrow b$ ,  $F_2: a$ ,  $G: b$
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$a$	$b$	$a \rightarrow b$	$(a \rightarrow b) \wedge a$	$((a \rightarrow b) \wedge a) \rightarrow b$
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  - $F_1: a \rightarrow b$ ,  $F_2: a$ ,  $G: b$
- The final formula for deduction  $(F_1 \wedge F_2) \rightarrow G$  that is  $((a \rightarrow b) \wedge a) \rightarrow b$

$a$	$b$	$a \rightarrow b$	$(a \rightarrow b) \wedge a$	$((a \rightarrow b) \wedge a) \rightarrow b$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

## Deduction using Propositional Logic: Example 2

- If I am the President then I am well-known. I am **not** the President. So I am **not** well-known.

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# Deduction using Propositional Logic: Example 2

- If I am the President then I am well-known. I am **not** the President. So I am **not** well-known.
- Coding: Variables
  - $a$ : I am the President,  $b$ : I am well-known
- Coding sentences

# Deduction using Propositional Logic: Example 2

- If I am the President then I am well-known. I am **not** the President. So I am **not** well-known.
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  - $a$ : I am the President,  $b$ : I am well-known
- Coding sentences
  - $F_1: a \rightarrow b$ ,

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- Coding: Variables
  - $a$ : I am the President,  $b$ : I am well-known
- Coding sentences
  - $F_1: a \rightarrow b$ ,  $F_2: \sim a$ ,

# Deduction using Propositional Logic: Example 2

- If I am the President then I am well-known. I am **not** the President. So I am **not** well-known.
- Coding: Variables
  - $a$ : I am the President,  $b$ : I am well-known
- Coding sentences
  - $F_1: a \rightarrow b$ ,  $F_2: \sim a$ ,  $G: \sim b$

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- Coding sentences
  - $F_1: a \rightarrow b$ ,  $F_2: \sim a$ ,  $G: \sim b$
- The final formula for deduction  $(F_1 \wedge F_2) \rightarrow G$  that is  $((a \rightarrow b) \wedge \sim a) \rightarrow \sim b$

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  - $a$ : I am the President,  $b$ : I am well-known
- Coding sentences
  - $F_1: a \rightarrow b$ ,  $F_2: \sim a$ ,  $G: \sim b$
- The final formula for deduction  $(F_1 \wedge F_2) \rightarrow G$  that is  $((a \rightarrow b) \wedge \sim a) \rightarrow \sim b$

$a$	$b$	$a \rightarrow b$	$(a \rightarrow b) \wedge \sim a$	$((a \rightarrow b) \wedge \sim a) \rightarrow \sim b$
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# Deduction using Propositional Logic: Example 2

- If I am the President then I am well-known. I am **not** the President. So I am **not** well-known.
- Coding: Variables
  - $a$ : I am the President,  $b$ : I am well-known
- Coding sentences
  - $F_1: a \rightarrow b$ ,  $F_2: \sim a$ ,  $G: \sim b$
- The final formula for deduction  $(F_1 \wedge F_2) \rightarrow G$  that is  $((a \rightarrow b) \wedge \sim a) \rightarrow \sim b$

$a$	$b$	$a \rightarrow b$	$(a \rightarrow b) \wedge \sim a$	$((a \rightarrow b) \wedge \sim a) \rightarrow \sim b$
T	T	T	F	T
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T



# Deduction using Propositional Logic: Example 3

- If I am the President then I am well-known. I am the President. So I am well-known.
- Coding: Variables
  - $a$ : I am the President,
  - $b$ : I am well-known
- Coding sentences
  - $F_1: a \rightarrow b$ ,  $F_2: a$ ,  $G: b$
- The final formula for deduction  $(F_1 \wedge F_2) \rightarrow G$  that is  $((a \rightarrow b) \wedge a) \rightarrow b$
- If Rajat is the President then Rajat is well-known. Rajat is the President. So Rajat is well-known.

$a$	$b$	$a \rightarrow b$	$(a \rightarrow b) \wedge a$	$((a \rightarrow b) \wedge a) \rightarrow b$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

# Deduction using Propositional Logic: Example 3

- If I am the President then I am well-known. I am the President. So I am well-known.

- Coding: Variables

- $a$ : I am the President,
- $b$ : I am well-known

- Coding sentences

- $F_1: a \rightarrow b$ ,  $F_2: a$ ,  $G: b$

- The final formula for deduction  $(F_1 \wedge F_2) \rightarrow G$  that is  $((a \rightarrow b) \wedge a) \rightarrow b$

$a$	$b$	$a \rightarrow b$	$(a \rightarrow b) \wedge a$	$((a \rightarrow b) \wedge a) \rightarrow b$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

- If Rajat is the President then Rajat is well-known. Rajat is the President. So Rajat is well-known.

- Coding: Variables

- $a$ : Rajat is the President,
- $b$ : Rajat is well-known

- Coding sentences

- $F_1: a \rightarrow b$ ,  $F_2: a$ ,  $G: b$

- The final formula for deduction  $(F_1 \wedge F_2) \rightarrow G$  that is  $((a \rightarrow b) \wedge a) \rightarrow b$

# Deduction using Propositional Logic: Example 4,5

- If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore Asha is not elected VP.

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- If Asha is elected VP then Rajat is chosen as G-Sec and Bharati is chosen as Treasurer. Rajat is chosen as G-Sec. Therefore Asha is elected VP.

## More examples

- If Asha is elected VP then Rajat is chosen as G-Sec or Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore if Asha is elected VP then Bharati is chosen as Treasurer.

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- If Asha is elected VP then Rajat is chosen as G-Sec or Bharati is chosen as Treasurer. Rajat is not chosen as G-Sec. Therefore if Asha is elected VP then Bharati is chosen as Treasurer.
- If Asha is elected VP then either Rajat is chosen as G-Sec or Bharati is chosen as Treasurer but not both. Rajat is not chosen as G-Sec. Therefore if Asha is elected VP then Bharati is chosen as Treasurer.

*Thank you!*